OmniaScience

Journal of Industrial Engineering and Management
JIEM, 2026 – 19(2): 406-426 – Online ISSN: 2013-0953 – Print ISSN: 2013-8423
https://doi.org/10.3926/jiem.8790

Dynamic Game of Closed-Loop Supply Chain with Remanufacturer Participation Considering Product Design for Recycling

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Received: March 2025
Accepted: May 2026

Abstract:

Purpose: The change in recovery rate is dynamic, while the front and back states are related and evolve continuously until they reach a steady state. This paper examines the dynamic balancing strategies employed by members of a closed-loop supply chain system (CLSC), specifically the original equipment manufacturer (OEM) and the Third-party remanufacturer (TPR).

Design/methodology/approach: Using the Itô process, the characteristics of the stochastic evolution of the recovery rate of the supply chain system are described. Based on the behavior of OEM investment in product recyclable design, the profit target function of OEM seeking to maximize profit is constructed. According to the composition of the retailer's profit, the profit target function of the retailer seeking to maximize profit is constructed. The behavior of recycling and remanufacturing activities of TPR provided a basis for the profit target function of TPR constructed to pursue profit maximization. The evolution of optimal decision-making and profit of a closed-loop supply chain under centralized decision-making, decentralized decision-making, and TPR cost-sharing coordination contract mechanism is also discussed. The evolution process of recovery rate under decentralized decision making, centralized decision making, and coordinated contract mechanism of different TPRs sharing the proportion of product recoverable design investment cost is studied.

Findings: The results show that the recovery rate of a closed-loop supply chain under a coordination contract mechanism is higher than that under decentralized decision-making but lower than that under centralized decision-making. The higher proportion of TPR cost-sharing leads to a higher recovery rate.

Originality/value: This study introduces a dynamic game-theoretic model for closed-loop supply chains, incorporating third-party remanufacturers and product recyclable design. It uses the Itô process to model stochastic recovery rates and explores centralized, decentralized, and cost-sharing mechanisms. Findings highlight that cost-sharing improves recovery rates, offering practical insights for sustainable supply chain coordination.

Keywords: closed-loop supply chain, stochastic differential game, third-party remanufacturing, stackelberg game

To cite this article:

Sun, J. (2026). Dynamic game of closed-loop supply chain with remanufacturer participation considering product design for recycling. *Journal of Industrial Engineering and Management*, *19*(2), 406–426. https://doi.org/10.3926/jiem.8790

1. Introduction

In recent years, growing concerns about resource consumption and ecological degradation have drawn worldwide attention. Among the various strategies for achieving sustainable production, the closed-loop supply chain (CLSC) has become an effective mechanism to enhance resource utilization and reduce waste (Govindan & Soleimani, 2017). CLSCs integrate product collection, remanufacturing, and resale processes, linking upstream manufacturing with downstream recovery. Within this system, the dynamic relationship between Original Equipment Manufacturers (OEMs) and Third-Party Remanufacturers (TPRs) plays a central role in determining both environmental performance and economic outcomes (Zhang et al. 2020; Zou et al., 2016).

In practice, OEMs often face challenges related to remanufacturing cost, technological capability, and potential brand dilution. To mitigate these concerns, many firms have chosen to outsource remanufacturing operations to specialized third-party remanufacturers. This arrangement allows OEMs to focus on design and market development while leveraging the TPR's expertise in collection and remanufacturing (Wu & Zhou, 2019). For instance, Land Rover collaborates with Caterpillar for remanufacturing operations, illustrating the flexibility and efficiency of such partnerships. However, this outsourcing model also introduces complex strategic interactions, as both OEM and TPR pursue profit maximization under uncertain recovery conditions.

The effectiveness of remanufacturing depends critically on the product's recyclable design level, which is determined by the OEM during the design stage (Örsdemir et al., 2014). Although recyclable design can reduce lifecycle costs and improve sustainability, it requires significant investment and may intensify market competition with low-cost remanufacturers. Thus, OEMs face a trade-off between design investment and potential profit erosion caused by third-party competition (Huang et al., 2019).

