




Developing an Integrated Optimization Inspection Scheme with A Flexible Sampling Mechanism for Quality Determination Based on the Process Loss Index

Armin Darmawan* , Afirah Armayfa , Meti Sesa 

Universitas Hasanuddin (Indonesia)

*Corresponding author: darmawanarmin@gmail.com
afiraharmayfa@gmail.com, metisesa13796@gmail.com

Received: August 2025

Accepted: January 2026

Abstract:

Purpose: This study examines the quick switching sampling (QSS) system. This well-established sampling scheme incorporates two single sampling plans (SSPs) with adaptive transition rules between normal and tightened inspections. The QSS system dynamically adjusts inspection stringency in response to fluctuations in product quality, implementing normal inspection when quality meets satisfactory standards, and tightened inspection when quality deterioration is detected.

Design/methodology/approach: Traditional acceptance sampling plans often evaluate product quality based on process yield, which overlooks subtle variations within specification limits. To address this limitation, a novel performance metric, the process loss index L_{ϵ} , was developed to quantify quality loss. This index is calculated as the ratio of expected quadratic loss to the square of half the specification width. Utilizing this index, two models of QSS sampling schemes were constructed by solving nonlinear optimization mathematical models and evaluated using general metrics. The efficacy and characteristics of these schemes were investigated, compared, and discussed.

Findings: The results highlight the potential of QSS systems to enhance the effectiveness of quality control while maintaining stringent quality standards. Besides, the proposed plan demonstrates superiority over the conventional plan in terms of adaptability, particularly with sample size adjustments, when switching to a stricter inspection plan in response to deteriorating lot quality and improved efficiency.

Originality/value: This study presents a novel approach to quality control by integrating the process loss index into the QSS system, offering a fresh perspective on sampling methodologies. The integration of QSS with the process loss index L_{ϵ} marks a significant contribution to the field of quality control, enabling more nuanced evaluations of product quality and providing a groundbreaking framework for optimizing quality control processes while minimizing sample sizes, thereby enhancing efficiency and effectiveness.

Keywords: quality control and assurance, normal inspection, process loss index, quick switching rules, tightened inspection, optimization

To cite this article:

Darmawan, A., Armayfa, A., & Sesa, M. (2026). Developing an integrated optimization inspection scheme with a flexible sampling mechanism for quality determination based on the process loss index. *Journal of Industrial Engineering and Management*, 19(1), 120-134. <https://doi.org/10.3926/jiem.9061>

1. Introduction

The advent of rapid technological advancements has necessitated that industries prioritize quality control and performance optimization to maintain competitiveness. Empirical evidence suggests that consistency in production and adherence to stringent quality standards are crucial determinants of customer satisfaction and market success. Acceptance sampling plans, as a statistical quality control tool, play a vital role in ensuring product reliability and facilitating data-driven decision-making. By leveraging these plans, organizations can identify areas for improvement, reduce variability, and enhance the overall quality of their products. Furthermore, acceptance sampling plans provide valuable insights into quality history and process control, enabling organizations to refine their quality control protocols and drive innovation. Ultimately, the effective implementation of acceptance sampling plans can lead to enhanced customer satisfaction, reduced costs, and improved competitiveness in the market (Wu, Darmawan & Liu, 2025; Wu & Darmawan, 2025).

The primary objective of a sampling plan is to provide decision-makers with a structured framework for determining the disposition of product lots based on predefined quality and risk criteria. The efficacy of a sampling plan can be assessed utilizing the operational characteristic (OC) curve and average sample number (ASN). The OC curve illustrates the probability of acceptance across varying quality levels, thereby demonstrating a sampling plan's discriminatory capability. A steeper OC curve indicates superior discriminatory power, enabling more accurate differentiation between acceptable and unacceptable product lots. Furthermore, a sampling system can comprise multiple plans with switching rules, allowing for the tracking of inspection history and optimization of sampling plan effectiveness. These switching rules facilitate adaptive inspection protocols, where normal inspection plans are employed for products exhibiting excellent quality, while tightened inspection plans are implemented in response to significant declines in product quality. By integrating multiple sampling plans with switching rules, organizations can enhance the precision and efficiency of their quality control processes, ultimately ensuring the delivery of high-quality products that meet stringent quality standards. This approach enables organizations to respond dynamically to changes in product quality, thereby minimizing the risk of defective products.

The normal-tightened-normal sampling scheme, commonly referred to as quick switching systems (QSS), is a type of sampling system that dynamically adjusts inspection stringency in response to fluctuations in product quality. The genesis of this quick sampling scheme can be attributed to MIL-STD-105D, with seminal contributions from Dodge (1967). Hald and Thyregod (1965) proposed normal and tightened sampling inspection by attributes. Romboski (1966) conducted an in-depth analysis of the system's properties utilizing two distinct process models ($QSS(n, \kappa_N, \kappa_T)$) and subsequently proposed key recommendations for effective implementation. Soundararajan and Arumainayagam (1990) further expanded on this concept by developing some enhanced modifications of QSS that incorporated master tables. Subsequent research by Govindaraju and Ganesalingam (1998) introduced a two-plan sampling system with a zero acceptance number for inspection, which required a smaller sample size while maintaining discrimination power.

Soundararajan and Palanivel (2000) developed a variable quick switching sampling technique (VQSS) for quality characteristics evaluated by shifted inspection. Known as the QSVSS system, this scheme has been developed for quality characteristics with double specification limits and normal distributed data. This technique use same sample size n for both normal and tightened inspections, with distinct critical values (κ_T and κ_N). Balamurali and Usha (2012) formulated VQSS by using dual specification limits. Balamurali and Usha (2014) established and enlarged a comparable VQSS scheme by including capability indicators into the model, hence augmenting its efficacy and application across an additional spectrum of industrial contexts.

As research on sampling plans incorporating capability indices continued to evolve, the VQSS scheme was further developed to integrate various capability indices. Notably, Liu and Wu (2016) incorporated the S_{pk} index into VQSS, while Wu, Lee, Liu and Shih (2017) proposed two VQSS variants ($(n; \kappa_N, \kappa_T)$ and $(n_N, n_T; \kappa)$) utilizing the C_{pk} index. Balamurali and Usha (2017a) developed $VQSS(n; \kappa_N, \kappa_T)$ with consideration of process loss functions. In addition, Balamurali and Usha (2017b) have contributed to this area by integrating the C_{pmk} index into $VQSS(n; \kappa_N, \kappa_T)$, demonstrating the ongoing evolution of this methodology. These pioneering studies have laid the groundwork for the development of more sophisticated QSS schemes, enabling organizations to optimize their quality control

processes and respond adaptively to changes in product quality. By leveraging these advanced sampling schemes, organizations can enhance the efficacy of their quality control protocols.

Significant research has been conducted in recent years to advance VQSS from many viewpoints, including contributions from Wang, Wu and Jhu (2021), Wang (2022), Wang and Shu (2023), Liu, Wang and Wang (2023), Wu, Shu, Wang and Chen (2024), Wang, Wu and Wang (2025), and Wang and Wu (2025). Mostly, VQSS often rely on process yield and high yield to evaluate product quality, but this approach does not account for variations within specification limits. The process loss index (L_e) offers a more nuanced assessment of process performance by considering the quality loss function. Balamurali and Usha (2017a) have developed the VQSS model for a single sample size and two critical values (n ; k_N , k_T), considering process loss functions. Therefore, this study aims to create two primary types of VQSS for double sample size and single critical value ($(n_N, n_T; k)$ and $(n_N, mn_N; k)$) that utilize the L_e index, and to further explore, analyze, and contrast their behavior and effectiveness.

The subsequent sections of this work are organized as follows. Section 2 delineates the process loss index and examines the statistical characteristics of its estimator. Section 3 delineates the operating mechanism of the proposed scheme, encompassing the OC and ASN functions, along with a mathematical model for ascertaining plan parameters. Section 4 presents a comprehensive analysis and discussion of plan parameters under various scenarios, followed by a comparative examination of the devised sample methods and a case study that demonstrates the applicability and viability of the proposed sampling strategy. Ultimately, Section 5 culminates this investigation by encapsulating the principal results and conclusions.