While prior studies have examined closed-loop supply chains, most focus on static equilibria and overlook the dynamic evolution of recycling behaviors (Ji et al., 2019; Mohsin et al., 2021). In reality, recovery rates fluctuate due to random environmental and behavioral factors, making static analysis insufficient. Drawing on the intuition of Nerlove and Arrow (1962), the recovery rate can be viewed similarly to advertising capital: it accumulates over time according to TPRs' collection efforts and OEMs' recyclable design investments, while being subject to external stochastic influences such as changes in consumer environmental awareness, facility aging, or unexpected operational disturbances. Representing the recovery rate as an Itô process captures both the deterministic accumulation of recovery efforts and the stochastic fluctuations arising from these external factors. This study therefore employs a stochastic differential Stackelberg game to capture the leader–follower dynamics between OEMs and TPRs (Cheng & Ding, 2021). A closed-loop supply chain model comprising an OEM, a TPR, and a retailer is developed, where the recovery rate follows an Itô process reflecting these stochastic dynamics. The study aims to investigate: (1) how OEM and TPR determine optimal feedback strategies under uncertainty, and (2) how different cost-sharing ratios affect recovery rates, equilibrium decisions, and overall profitability.

This work contributes to the literature by integrating third-party remanufacturing, recyclable product design, and stochastic dynamic modeling into a unified analytical framework. The remainder of the paper is organized as follows: Section 2 reviews relevant literature, Section 3 formulates the model, Section 4 and 5 analyze decentralized, centralized, and coordinated decisions, and section 6 concludes with implications for sustainable supply chain coordination.

2. Literature Review

The literature review is divided into two subsections. The first subsection presents the literature review related to closed-loop supply chain management. Whereas, subsection two describes the literature review related to the remanufacturing model of a closed-loop supply chain.

2.1. Literature Related to Closed-Loop Supply Chain Management

The research results of remanufacturing closed-loop supply chain management can be roughly divided into three categories according to their research contents: the first is the research of network design; the second is the research of recycling channels, pricing strategies, and coordination mechanisms; the third is the research of inventory management. Because the research on recycling channels, pricing strategies, and coordination mechanisms is directly related to this paper, we focus on the current situation of this kind of research.

In the closed-loop supply chain recycling channel selection issues. Savaskan and Van Wassenhove (2004) studied the problem of selecting a suitable reverse channel structure when collecting waste products from customers and proposed a model for recycling waste products by manufacturers, retailers, and third-party logistics service providers. Savaskan and Van Wassenhove (2006) discuss the price decision of the product in the forward logistics when the manufacturer chooses different recycling channels (manufacturer recycling, retailer recycling, third-party recycling) to recycle waste products, and establishes a mixed recycling model in which the manufacturer directly recycles the product and the retailer indirectly recycles the product. With the continuous development of waste product recycling mode, more and more enterprises can not only recycle waste products through retailers or third parties, but also directly recycle waste products through network channels, so the single recycling channel is gradually being replaced by dual-channel, mixed channel or multi-channel recycling mode. The rational use of these recycling models by enterprises will bring more economic benefits to themselves.

The optimal coordination of a closed-loop supply chain has been interpreted by many experts (Peng et al., 2020). After studying the impact of customers' intuitive impression on the efficiency of the competitive recycling closed-loop supply chain, He, Wang et al. (2019) believed that retailers' entering the recycling competition could reduce the costs but not improve the recovery rate. In contrast, establishing a contract and authorization mechanism could improve recycling efficiency. Panda et al. (2017) discussed the optimal decision of a closed-loop supply chain from the perspective of profit maximization and member social responsibility in the context of corporate social responsibility. Chen et al. (2020) interpreted the impact of carbon emission and subsidy policies on remanufacturing quantity decisions. Herein, He, He and Xu (2019) further interpreted the dual-channel closed-loop supply chain in which manufacturers sell new products through retailers and the channel structure and pricing decisions when manufacturers sell remanufactured products through third parties or platforms, as well as the government subsidy policies when they compete for new products and remanufactured products. Considering the manufacturer competition, Hong et al. (2019) interpreted the influence of technology authorization on the two-cycle closed-loop supply chain in which manufacturers and remanufacturers compete to recover waste products. Comparing the two authorization modes of fixed fee and commission, it is shown that the manufacturer's optimal authorization strategy is affected by fixed fees. Furthermore, Zhang and Ren (2016) interpreted the impact of competitive sales between the original manufacturer and the patent-authorized third-party remanufacturer on product prices and profits of supply chain members. They designed a revenue-sharing contract to determine a series of distribution factors to achieve supply chain coordination.

As a key factor affecting the decision-making of the remanufacturing supply chain, product design issues have also received much attention. In terms of product design, Wu (2013) studied the manufacturer's product design decision and the impact of product design on the competition between the manufacturer and the remanufacturer in the competitive environment. The degree of resource savings and cost reduction in the remanufacturing process is mainly affected by the level of product recyclable design, which is mainly determined by the original equipment manufacturer at the product design stage (Örsdemir et al. (2014). However, adopting a product design approach conducive to remanufacturing not only requires significant investment but also intensifies the competitive threat Oems faces from low-cost remanufactured products. As the Gartner report points out, the competition for low-cost remanufacturing cartridges has taken a heavy toll on printer manufacturers, which has led some printer manufacturers to reduce product manufacturability to discourage remanufacturing.

The closed-loop supply chain models are static though, establishing a recovery rate can be dynamic. Therefore, the closed-loop supply chain tends to show long-term and dynamic characteristics continuously, leading to the obsolescence of the traditional closed-loop supply chain decision-making method of finding extreme value points in the short term. Hence, the closed-loop supply chain changes from static to dynamic, with the research focus shifting from a short-term search for extreme points to a long-term search for an optimal decision-making path.

Ji et al. (2019) analyzed the cooperation trend between suppliers and manufacturers in green procurement based on an evolutionary game model for dynamic supply chain optimization. Sun et al. (2019) interpreted the green investment of two-level supply chain members. Mohsin et al. (2021) interpreted the secondary green supply chain of green technology research and development based on the differential game model of the green supply chain. Kang and Tan (2019) interpreted an evolutionary game model of investment decisions of suppliers and manufacturers under cap-and-trade regulation. Cheng and Ding (2021) used the difference game theory to construct the social responsibility decision-making of supply chain enterprises under different supply chain structures. When uncertainty is involved in closed-loop supply chain recycling, how manufacturers collect waste products from customers is a crucial issue (Cao et al., 2020). Wen et al. (2020) interpreted the price and recovery rate decisions of closed-loop supply chain members under retailer differential pricing and average pricing, considering heterogeneous consumers with environmental responsibility. The influence of member competition on dynamic closed-loop supply chain decision-making is further considered. Zhang et al. (2020) interpreted the competitive closed-loop supply chain formed by two leading OEMs and two third-party remanufacturers by using the evolutionary game model in the outsourcing model and the third-party remanufacturing model of the licensing model. Govindan and Soleimani (2017) further interpreted the influence of joint incentives of two competing retailers on manufacturers' recycling input decisions in the closed-loop supply chain where manufacturers are responsible for recycling and summarized recent achievements and possible development directions in this field. Our research aims to deal with three aspects of literature: reverse channel management, dynamic recycling, and third-party remanufacturing. In summary, the supply chain system is influenced by various uncontrollable factors (Kalish, 1983), and its state evolution is inherently uncertain. Previous studies have primarily analyzed static or deterministic settings, while few have incorporated the stochastic evolution of recovery rates or examined the interplay between OEMs and TPRs in recyclable design decisions. To fill this gap, the present study extends the closed-loop supply chain literature by introducing stochastic dynamics and analyzing strategic interactions using a differential game approach.

2.2. Literature Related to the Remanufacturing Model of Closed-Loop Supply Chain

In terms of remanufacturing model choice, Zou (2016) compared the two modes in which original equipment manufacturers outsource or license remanufacturing to third-party remanufacturing enterprises, and found that OEMs and TPR are more inclined to choose the outsourcing or licensing mode. Previous studies on third-party remanufacturing focused on buyer-specific pricing and uniform pricing of wholesale supplier prices (Wu & Zhou, 2019), comparison of internal remanufacturing and outsourced remanufacturing (Wang et al., 2017), future supply chain performance (Jin et al. 2019). Third-party remanufacturing has been regarded as an important method for OEMs to balance the operations' economic benefits and environmental impacts effectively (Zhang et al. 2021). Therefore, given the uncertainty of the environment, the main task of this paper is to consider the third-party remanufacturing model and product recyclable design and obtain the feedback dynamic equilibrium strategy of supply chain members.

3. Problem Formulation and Model Building

The closed-loop supply chain consists of an Original Equipment Manufacturer (OEM), a Third-Party Remanufacturer (TPR), and a retailer. The OEM produces new products from raw materials and outsources the remanufacturing of used products to the TPR. The retailer sets the retail price and sells both new and remanufactured goods.