2. Process Loss Index (L_e)

The process performance evaluation often relies on the yield index as a primary measure, which indicates the percentage of products that meet specifications. However, this index has a limitation in distinguishing between products that fall within different specification limits. To address this limitation, a quadratic loss function is frequently employed to identify products within the limits, penalizing deviations from the target value. The concept of applying loss functions to quality improvement was first introduced by Hsiang and Taguchi (1985), focusing on reducing variation around the target value. Nevertheless, relying solely on a quadratic loss function fails to compare performance with specification limits due to its dependence on the unit of the quality characteristic.

Johnson (1992) subsequently introduced the notion of relative expected squared error loss (L_e) for scenarios with symmetric tolerance, evaluating process capability in terms of quality loss. Tsui (1997) further refined the process loss index L_e by introducing two sub-indices, L_{pe} and L_{ot} , which enable the assessment of potential relative expected loss and relative off-target squared deviation, respectively. The mathematical representation of this relationship is given by:

$$L_e = \int_{-\infty}^{\infty} \left[\frac{(x-T)^2}{d^2} \right] f(x) dx = \frac{\sigma^2 + (\mu - T)^2}{d^2} \quad (1)$$

$$= \frac{\sigma^2}{d^2} + \frac{(\mu - T)^2}{d^2} = L_{pe} + L_{ot},$$

The probability density function of the measured characteristic X , denoted by $f(x)$, is a critical component in evaluating process capability, where the process mean (μ) and standard deviation (σ) play pivotal roles. The target value of the quality characteristic is represented by T , and the half-length of the specification interval, defined by the upper and lower specification limits (USL and LSL), is denoted by $d = (USL - LSL)/2$. This formulation enables the derivation of mathematical relationships, specifically $L_e = (3C_{pm})^{-2}$, $L_{pe} = (3C_p)^{-2}$ and $L_{ot} = (1 - C_a)^2$, can be formulated, where $C_p = d/(3\sigma)$, $C_{pm} = d/\{3[\sigma^2 + (\mu - T)^2]^{1/2}\}$, and $C_a = 1 - |\mu - T|/d$. These are rooted in three fundamental capability indices previously established by Kane (1986), Chan, Cheng and Spiring (1988), and Pearn, Kotz and Johnson (1992), respectively. Furthermore, the L_e index has been extensively built upon in recent decades, with notable contributions from researchers such as Pearn, Chang and Wu (2004), Pearn, Chang and Wu (2006), Yen and Chang (2009), Aslam, Yen and Jun (2011), Wu and Shu (2011), Aslam, Yen, Chang, Jun, Ahmad and Rasool (2012), Aslam, Yen, Chang and Jun (2013), Aslam, Yen, Chang and Jun (2014), Balamurali and Usha

(2017b), Erfanian and Gildeh (2021), Darmawan, Wu, Wang and Chiang (2025), Darmawan and Wu (2025), and Darmawan, Bahri, Amar, Do-Bagus and Tahir (2025).

Pearn et al. (2006) established a framework for evaluating process performance based on the L_e index, categorizing it into five distinct levels. According to this framework, a process with a L_e value of 0.11 or higher is deemed “inadequate,” indicating a need for adjustment in the process mean or reduction in process variation to achieve improvement. Processes with L_e values between 0.06 and 0.11 are considered “capable,” suggesting that quality managers should implement stringent quality control measures. L_e value between 0.05 and 0.06 is classified as “satisfactory,” while values ranging from 0.03 to 0.05 indicate an “excellent” quality condition, where no immediate quality improvement is necessary. The highest level of process performance is achieved when the L_e value is 0.03 or less, categorized as “super,” signifying exceptional process capability.

The L_e index is typically unknown in practice due to the presence of two commonly unknown parameters, the process mean (μ) and standard deviation (σ). To overcome this challenge, Pearn et al. (2004) proposed a statistical estimator for the L_e index, enabling practitioners to estimate process capability with greater accuracy.

$$\hat{L}_e = \frac{1}{n} \sum_{i=1}^n \frac{(X_i - T)^2}{d^2} = \frac{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2}{d^2} + \frac{(\bar{X} - T)^2}{d^2} = \frac{S_n^2}{d^2} + \frac{(\bar{X} - T)^2}{d^2}, \quad (2)$$

With $\bar{X} = \sum_{i=1}^n X_i / n$ and $S_n^2 = \sum_{i=1}^n (X_i - \bar{X})^2 / n$ being the maximum likelihood estimates of μ and σ^2 , respectively, which were calculated based on the data observation.

Then,

$$\frac{\hat{L}_e}{L_e} = \frac{\sum_{i=1}^n (X_i - T)^2}{n[\sigma^2 + (\mu - T)^2]} = \frac{\frac{\sum_{i=1}^n (X_i - T)^2}{\sigma^2}}{n\left[1 + \frac{(\mu - T)^2}{\sigma^2}\right]} = \frac{\frac{\sum_{i=1}^n (X_i - T)^2}{\sigma^2}}{n + \delta}. \quad (3)$$

Under normality situation, \hat{L}_e has a non-central chi-squared distribution with n degrees of freedom and non-centrality parameter $\delta = n\xi^2 = n(\mu - T)^2 / \sigma^2$, i.e., $\hat{L}_e \sim L_{e,\chi_{n,\delta}^2} / (n + \delta)$ (Pearn et al., 2006). Notably, $\delta = 0$ implies the process mean is at the target value. Therefore, the \hat{L}_e 's cumulative distribution function (CDF) can then be represented as

$$F_{\hat{L}_e}(y) = P(\hat{L}_e \leq y) = P\left(\chi_{n,\delta}^2 \leq \frac{(n + \delta)y}{L_e}\right). \quad (4)$$

3. Proposed Model of VQSS Based on the Process Loss Index (L_e)

In this study, the VQSS framework comprises two single-sampling plans (VSSPs) — one for normal inspection and one for tightened inspection — with predefined switching rules between them. Two principal variants are considered: VQSS: VQSS(n_N , n_T ; k) and VQSS(n_N , $n_T = mn_N$; k). The first variant utilizes distinct sample sizes (n_N and n_T) for normal and tightened inspections, respectively, while maintaining a common critical threshold k . By contrast, the second variant employs a single base sample size for normal inspection and a tightened inspection sample size by a factor m .

3.1. VQSS(n_N , n_T ; k)

Under the assumption that the quality attribute follows a normal distribution with two-sided specification limits, Figure 1 presents the flow chart for implementing VQSS(n_N , n_T ; k) based on the estimated Process Loss Index (L_e). The following steps summarize the operational procedure depicted in the flow chart (Figure 1).

Specify producer's risk (α) and consumer's risk (β). Define the acceptable quality level (l_{AQL}) and the rejectable quality level (l_{RQL}). Determine the critical value k based on the sampling distribution of the estimated Process Loss Index and the chosen risks (α and β) and the sample size (n_N and n_T).

1. Normal Inspection:

- Draw a sample size of n_N items from the lot and employing a critical value of k for lot sentencing.
- Measure each unit and compute the sample statistics (\bar{X} and S_n^2) needed to estimate the Process Loss Index (\hat{L}_e).
- Compare the estimate to the critical threshold k : If $\hat{L}_e \leq k$, accept the lot. If $\hat{L}_e > k$, reject the lot, and switch to Tightened Inspection for the next lot.

2. Tightened Inspection:

- Draw a sample size of n_T items from the lot and employing a critical value of k for lot sentencing.
- Recalculate the estimated Process Loss Index (\hat{L}_e) using the new measured data.
- If $\hat{L}_e \leq k$, accept the lot and back to Normal Inspection for the next lot. If $\hat{L}_e > k$, reject the lot and proceed with Tightened Inspection for the next lot or halt production if quality deterioration persists.

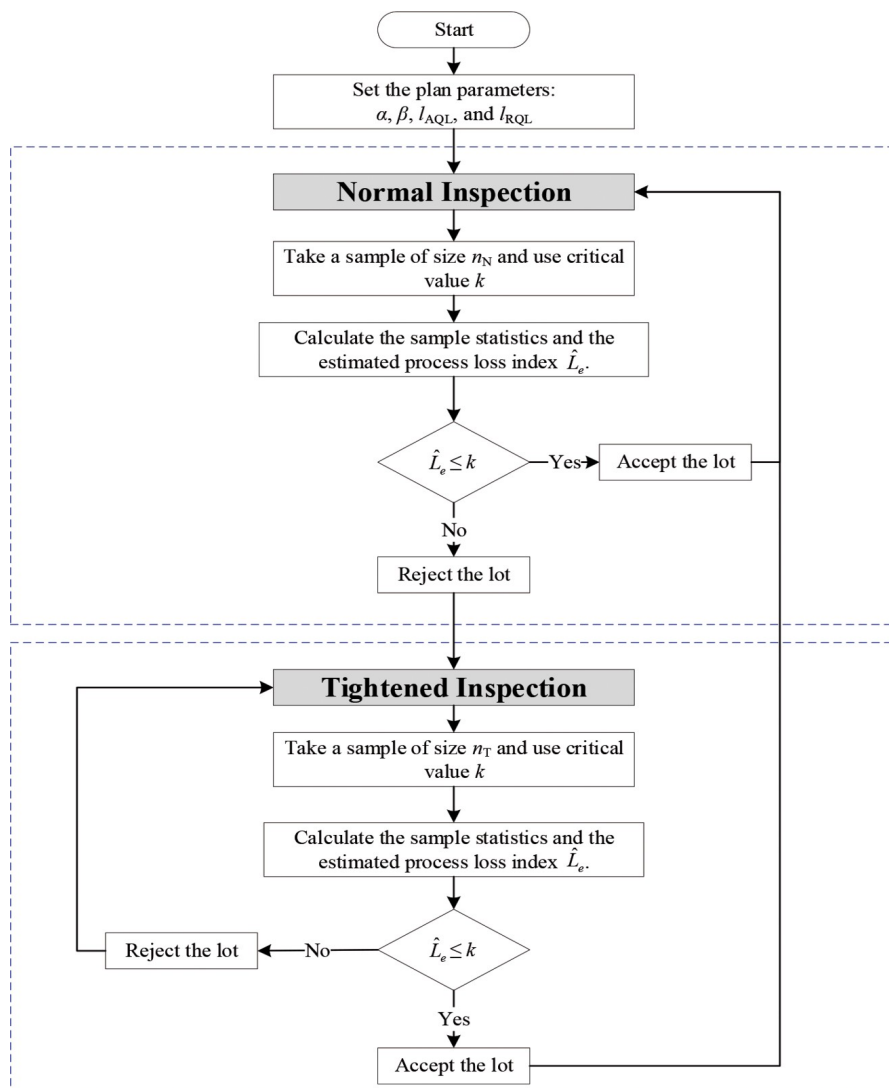


Figure 1. Flow chart of VQSS(n_N , n_T ; k).

Applying Romboski's (1966) AQSS framework, the lot acceptance probability $P^I(L_e)$ -or the OC function for the VQSS($n_N, n_T; k$) plan based on the process loss index L_e is defined as:

$$P_T^I(L_e) = P\left(\chi_{n_T, \delta}^2 \leq \frac{(n_T + \delta) \times k}{L_e}\right), \quad (5)$$

$$P_N^I(L_e) = P\left(\chi_{n_N, \delta}^2 \leq \frac{(n_N + \delta) \times k}{L_e}\right). \quad (6)$$

To satisfy both producer's and consumer's risk requirements, the OC function must attain the values $(l_{AQL}, 1 - \alpha)$ and (l_{RQL}, β) . Imposing these two-point condition on the above expression (Equations (5) and (6)) yields:

$$\pi_A^I(l_{AQL}) = \frac{P_T^I(l_{AQL})}{1 - P_N^I(l_{AQL}) + P_T^I(l_{AQL})}, \quad (7)$$

$$\pi_A^I(l_{RQL}) = \frac{P_T^I(l_{RQL})}{1 - P_N^I(l_{RQL}) + P_T^I(l_{RQL})}, \quad (8)$$

Because VQSS employs two inspection modes with distinct sample sizes, the Average Sample Number (ASN)—the expected count of inspected units before a decision—is a more appropriate performance metric. The ASN is given by:

$$ASN(L_e) = \frac{P_T^I(L_e) \times n_N + [1 - P_N^I(L_e)] \times n_T}{1 - P_N^I(L_e) + P_T^I(L_e)}. \quad (9)$$

Determining the plan parameters (n_N, n_T, k) requires solving the two-point OC equations simultaneously. Since multiple parameter sets may satisfy these constraints, the ASN serves as the objective function to be minimized. Hence, an optimization model is formulated to identify the combination of n_N, n_T , and k that minimizes ASN while adhering to the specified producers' and consumers' risk levels.

$$\text{Min}_{n_T, n_N, k} \text{ASN}(l_{AQL}) = \frac{P_T^I(l_{AQL}) \times n_N + [1 - P_N^I(l_{AQL})] \times n_T}{1 - P_N^I(l_{AQL}) + P_T^I(l_{AQL})}$$

Subject to

$$\pi_A^I(l_{AQL}) \geq 1 - \alpha, \quad (10)$$

$$\pi_A^I(l_{RQL}) \leq \beta,$$

$$n_T > n_N > 1, \quad l_{AQL} \leq k \leq l_{RQL}.$$

3.2. VQSS($n_N, n_T = mn_N; k$)

To streamline parameter determination and ease practical implementation, a specialized form of VQSS($n_N, n_T; k$) assumes that the tightened-inspection sample size n_T is an integer multiple m of the normal-inspection sample size n_N (i.e., $n_T = m \times n_N$ with $m > 1$). This variant, denoted VQSS($n_N, n_T = mn_N; k$), retains the same operational logic, acceptance probability function, and mathematical formulation as the general VQSS model. Under this constraint, only two parameters (n_N and k) must be determined via the optimization procedure.

Moreover, when $m = 1$, the VQSS($n_N, n_T = mn_N; k$) model simplifies to the conventional VSSP; thus, the proposed scheme can also be regarded as a generalized extension of VSSP, broadening its applicability and functionality.

4. Results and Analysis

4.1. Determination of Plan Parameters

To determine the plan parameters of the VQSS based on the framework established in this study, the sequential quadratic programming algorithm was utilized for this purpose using the “*fmincon*” function provided by MATLAB R2019a.

4.1.1. Plan Parameters of VQSS($n_N, n_T; k$)

To implement VQSS($n_N, n_T; k$), three plan parameters are necessary to be determined concurrently: the sample sizes for tightened inspection (n_T) and normal inspection (n_N), along with the critical value (k). Table 1 presents the plan parameters for VQSS($n_N, n_T; k$) under various combinations that can be used by practitioners to carry out the two-plan sampling system. For example, if the producer and the consumer have predetermined the conditions of $(I_{AQL}, I_{RQL}) = (0.03, 0.05)$ and $(\alpha, \beta) = (0.10, 0.05)$, plan parameters $(n_N, n_T; k) = (25, 164, 0.0406)$ can be obtained from Table 1. This means that 25 samples should be taken under normal inspection and 164 samples should be taken under tightened inspection, with a critical value of 0.0406. Afterward, the \hat{L}_e can be calculated to decide whether to accept or reject the inspected lot. If \hat{L}_e exceeds the critical value $k = 0.0416$, the lot will be rejected; otherwise, it will be accepted. Moreover, if the lot is rejected under normal inspection, the inspection system has to be switched to tightened inspection. However, when the lot is accepted during tightened inspection, normal inspection must be carried out for the next submitted lot to ensure the quality of the delivered products.