Assumption 1: The OEM's unit production cost for new products is c_n; the TPR's unit remanufacturing cost is c_r; and the OEM pays an outsourcing price μ to the TPR. It is assumed that $c_n > \mu > c_r$, ensuring that remanufacturing saves cost for the OEM and yields profit for the TPR (Huang et al., 2019; Savaskan & Van Wassenhove, 2006). Products made from used or raw materials are identical in quality, so no differentiation is considered.

The planning horizon is infinite, $t \in [0, \infty)$. The recovery rate $\theta(t)$ represents the proportion of remanufactured products. The TPR chooses its recycling effort $A(t)$; the OEM chooses the recyclable-design level $h(t)$.

Assumption 2: The TPR's recycling cost is $C_A = kA(t)^2/2$; the OEM's design cost is $C_h = k_1h(t)^2/2$, where k, $k_1 > 0$. This standard quadratic form reflects increasing marginal costs and follows Cheng and Ding (2021).

The recycling process is influenced by operational and environmental uncertainty. Following the dynamic-adjustment idea of Nerlove and Arrow (1962), the recycling effort $A(t)$ and recyclable-design level $h(t)$ determine the drift of the recovery-rate process: $d\theta(t) = (\alpha A(t) + \beta h(t))dt$.

Assumption 3: Consumer environmental awareness and facility aging cause natural decay in the recovery rate: $d\theta(t) = -\delta\theta(t)dt$, $\theta(0) \in [0,1]$.

Assumption 4: The recovery process is subject to random fluctuations represented by a standard Wiener process $z(t)$, and its volatility is proportional to the square root of $\theta(t)$ (Prasad & Sethi, 2004): $d\theta(t) = \sigma\sqrt{\theta(t)}dz(t)$.

Combining these components, the recovery rate evolves according to the Itô process

$$d\theta(t) = \big(\alpha A(t) + \beta h(t) - \delta\theta(t)\big)dt + \sigma\sqrt{\theta(t)}dz(t) \tag{1}$$

which captures the stochastic uncertainty of recycling as a continuously adjusting state variable.

Market demand depends on price and recyclable design. Using the standard linear form (Govindan & Soleimani, 2017):

$$D(t) = d - bp(t) + mh(t) \tag{2}$$

Here d is market potential, b is price sensitivity, and m measures how recyclable design enhances consumer preference for eco-friendly products.

Table 1 summarizes all notations used in the model.

Notations	Definition
d	The potential market size of the product
b	Price sensitivity of consumers
$\omega(t)$	OEM production unit wholesale price of new products
$p(t)$	Unit retail price of a product
c_n	The unit marginal cost of OEM manufacturing a new product from materials
c_r	The unit marginal cost of remanufacturing TPR into a new product
m	The influence coefficient of product recyclable design on market demand
$h(t)$	OEM product recyclable design level
$\theta(t)$	Product recovery rate
$A(t)$	TPR recycling effort level
$dz(t)$	Wiener process
α	The influence coefficient of recovery effort level on recovery rate
β	The effect coefficient of OEM product recyclable design level on recovery rate
δ	Attenuation coefficient of waste recovery
$\sigma(\theta(t))$	The volatility of the waste recovery rate
r	Market interest rate
k	Influence coefficient of the fixed cost of product recoverable design
k_1	Recovery fixed cost influence coefficient
γ	TPR unit product recovery price
μ	Unit outsourcing price paid by OEM to TPR.

Table 1. Information about symbols' definitions

Let $J_i^x (i = M, R, T; x = d, c, sc)$ denote the objective function of player i (OEM, retailer, or TPR) under decision mode x: decentralized (d), centralized (c), and coordinated (sc).

4. Decentralized Decisions

This section examines the decentralised decision-making problem in a closed-loop supply chain under stochastic disturbances. The game follows a Stackelberg structure, in which the OEM acts as the leader and both the retailer and the TPR serve as followers. The OEM determines the wholesale price and recyclable design level, anticipating the optimal responses of the TPR and retailer. The OEM's profit consists of the revenue from wholesaling both new and remanufactured products to retailers, minus the production, outsourcing, and recyclable design costs. Based on Equation (2), the OEM's objective function is given by:

$$max J_M^d = E\left(\int_0^{\infty} e^{-rt}((\omega(t) - c_n)D(t) + (c_n - \mu)\theta(t)D(t) - C_h)dt\right) \tag{3}$$

In Equation (3), the first term represents the OEM's immediate profit from selling new products, while the second addend $(c_n - \mu)\theta(t)D(t)$ reflects the additional benefit from outsourcing remanufacturing activities, which is central to the supply chain's efficiency objective.