α	β	$I_{AQL} = 0.03,$ $I_{RQL} = 0.04$			$I_{AQL} = 0.03,$ $I_{RQL} = 0.05$			$I_{AQL} = 0.04,$ $I_{RQL} = 0.06$			$I_{AQL} = 0.06,$ $I_{RQL} = 0.11$		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	178	4190	0.0379	53	1319	0.0453	86	2101	0.0555	36	934	0.0978
	0.050	166	3155	0.0382	49	994	0.0460	80	1583	0.0562	34	704	0.0995
	0.100	159	2600	0.0384	47	821	0.0464	77	1306	0.0566	32	582	0.1005
0.050	0.010	130	1280	0.0363	39	410	0.0419	63	648	0.0522	27	292	0.0891
	0.050	114	898	0.0367	34	288	0.0428	55	454	0.0531	23	205	0.0913
	0.100	105	705	0.0370	31	227	0.0434	51	357	0.0537	21	162	0.0928
0.100	0.010	103	750	0.0352	30	243	0.0397	50	381	0.0500	21	74	0.083
	0.050	86	505	0.0357	25	164	0.0406	42	257	0.0510	18	117	0.0858
	0.100	77	387	0.0360	23	126	0.0413	37	197	0.0517	16	90	0.0875

Table 1. Plan parameters of VQSS($n_N, n_T; k$)

4.1.2. Plan Parameters of VQSS($n_N, n_T = mn_N; k$)

To simplify the determination of necessary parameters and implementation of VQSS in practical scenarios, an alternative type of VQSS($n_N, n_T; k$) is proposed. This type assumes that the sample size for tightened inspection (n_T) is equal to m times of the sample size for normal inspection (n_N), represented as $m \times n_N$, where $m > 1$. It is important to note that when $m = 1$, VQSS and VSSP are identical. The solved plan parameters for the VQSS($n_N, n_T = mn_N; k$) under various conditions are provided in Tables 2-5, with m values of 1.5, 2.0, 2.5, and 3.0. These results offer practical guidance for practitioners and simplify the implementation of VQSS in their inspection processes.

For instance, if a contract specifies conditions such as $(I_{AQL}, I_{RQL}) = (0.04, 0.06)$, $(\alpha, \beta) = (0.05, 0.10)$, and $m = 1.5$, then plan parameters $(n_N, n_T; k) = (86, 129, 0.0504)$ can be obtained by referring to Table 2. This indicates that 129 samples must be collected under tightened inspection, whereas 86 samples are required to be taken under normal inspection. Then, \hat{L}_e can be calculated based on the collected samples and compared with the critical value for lot sentencing. If $\hat{L}_e > 0.0504$, the lot is rejected; otherwise, if $\hat{L}_e \leq 0.0504$, the lot is accepted.

α	β	$I_{AQL} = 0.03,$ $I_{RQL} = 0.04$			$I_{AQL} = 0.03,$ $I_{RQL} = 0.05$			$I_{AQL} = 0.04,$ $I_{RQL} = 0.06$			$I_{AQL} = 0.06,$ $I_{RQL} = 0.11$		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	428	642	0.0350	135	203	0.0391	215	322	0.0495	96	144	0.0820
	0.050	319	479	0.0358	100	150	0.0408	159	239	0.0512	71	106	0.0860
	0.100	268	402	0.0364	83	125	0.0419	133	200	0.0523	59	88	0.0888
0.050	0.010	304	456	0.0341	98	146	0.0374	154	231	0.0477	70	104	0.0775
	0.050	214	321	0.0349	68	102	0.0389	108	161	0.0493	48	72	0.0813
	0.100	173	259	0.0355	54	81	0.0400	86	129	0.0504	39	58	0.0840
0.100	0.010	246	369	0.0334	80	119	0.0361	125	188	0.0465	57	86	0.0745
	0.050	166	249	0.0342	53	80	0.0375	84	126	0.0480	38	57	0.0779
	0.100	130	194	0.0348	41	62	0.0386	65	98	0.0490	30	44	0.0805

Table 2. Plan parameters of VQSS($n_N, n_T; k$) under $m = 1.5$

α	β	$I_{AQL} = 0.03,$ $I_{RQL} = 0.04$			$I_{AQL} = 0.03,$ $I_{RQL} = 0.05$			$I_{AQL} = 0.04,$ $I_{RQL} = 0.06$			$I_{AQL} = 0.06,$ $I_{RQL} = 0.11$		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	375	750	0.0353	118	235	0.0398	188	375	0.0502	84	167	0.0837
	0.050	288	575	0.0361	89	178	0.0414	143	286	0.0518	63	126	0.0877
	0.100	246	492	0.0366	76	151	0.0425	122	243	0.0529	53	106	0.0904
0.050	0.010	259	518	0.0344	83	165	0.0380	131	261	0.0484	59	117	0.0791
	0.050	187	374	0.0352	59	118	0.0395	94	187	0.0500	42	83	0.0829
	0.100	154	308	0.0358	48	96	0.0406	77	153	0.0511	34	68	0.0856
0.100	0.010	205	409	0.0337	66	131	0.0367	104	207	0.0471	47	94	0.0758
	0.050	142	283	0.0345	45	90	0.0381	71	142	0.0485	32	64	0.0793
	0.100	113	225	0.0351	36	71	0.0392	57	113	0.0496	25	50	0.0818

Table 3. Plan parameters of VQSS($n_N, n_T; k$) under $m = 2.0$

α	β	$I_{AQL} = 0.03,$ $I_{RQL} = 0.04$			$I_{AQL} = 0.03,$ $I_{RQL} = 0.05$			$I_{AQL} = 0.04,$ $I_{RQL} = 0.06$			$I_{AQL} = 0.06,$ $I_{RQL} = 0.11$		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	342	854	0.0356	107	266	0.0404	171	426	0.0508	75	188	0.0851
	0.050	267	667	0.0364	83	206	0.0419	133	331	0.0523	58	144	0.0890
	0.100	231	578	0.0369	71	177	0.0430	114	285	0.0533	50	124	0.0916
0.050	0.010	231	577	0.0347	73	182	0.0385	116	289	0.0489	52	129	0.0804
	0.050	170	425	0.0355	53	133	0.0400	85	212	0.0505	38	94	0.0842
	0.100	142	354	0.0360	44	110	0.0411	71	176	0.0515	31	77	0.0868
0.100	0.010	179	447	0.0340	57	142	0.0371	90	225	0.0475	41	102	0.0769
	0.050	126	315	0.0348	40	99	0.0386	63	158	0.0490	28	70	0.0804
	0.100	102	254	0.0353	32	79	0.0396	51	127	0.0501	23	56	0.0830

Table 4. Plan parameters of VQSS($n_N, n_T; k$) under $m = 2.5$

α	β	$I_{AQL} = 0.03,$ $I_{RQL} = 0.04$			$I_{AQL} = 0.03,$ $I_{RQL} = 0.05$			$I_{AQL} = 0.04,$ $I_{RQL} = 0.06$			$I_{AQL} = 0.06,$ $I_{RQL} = 0.11$		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	318	953	0.0358	99	296	0.0408	158	474	0.0512	70	208	0.0862
	0.050	252	756	0.0366	78	232	0.0423	125	374	0.0527	55	163	0.0900
	0.100	221	661	0.0370	67	201	0.0433	109	326	0.0537	47	141	0.0925
0.050	0.010	211	633	0.0349	67	199	0.0389	106	317	0.0493	47	141	0.0814
	0.050	158	474	0.0357	49	147	0.0405	79	236	0.0509	35	104	0.0852
	0.100	134	400	0.0362	41	123	0.0415	66	198	0.0519	29	86	0.0878
0.100	0.010	161	483	0.0342	51	153	0.0375	81	243	0.0479	37	109	0.0778
	0.050	116	346	0.0349	36	108	0.0389	58	173	0.0494	26	76	0.0814
	0.100	94	282	0.0355	29	87	0.0400	47	140	0.0504	21	62	0.0839

Table 5. Plan parameters of VQSS($n_N, n_T; k$) under $m = 3.0$

4.2. Operating Characteristics (OC) Curve

In this section, we assess and compare the performance of the proposed VQSS and VSSP by examining the OC and ASN curves. Initially, we conduct a performance comparison of the two types of VQSS and VSSP using the OC curve.