The retailer's income is mainly the product sales revenue, and the cost is the product purchase cost. According to Equation (2), the retailer's profit objective function can be expressed as

$$max J_R^d = E\left(\int_0^{\infty} e^{-rt}((p(t) - \omega(t))D(t))dt\right) \tag{4}$$

The TPR receives an outsourcing fee μ per unit from the OEM and incurs remanufacturing, recycling, and collection costs. According to Equation (2), the objective function of TPR can be expressed as

$$max J_T^d = E\left(\int_0^{\infty} e^{-rt}((\mu - \gamma - c_r)\theta(t)D(t) - C_A)dt\right) \tag{5}$$

In Equation (5), the first term denotes the TPR's operating profit per remanufactured product, and the second addend captures the quadratic recycling cost, which rises with effort level. This structure reflects the diminishing returns to recycling effort under uncertainty.

Based on the stochastic recovery model (Equation (1)), the system's evolution depends on OEM wholesale pricing, recyclable design, the retailer's pricing, the TPR's recycling effort, and exogenous environmental uncertainty. Each participant's profit in Equations (3)-(5) is linked to the dynamic recovery rate $\theta(t)$. Given the Stackelberg power structure with the OEM as the channel leader, and the retailer and TPR as followers, the following subsection derives the feedback equilibrium strategies for the stochastic differential Stackelberg game.

4.1. Feedback Equilibrium Strategy

The Stackelberg equilibrium is defined as a strategy profile in which the leader (OEM) maximises its expected profit, anticipating the optimal reactions of the followers (TPR and retailer). The solution concept is first specified, followed by the computational derivation through backward induction. Using this method, the retailer's optimal price and the TPR's effort response are derived first, after which the OEM's optimal wholesale price and recyclable design level are determined based on these responses.

$V_i^d (i = M, R, T)$ is expressed as the value function of member OEM, retailer, and TPR under decentralized decision-making, V_θ^i is the first-order partial derivative of the optimal profit of each member of the closed-loop supply chain concerning the recovery rate and $V_{\theta\theta}^i$ is the second-order partial derivative of the optimal profit of each member concerning the recovery rate. According to Equation (3) and Equation (5) and continuous dynamic programming theory of optimal control, the partial differential equation of HJB (Hamilton-Jacobi-Bellman) in the OEM, retailer, and TPR decision stage is expressed as follows. For simplicity, the time variable *is* omitted in the equation below.

$$rV_R^d = \max_p\{(p-\omega)D + V_\theta^R(\theta)(\alpha A + \beta h - \delta\theta) + \sigma^2\theta V_{\theta\theta}^R/2\} \tag{6}$$

$$rV_T^d = \max_A\{(\mu - \gamma - c_r)\theta D - kA^2/2 + V_\theta^T(\theta)(\alpha A + \beta h - \delta\theta) + \sigma^2\theta V_{\theta\theta}^T/2\} \tag{7}$$

Solve the first-order conditions of Equation (6) and Equation (7) to obtain the recovery effort level of TPR and the retail price response strategy of retailers:

$$\begin{cases} A = \dfrac{\alpha V_\theta^T}{k} \\ p = \dfrac{d + hm + b\omega}{2b} \end{cases} \tag{8}$$

Equation (8) shows that the TPR's effort depends positively on the marginal value of recovery, while the retailer's price increases with both the product's design level and the OEM's wholesale price. OEM's HJB equation can be described as follows:

$$rV_M^d = \max\{(\omega - c_n)D + (c_n - \mu)\theta D - k_1h^2/2 + V_\theta^M(\alpha A + \beta h - \delta\theta) + \sigma^2\theta V_{\theta\theta}^M/2\} \tag{9}$$