The Operating Characteristic (OC) curve illustrates the performance of a sampling plan's acceptance probability (y-axis) across various quality levels (z-axis). A steeper OC curve slope indicates better discriminatory power. The Operating Characteristic (OC) curve behavior of the proposed Variable Quick Switching Sampling (VQSS) system is further investigated. This system combines two sampling plans: the Normal-VSS plan with $(n_N, k) = (41, 0.0415)$ and the Tightened-VSS plan with $(n_T, k) = (123, 0.0415)$. The OC curves of these plans and the VQSS system are compared in Figure 2. When the submitted lot's quality is poor (above the critical value $k = 0.0415$), the VQSS system's OC curve closely resembles the Tightened-VSS plan's curve. Conversely, as the lot's quality improves, the VQSS system's OC curve approaches the Normal-VSS plan's curve. This demonstrates the VQSS system's flexibility in selecting the appropriate inspection based on the actual quality level, while maintaining discriminatory power. The VQSS system adapts to changing quality levels, making it a robust and efficient sampling system. By leveraging this flexibility, the VQSS system can provide effective quality control while minimizing unnecessary inspections.

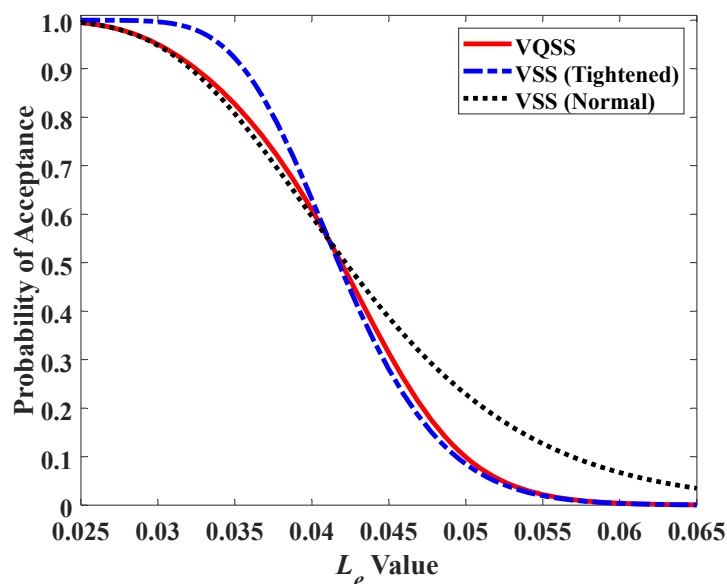


Figure 2. The OC curves of Normal-VSS plans, Tightened-VSS plans, and the VQSS system

Figure 3 and 4 compares the OC curves of the Variable Single Sampling (VSSP) plan and the Variable Quick Switching Sampling (VQSS) system, both based on process loss index, under specific conditions: $(l_{AQL}, l_{RQL}) = (0.03, 0.05)$ with $(\alpha, \beta) = (0.05, 0.05)$ and $(\alpha, \beta) = (0.10, 0.05)$. As shown in Figures 3 and 4, both OC curves pass through the designated points $(l_{AQL}, 1 - \alpha)$ and (l_{RQL}, β) , meeting the required conditions.

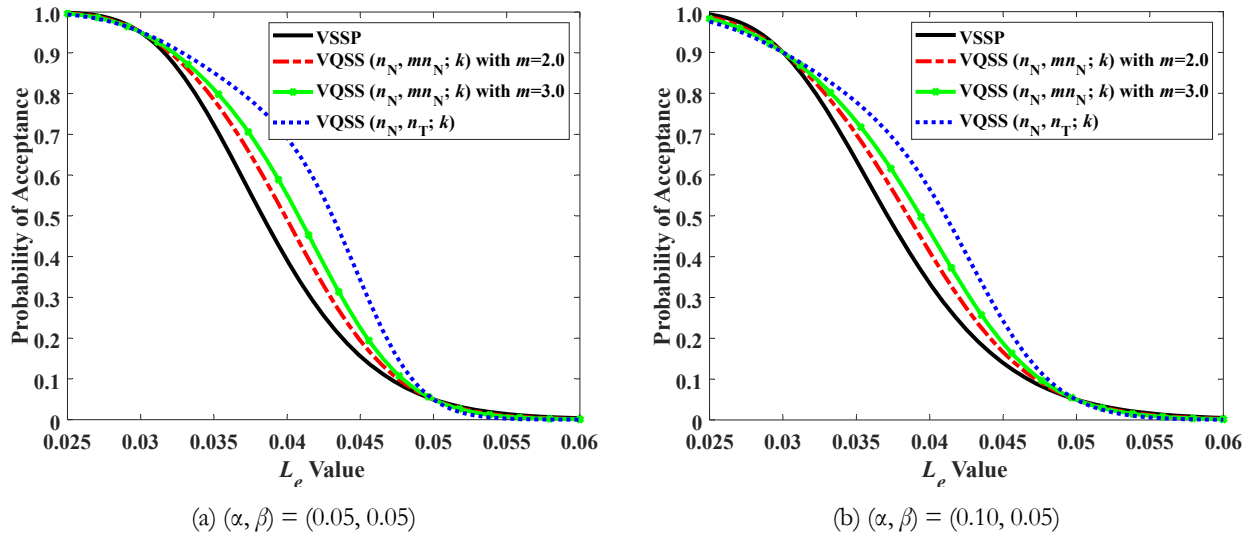


Figure 3. The OC curves of the VSSP, VQSS($n_N, n_T = mn_N; k$) with $m = 2.0, 3.0$, VQSS($n_N, n_T; k$), and $(l_{AQL}, l_{RQL}) = (0.03, 0.05)$

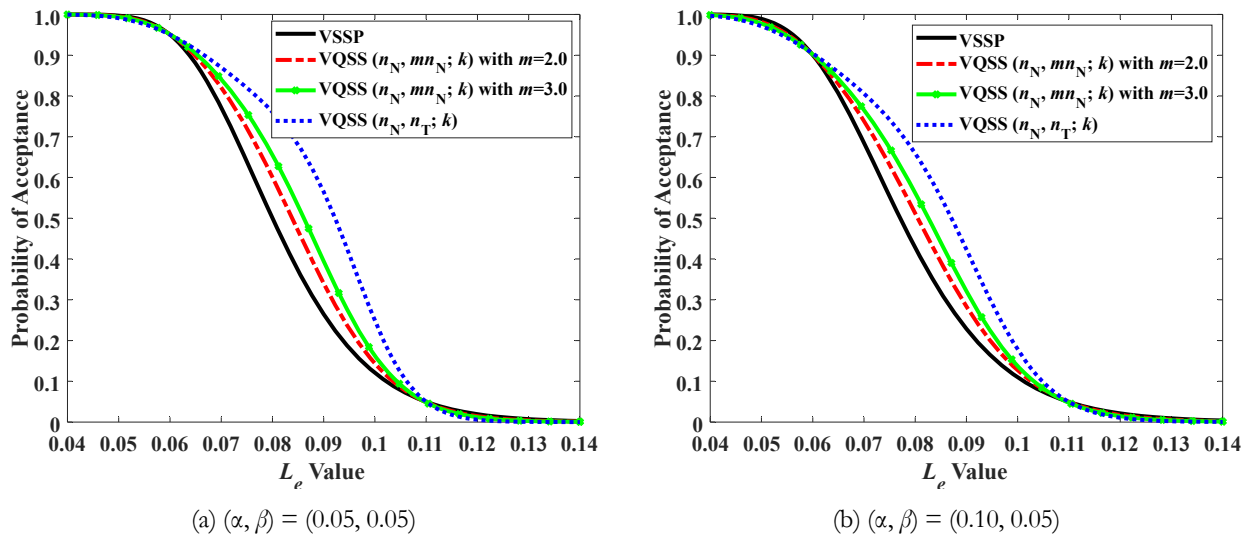


Figure 4. The OC curves of the VSSP, VQSS($n_N, n_T = mn_N; k$) with $m = 2.0, 3.0$, VQSS($n_N, n_T; k$), and $(l_{AQL}, l_{RQL}) = (0.06, 0.11)$

Notably, the VQSS system's OC curve exhibits a shape closer to the ideal OC curve, demonstrating its superior discriminatory power compared to the VSS plan. This suggests the VQSS system can more effectively distinguish between acceptable and unacceptable quality levels. The figures indicate that for the VQSS system with parameters VQSS($n_N, n_T = mn_N; k$), increasing the value of m leads to a steeper OC curve slope, which translates to improved discriminatory power. In other words, larger values of m enhance the system's ability to distinguish between high- and low-quality lots.

4.3. Average Sample Number (ASN)

Another general measurement system, we generate the ASN curves for VSSP and the two types of VQSS to evaluate their sampling efficiency from an economic standpoint. Figures 5-6 present the ASN curves for VSSP, VQSS($n_N, n_T; k$), VQSS($n_N, n_T = mn_N; k$) with different m values ($m = 2, 3$), and risk levels $(\alpha, \beta) = (0.05, 0.05), (0.10,$

0.05), quality levels $(l_{AQL}, l_{RQL}) = (0.03, 0.05)$ and $(0.06, 0.11)$. It is evident that VSSP and $VQSS(n_N, n_T = mm_N; k)$ with different m values and $VQSS(n_N, n_T; k)$ heavily depend on the lot's quality. When the lot is of excellent quality, both $VQSS(n_N, n_T = mm_N; k)$ (regardless of the value of m) and $VQSS(n_N, n_T; k)$ require a smaller sample size than VSSP. In addition, when the lot's quality is not satisfactory (with a relatively large value of L_e), $VQSS(n_N, n_T = mm_N; k)$ and $VQSS(n_N, n_T; k)$ tend to require a larger number of sample items for inspection as tightened inspection may be necessary to ensure the lot's quality.

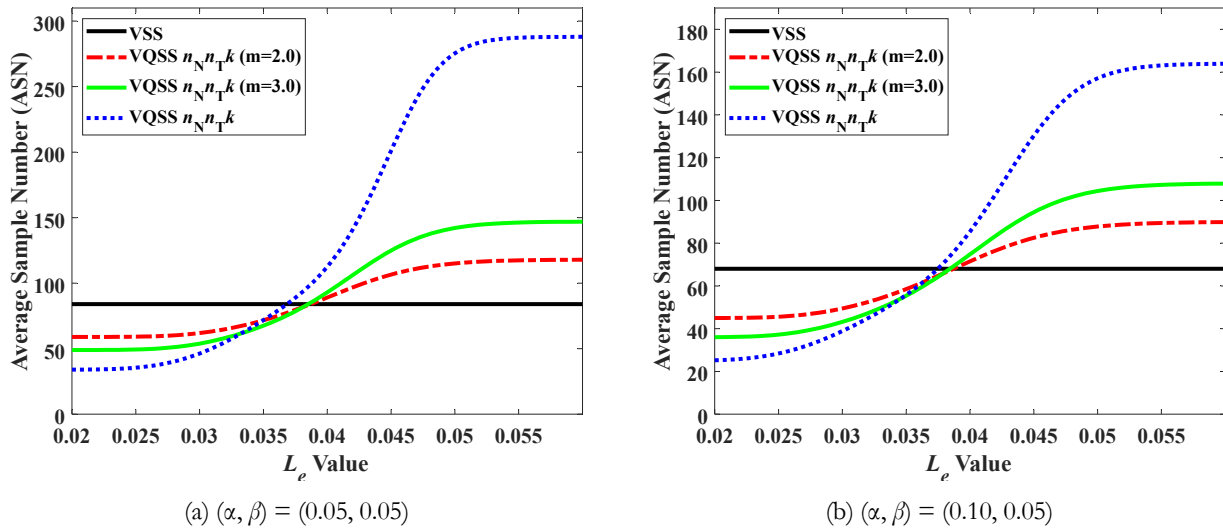


Figure 5. The ASN curves of the VSS, $VQSS(n_N, n_T = mm_N; k)$ with $m = 2.0, 3.0$, $VQSS(n_N, n_T; k)$, under $(l_{AQL}, l_{RQL}) = (0.03, 0.05)$

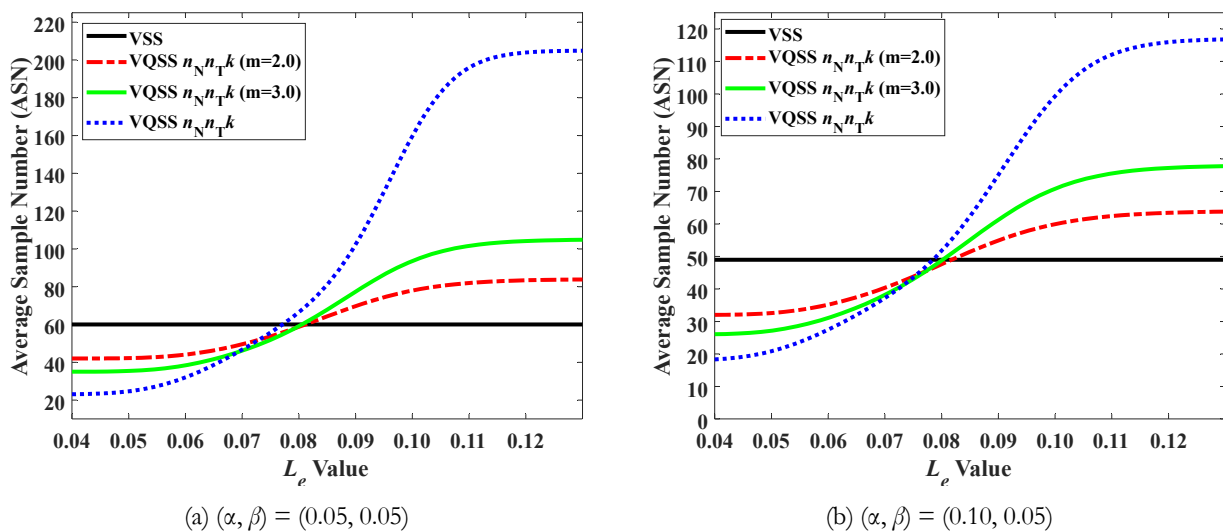


Figure 6. The ASN curves of the VSS, $VQSS(n_N, n_T = mm_N; k)$ with $m = 2.0, 3.0$, $VQSS(n_N, n_T; k)$, under $(l_{AQL}, l_{RQL}) = (0.06, 0.11)$

4.4. Example Demonstration

The proposed methodology was validated through a case study of an amplified pressure sensor, sourced from Yen and Chang (2009), which is a representative example of sensors used in electronic device modules. The study emphasizes the importance of precise span control and monitoring in amplified pressure sensors, highlighting the need for consistent and reliable performance in these critical components.

The specification limit for this particular case is set at 2.0 ± 0.1 V, which translates to a target value (T) of 2.0, a lower specification limit (LSL) of 1.9, and an upper specification limit (USL) of 2.1. According to the agreement, the multiplication number sample size for tightened inspection (m) is assumed to be 2. The quality level requirements are specified as $(l_{AQL}, l_{RQL}) = (0.06, 0.11)$, while the risk levels are set at $(\alpha, \beta) = (0.01, 0.05)$.