The response strategy of retailers and TPR Equation (8) is introduced into the value function of OEM Equation (9) to calculate the optimal wholesale price and optimal product recyclable design level control strategy:

$$\begin{cases} \omega^* = \dfrac{(1-\theta)(2bc_nk_1 - c_nm^2) + 2dk_1 - m^2\theta\mu + 2\beta mV_\theta^M + 2b\theta\mu k_1}{-m^2 + 4bk_1} \\ h^* = \dfrac{dm + 4\beta bV_\theta^M - bmc_n + bm\theta c_n - bm\theta\mu}{-m^2 + 4bk_1} \end{cases} \tag{10}$$

By substituting the optimal wholesale price and the optimal product recyclable design level control strategy Equation (10) into Equation (8), the equilibrium control strategy of TPR and retailers is derived as:

$$\begin{cases} A^* = \dfrac{\alpha V_\theta^T}{k} \\ p^* = \dfrac{3dk_1 - c_nm^2 - \theta\mu m^2 + 3m\beta V_\theta^M + bc_nk_1 + \theta c_nm^2 + b\theta\mu k_1 - b\theta c_nk_1}{-m^2 + 4bk_1} \end{cases} \tag{11}$$

Equation (10) and (11) express the OEM's and followers' equilibrium control strategies: $\{\omega, h, A, p\}$. These expressions form the foundation for deriving the value functions of each player and the subsequent feedback Stackelberg–Markov equilibrium.

After inserting the equilibrium control strategy Equation (10) and Equation (11) into the HJB Equation (6), Equation (7) and Equation (9), substituting the equilibrium control strategy into the value function, since the value function is a quadratic function of recovery rate, it is assumed that the value functions of OEM and TPR are $V_M^d = f_1\theta^2 + f_2\theta + f_3$, $V_T^d = e_1\theta^2 + e_2\theta + e_3$ respectively. The first and second partial derivatives of the optimal profit of OEM and TPR concerning the recovery rate can be expressed as $V_\theta^M = 2f_1\theta + f_2$, $V_{\theta\theta}^M = 2f_1$, $V_\theta^T = 2e_1\theta + e_2$, and $V_{\theta\theta}^T = 2e_1$. By substituting the value function and its derivative into the HJB equation, the coefficient equation to be solved can be obtained:

$$\begin{cases} rf_1 = \dfrac{16n_7f_1^2 + 4n_9f_1 + kk_1b^2n_1^2}{2kn_2} \\ rf_2 = \dfrac{(2n_9 + 16n_7f_1)f_2 + \big((2n_2n_4 - 4k\beta mn_3)f_1 - 2bkk_1n_1n_3\big)}{2kn_2} \\ rf_3 = \dfrac{4n_7f_2^2 - 2km\beta f_2n_3 + 2e_2\alpha^2f_2n_2 + kk_1n_3^2}{2kn_2} \end{cases} \tag{12}$$

$$\begin{cases} re_1 = \dfrac{4\alpha^2 n_2 e_1^2 + 4n_{10}e_1 + 2kb^2k_1n_1n_6 + 4mf_1n_8n_6}{2kn_2} \\ re_2 = \dfrac{(4\alpha^2 e_1 n_2 + 2n_{10})e_2 + (2k\sigma^2 n_2 - 4km\beta n_3 + 16f_2n_7)e_1 + 2mf_2n_6n_8 - 2bkk_1n_3n_6}{2kn_2} \\ re_3 = \dfrac{\alpha^2 e_2^2 n_2 - 2km\beta e_2 n_3 + 8e_2f_2n_7}{2kn_2} \end{cases} \tag{13}$$

$n_1 = c_n - \mu$, $n_2 = 4bk_1 - m^2$, $n_3 = bc_n - d$, $n_4 = k\sigma^2 + 2\alpha^2 e_2$, $n_5 = -\delta k + 2\alpha^2 e_1$ $n_6 = \mu - \gamma - c_r$,$n_7 = bk\beta^2$,$n_8 = bk\beta$,$n_9 = mn_1n_8 + n_2n_5$, $n_{10} = 8n_7f_1 + mn_1n_8 - k\delta n_2$

According to Equation (12) and Equation (13), the coefficients can be calculated as follows:

$$\begin{cases} f_1 = \dfrac{-(4n_9 - 2kn_2r) - \sqrt{(4n_9 - 2kn_2r)^2 - 64n_7kk_1b^2n_1^2}}{32n_7} \\ f_2 = \dfrac{(n_2n_4 - 2k\beta mn_3)f_1 - bkk_1n_1n_3}{kn_2r - n_9 - 8n_7f_1} \\ f_3 = \dfrac{4n_7f_2^2 - 2km\beta f_2n_3 + 2e_2\alpha^2 f_2n_2 + kk_1n_3^2}{2krn_2} \end{cases} \tag{14}$$

$$\begin{cases} e_1 = \dfrac{-(4n_{10} - 2kn_2r) - \sqrt{(4n_{10} - 2kn_2r)^2 - 16\alpha^2 n_2(2kb^2k_1n_1n_6 + 4mf_1n_8n_6)}}{8\alpha^2 n_2} \\ e_2 = \dfrac{(2k\sigma^2 n_2 - 4km\beta n_3 + 16f_2n_7)e_1 + 2mf_2n_6n_8 - 2bkk_1n_3n_6}{2rkn_2 - 4\alpha^2 e_1n_2 - 2n_{10}} \\ e_3 = \dfrac{\alpha^2 e_2^2 n_2 - 2km\beta e_2n_3 + 8e_2f_2n_7}{2krn_2} \end{cases} \tag{15}$$

Since the function of f_1 and e_1 is a quadratic function, two solutions will appear in the process of solving f_1 and e_1, so how to judge the two solutions and get the feedback Stackelberg-Markov equilibrium of the closed-loop supply chain will be explained in Theorem 1.

Theorem 1. When $k_1 > B_1$ and $k > B_3$, there exists a unique Stackelberg–Markov equilibrium with feedback. The equilibrium wholesale price, recyclable design level, retailer price, and TPR recycling effort are given by Equation (16). **Proof** See Appendix A.

$$\begin{cases} \omega^* = \dfrac{(1-\theta)(2bc_nk_1 - c_nm^2) + 2dk_1 - m^2\theta\mu + 2\beta m(2f_1\theta + f_2) + 2b\theta\mu k_1}{-m^2 + 4bk_1} \\ h^* = \dfrac{dm + 4\beta b(2f_1\theta + f_2) - bmc_n + bm\theta c_n - bm\theta\mu}{-m^2 + 4bk_1} \\ p^* = \dfrac{3dk_1 - c_nm^2 - \theta\mu m^2 + 3m\beta(2f_1\theta + f_2) + bc_nk_1 + \theta c_nm^2 + b\theta\mu k_1 - b\theta c_nk_1}{-m^2 + 4bk_1} \\ A^* = \dfrac{\alpha(2e_1\theta + e_2)}{k} \end{cases} \tag{16}$$

4.2. The Evolutionary Path of the Recovery Rate

Based on Equation (1), the dynamic recovery process of the closed-loop supply chain depends on the OEM's pricing and design decisions, the retailer's pricing strategy, the TPR's recycling effort, and exogenous random disturbances. This subsection analyses the evolution of the recovery rate under the equilibrium control strategies.

By substituting the equilibrium levels of recycling effort and recyclable design into Equation (1), the recovery-rate dynamics are expressed as: $d\theta(t) = \left(\left(\frac{2e_1\alpha^2}{k} + \frac{\beta(8\beta bf_1 + bmc_n - bm\mu)}{-m^2+4bk_1} - \delta\right)\theta + \left(\frac{\alpha^2 e_2}{k} + \frac{\beta(dm - bmc_n + 4\beta bf_2)}{-m^2+4bk_1}\right)\right)dt + \sigma\sqrt{\theta}dz(t), \theta(0) = \theta_0$.

It describes the stochastic evolution of the recovery rate. The first term inside the parentheses captures the drift—driven by design and effort, while the second addend represents the deterministic growth component affected by random shocks.

When $k > B_3$, $k_1 > B_1$, the drift term is negative, ensuring system stability. Let $M = 2e_1\alpha^2/k + \beta(8\beta bf_1 + bmc_n - bm\mu)/(-m^2 + 4bk_1) - \delta$ and $N = \alpha^2 e_2/k + \beta(dm + bmc_n + 4\beta bf_2)/(-m^2 + 4bk_1)$.Then: $d\theta(t) = (M\theta + N)dt + \sigma\sqrt{\theta}dz(t),\ \ \theta(0) = \theta_0$.