1.9422	1.9651	2.0230	1.9712	1.9975	2.0164	1.9927	1.9566
1.9738	1.9541	1.9800	1.9596	1.9811	2.0088	1.9858	1.9677
2.0001	1.9659	1.9955	1.9842	1.9909	1.9829	1.9684	1.9942
1.9897	1.9836	1.9891	1.9608	2.0109	1.9912	2.0077	1.9803
2.0106	1.9885	1.9704	1.9882	1.9689	1.9553	1.9741	1.9825
1.9640	2.0187	1.9616	1.9865	1.9556	1.9817	1.9774	1.9316
1.9841	1.9919	1.9737	1.9958	2.0121	2.0021	1.9665	1.9773
1.9841	1.9570	1.9610	2.0015	1.9750	1.9825	1.9758	

Table 6. The submitted lot yielded 63 measurement data points

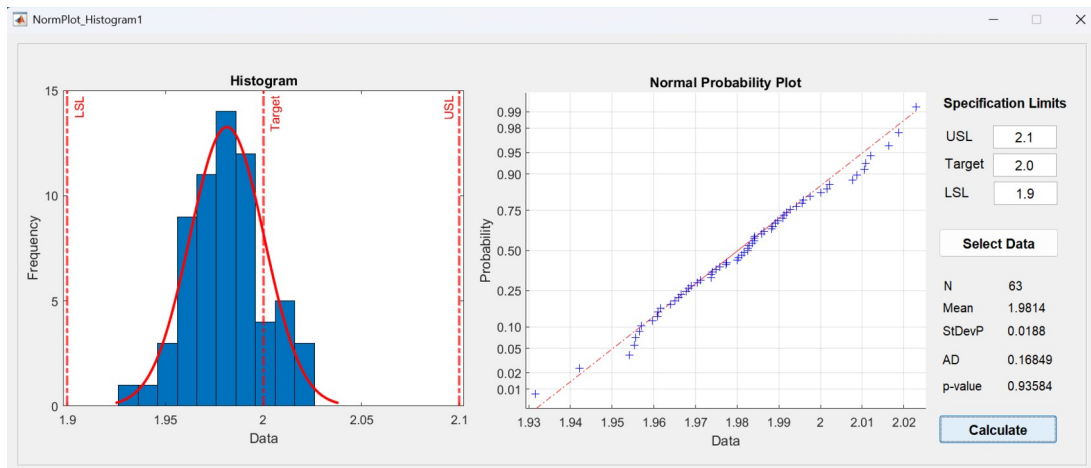


Figure 7. The histogram plot of the sample data and the normal probability plot of the sample data

The plan parameters, which are determined to be $(n_N, n_I; k) = (63, 126; 0.0877)$, can be ascertained by referencing Table 3. Specifically, a random sample of 63 units is selected from the submitted lot, and the measurements are compiled and presented in Table 6. The distribution of the sample data is illustrated in Figure 7 through a histogram and a probability plot, which yields a sample mean and standard deviation of $\bar{X} = 1.9814$ and $S = 0.0188$, respectively. Furthermore, the Anderson-Darling normality test indicates that the data conform to a normal distribution, with a p-value of 0.93584 (Figure 7). The calculated value is then compared to the critical value k . If the calculated value is less than or equal to $k = 0.0877$, the lot is accepted; otherwise, it is rejected. Following the outlined operating procedure in Section 3, the lot sentencing is carried out based on the calculation of \hat{L}_e . In this sample, the calculated value \hat{L}_e is 0.0708, which is less than the critical value $k = 0.0877$. Therefore, the lot is accepted based on the original sampling plan.

A comparison with traditional variables sampling plans (VSS) reveals that a larger sample size of $n = 84$ than the $VQSS(n_N, n_I = mn_N; k)$ plan is required for inspection, with a critical value of 0.0836. In contrast, the proposed $VQSS(n_N, n_I = mn_N; k)$ plan can achieve lot sentencing with a smaller sample size of $n_N = 63$ under identical conditions. Moreover, the proposed $VQSS(n_N, n_I; k)$ plan demonstrates greater efficiency, requiring a sample size of only $n_N = 34$, which is significantly lower than previous models. However, if product quality deteriorates and a switch to tightened inspection is necessary, the sample size would need to be substantially increased to 704 for thorough inspection. The proposed VQSS systems, $VQSS(n_N, n_I; k)$ and $VQSS(n_N, n_I = mn_N; k)$, require adjustments to sample sizes when switching plans, which may lead to additional inspection costs if lot quality declines. However, these systems offer valuable insights into lot quality, motivating suppliers to improve product quality and reduce potential costs. By providing more information, VQSS variants encourage suppliers to enhance their processes, ultimately leading to better quality products and reduced costs. This makes VQSS a beneficial approach for quality control and supplier improvement.

5. Conclusions

In today's competitive market landscape, businesses must prioritize product quality to meet the increasingly stringent expectations of their customers. Traditional acceptance sampling plans often evaluate product quality based on process yield, which fails to capture subtle variations within specification limits. To address this limitation, the process loss index L_e was developed to quantify process performance by accounting for quality loss. This study introduces a novel two-plan sampling system, VQSS, which leverages the L_e index to dynamically adjust inspection stringency in response to fluctuations in product quality. By incorporating both tightened and normal inspection protocols, VQSS offers enhanced flexibility compared to conventional single sampling plans (VSSP). Two variants of VQSS were developed and comprehensively evaluated using operating characteristic (OC) and average sample number (ASN) curves. The results demonstrate the superiority of VQSS over VSSP in terms of adaptability and efficiency. By adopting VQSS, organizations can optimize their quality control processes and respond more effectively to changes in product quality. This study contributes to the advancement of quality control methodologies and provides practical insights for industries seeking to enhance their product quality.

The VQSS of type $(n; k_N, k_T)$ as developed by Balamurali and Usha (2017a) emerges as a cost-effective strategy, characterized by a substantial reduction in required sample size and facile plan switching via critical value adjustments. Notwithstanding this, the proposed method VQSS($n_N, n_T; k$) and VQSS($n_N, n_T=mn_N; k$) offers an advantage in terms of sample size adjustment when switching to a more stringent inspection plan when lot quality deteriorates. In addition, these VQSS variants provide supplementary information for lot quality assessment, incentivizing suppliers to enhance product quality and mitigate potential costs. Each VQSS type offers distinct advantages, rendering them suitable for specific scenarios and enabling practitioners to select the most appropriate type for their objectives. To facilitate implementation, the study provides comprehensive tables of plan parameters for each VQSS type, accommodating diverse quality conditions and risk combinations. An illustrative example demonstrates the practical applicability of the proposed system, underscoring its potential to inform efficient and cost-effective decision-making in lot disposition. Organizations adopting VQSS can optimize their quality control processes and adapt to changing product quality requirements. The proposed system offers a valuable framework for quality control practitioners seeking to enhance their inspection protocols.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest concerning the research, authorship, and/or publication of this article.

Funding

This research is partially supported by Faculty of Engineering (Hasanuddin University) under scheme of Laboratory Based Education (LBE).

References

- Aslam, M., Yen, C.H., Chang, C.H., & Jun, C.H. (2013). Multiple states repetitive group sampling plans with process loss consideration. *Applied Mathematical Modelling*, 37(20-21), 9063-9075. <https://doi.org/10.1016/j.apm.2013.04.020>
- Aslam, M., Yen, C.H., Chang, C.H., & Jun, C.H. (2014). Multiple dependent state variable sampling plans with process loss consideration. *International Journal of Advanced Manufacturing Technology*, 71(5-8), 1337-1343. <https://doi.org/10.1007/s00170-013-5574-9>
- Aslam, M., Yen, C.H., Chang, C.H., Jun, C.H., Ahmad, M., & Rasool, M. (2012). Two-stage variables acceptance sampling plans using process loss functions. *Communications in Statistics - Theory and Methods*, 41(20), 3633-3647. <https://doi.org/10.1080/03610926.2011.563019>
- Aslam, M., Yen, C.H., & Jun, C.H. (2011). Variable repetitive group sampling plans with process loss consideration. *Journal of Statistical Computation and Simulation*, 81(11), 1417-1432. <https://doi.org/10.1080/00949655.2010.487826>
- Balamurali, S., & Usha, M. (2012). Variables quick switching system with double specification limits. *International Journal of Reliability, Quality and Safety Engineering*, 19(2), 1-17. <https://doi.org/10.1142/S0218539312500088>

- Balamurali, S., & Usha, M. (2014). Optimal designing of variables quick switching system based on the process capability index Cpk. *Journal of Industrial and Production Engineering*, 31(2), 85-94. <https://doi.org/10.1080/21681015.2014.893929>
- Balamurali, S., & Usha, M. (2017a). Designing of variables quick switching sampling system by considering process loss functions. *Communications in Statistics - Theory and Methods*, 46(5), 2299-2314. <https://doi.org/10.1080/03610926.2015.1041983>
- Balamurali, S., & Usha, M. (2017b). Developing and designing of an efficient variables sampling system based on the process capability index. *Journal of Statistical Computation and Simulation*, 87(7), 1401-1415. <https://doi.org/10.1080/00949655.2016.1267735>
- Chan, L.K., Cheng, S.W., & Spiring, F.A. (1988). A New Measure of Process Capability: Cpm. *Journal of Quality Technology*, 20(3), 162-175. <https://doi.org/10.1080/00224065.1988.11979102>
- Darmawan, A., Bahri, S., Amar, K., Do-Bagus, M.R., & Tahir, N. (2025). A flexible resubmitted variable sampling plan for product quality determination using the process loss index. *Production Engineering Archives*, 31(2), 201-211. <https://doi.org/10.30657/pea.2025.31.20>
- Darmawan, A., & Wu, C.W. (2025). Designing an enhanced acceptance sampling strategy with the process loss index. *European Journal of Industrial Engineering*, 20(2), 157-182. <https://doi.org/10.1504/EJIE.2025.10064829>
- Darmawan, A., Wu, C., Wang, Z., & Chiang, P. (2025). Developing variables two-plan sampling scheme with consideration of process loss for lot sentencing. *Quality Engineering*, 37(2), 273-291. <https://doi.org/10.1080/08982112.2024.2381012>
- Dodge, F.H. (1967). Evaluation of sampling inspection system having rules for switching between normal and tightened inspection. In *Technical Report 14. Rutgers State University: The Statistics Center*.
- Erfanian, M., & Gildeh, B.S. (2021). A new capability index for non-normal distributions based on linex loss function. *Quality Engineering*, 33(1), 76-84. <https://doi.org/10.1080/08982112.2020.1761026>
- Govindaraju, K., & Ganesalingam, S. (1998). Zero acceptance number quick switching system for compliance sampling. *Journal of Applied Statistics*, 25(3), 399-407. <https://doi.org/10.1080/02664769823124>
- Hald, A., & Thyregod, P. (1965). The composite operating characteristic under normal and tightened sampling inspection by attributes. *Bulletin of International Statistical Institute*, 41, 517-529.
- Hsiang, T.C., & Taguchi, G. (1985). A Tutorial on Quality Control and Assurance-The Taguchi Methods. *America Statistical Association, Annual Meeting*.
- Johnson, T. (1992). The Relationship of Cpm to Squared Error Loss. *Journal of Quality Technology*, 24(4), 211-215. <https://doi.org/10.1080/00224065.1992.11979402>
- Kane, V. E. (1986). Process capability indices. *Journal of Quality Technology*, 18(I), 41-52. <https://doi.org/10.1201/9780203741948>
- Liu, S.W., Wang, Z.H., & Wang, T.C. (2023). Developing a cost-efficient dual sampling system for lot disposition by considering process yield and quality loss. *Quality Engineering*, 35(2), 267-278. <https://doi.org/10.1080/08982112.2022.2124381>
- Liu, S.W., & Wu, C.W. (2016). A quick switching sampling system by variables for controlling lot fraction nonconforming. *International Journal of Production Research*, 54(6), 1839-1849. <https://doi.org/10.1080/00207543.2015.1084062>
- Pearn, W.L., Chang, Y.C., & Wu, C.W. (2004). Distributional and inferential properties of the process loss indices. *Journal of Applied Statistics*, 31(9), 1115-1135. <https://doi.org/10.1080/0266476042000280364>
- Pearn, W.L., Chang, Y.C., & Wu, C.W. (2006). Measuring process performance based on expected loss with asymmetric tolerances. *Journal of Applied Statistics*, 33(10), 1105-1120. <https://doi.org/10.1080/02664760600746871>
- Pearn, W.L., Kotz, S., & Johnson, N.L. (1992). Distributional and Inferential Properties of Process Capability Indices. *Journal of Quality Technology*, 24(4), 216-231. <https://doi.org/10.1080/00224065.1992.11979403>

- Romboski, L.D. (1966). *An Investigation of Quick Switching Acceptance Sampling Systems*. Ph.D. Thesis. Rutgers-The State University.
- Soundararajan, V., & Arumainayagam, S.D. (1990). Construction and selection of modified quick switching systems. *Journal of Applied Statistics*, 17(1), 83-114. <https://doi.org/10.1080/757582650>
- Soundararajan, V., & Palanivel, M. (2000). Quick switching variables single sampling (QSVSS) system indexed by AQL and AOQL. *Journal of Applied Statistics*, 27(6), 771-778. <https://doi.org/10.1080/02664760050081942>
- Tsui, K.L. (1997). Interpretation of process capability indices and some alternatives. *Quality Engineering*, 9(4), 587-596. <https://doi.org/10.1080/08982119708919080>
- Wang, T.C. (2022). Developing an adaptive sampling system indexed by Taguchi capability with acceptance-criterion-switching mechanism. *The International Journal of Advanced Manufacturing Technology*, 122, 2329-2342. <https://doi.org/10.1007/s00170-022-09996-2>
- Wang, T.C., & Shu, M.H. (2023). Development of an adaptive sampling system based on a process capability index with flexible switching mechanism. *International Journal of Production Research*, 61(21), 7233-7247. <https://doi.org/10.1080/00207543.2022.2147236>
- Wang, T.C., & Wu, C.W. (2025). Adaptive sampling system with quick-switching mechanism indexed by lifetime performance for product reliability verification. *Annals of Operations Research*. <https://doi.org/10.1007/s10479-025-06555-2>
- Wang, T.C., Wu, C.W., & Wang, H.Y. (2025). Developing generalized quick-switch sampling systems for high-yield product verification. *Computers & Industrial Engineering*, 206. <https://doi.org/10.1016/j.cie.2025.111202>
- Wang, Z., Wu, C.W., & Jhu, J.J. (2021). Design and construction of a quick-switching sampling system with a third-generation capability index. *Communications in Statistics - Theory and Methods*, 52(11), 3633-3651. <https://doi.org/10.1080/03610926.2021.1977959>
- Wu, C.W., & Darmawan, A. (2025). A modified sampling scheme for lot sentencing based on the third-generation capability index. *Annals of Operations Research*, 349(1), 25-46. <https://doi.org/10.1007/s10479-023-05328-z>
- Wu, C.W., Darmawan, A., & Liu, S.W. (2025). Developing a stage-independent multiple sampling plan with loss-based capability index for lot disposition. *Journal of the Operational Research Society*, 63(6), 426-437. <https://doi.org/10.1080/01605682.2024.2363264>
- Wu, C.W., Lee, A.H.I., Liu, S.W., & Shih, M.H. (2017). Capability-based quick switching sampling system for lot disposition. *Applied Mathematical Modelling*, 52, 131-144. <https://doi.org/10.1016/j.apm.2017.07.050>
- Wu, C.W., & Shu, M.H. (2011). A novel approach for measuring the maximum process-loss information in multiple production line conditions. *International Journal of Production Research*, 50(14), 3809-3820. <https://doi.org/10.1080/00207543.2011.588626>
- Wu, C.W., Shu, M.H., Wang, T.C., & Chen, Y.L. (2024). Integrating capability index and generalized rule-switching mechanism for enhanced quick-switch sampling systems. *International Journal of Production Economics*, 276. <https://doi.org/10.1016/j.ijpe.2024.109366>
- Yen, C.H., & Chang, C.H. (2009). Designing variables sampling plans with process loss consideration. *Communications in Statistics: Simulation and Computation*, 38(8), 1579-1591. <https://doi.org/10.1080/03610910903046809>