Proposition 1. Solving the above stochastic differential equation yields the expected recovery rate path:

$$E(\theta) = \theta_0 e^{Mt} + \int_0^t e^{Mt-Ms}\,ds = -\frac{N}{M} + e^{Mt}\left(\theta_0 + \frac{N}{M}\right) \tag{17}$$

The condition $-N/M < 1$ ensures that the long-term recovery rate remains bounded. In the decentralised model, the steady-state expected recovery rate is given by:

$$\lim_{t\to\infty} E(\theta) = -\frac{N}{M} \tag{18}$$

This result implies that despite random disturbances, the recovery process converges to a unique steady state, characterising the long-run equilibrium of the closed-loop supply chain.

Lemma 1. The rate of change of the expected recovery is given by $\frac{dE(\theta)}{dt} = -M(\theta_0 - \frac{N}{M})e^{-Mt}$. The sign of $\frac{dE(\theta)}{dt}$ determines whether the system approaches or diverges from its steady state, depending on the initial recovery rate.

When $\theta_0 > -N/M$, $dE(\theta)/dt < 0$; When $\theta_0 < -N/M$, $dE(\theta)/dt > 0$. Thus, the expected recovery rate converges over time, either increasing or decreasing toward the unique steady state. Even if the initial rate exceeds the steady level, the long-term equilibrium remains optimal for system stability.

The steady-state recovery rate is directly proportional to e_1, e_2, f_1, f_2. Since $A^* = \alpha(2e_1\theta + e_2)/k$ and $h^* = (dm + 4\beta b(2f_1\theta + f_2) - bmc_n + bm\theta c_n - bm\theta\mu)/(-m_2 + 4bk_1)$, higher TPR recycling effort and greater OEM recyclable design lead to higher expected recovery rates in the system.

Over time, the recovery rate evolves dynamically, and the optimal strategies of the OEM, retailer, and TPR fluctuate around their expected values due to random disturbances. Nevertheless, these strategies gradually stabilise, implying convergence to steady-state behaviour.

Proposition 2. An increase in the unit outsourcing price reduces the OEM's recyclable design level. The effect is more pronounced when the influence coefficient m is large, as consumers' preference for recyclable design amplifies this sensitivity.

Proof. Taking the derivative of h^* with respect to μ, $\frac{\partial h}{\partial \mu} = \frac{-bm\theta}{-m^2+4bk_1} < 0$, proves the inverse relationship between outsourcing cost and design effort.

Hence, when OEMs face higher outsourcing prices, they reduce investment in recyclable design. A balanced outsourcing price is necessary to coordinate the OEM's design incentive with the TPR's remanufacturing efficiency.

Proposition 3. As the unit outsourcing price rises, both the OEM's wholesale price and the retailer's selling price increase.

Proof. From Equations (10–11), $\frac{\partial \omega}{\partial \mu} = \frac{-m^2\theta+2b\theta k_1}{-m^2+4bk_1} > 0$ and $\frac{\partial p}{\partial \mu} = \frac{-\theta m^2+b\theta k_1}{-m^2+4bk_1} > 0$. Thus, both wholesale and retail prices rise with outsourcing cost, transferring part of the burden to consumers.

This outcome implies that higher outsourcing costs propagate upstream to downstream prices. As the OEM raises wholesale prices to offset cost increases, retailers adjust their prices accordingly, ensuring profitability across the supply chain.

4.3. Numerical Simulation Under Decentralized Decisions

Numerical simulations are conducted to illustrate the dynamic behaviour of the decentralised closed-loop supply chain. The parameter values are chosen based on three considerations:

(1) alignment with previous studies on stochastic differential games and closed-loop supply chains;

(2) economic plausibility, ensuring that prices, efforts and recovery rates remain within realistic ranges;

(3) consistency with model assumptions, particularly the conditions required for the existence and uniqueness of the Stackelberg–Markov equilibrium ($k_1 > B_1$ and $k > B_3$).

Accordingly, the benchmark parameters are set as follows: $c_n = 15$, $c_r = 1$, $d = 50$, $b = 0.5$, $m = 0.4$, $\alpha = 0.8$, $\beta = 0.5$, $\delta = 0.5$, $r = 0.15$, $k = 1100$, $k_1 = 1120$, x = 2, $\gamma = 2$, $\theta_0 = 0$.

4.3.1. Evolution of Recovery Rates

To study the influence of the initial recovery rate on the evolutionary path of recovery rate, $\theta(0) = 0$ and $\theta(0) = 0.45$ were selected, respectively, for numerical simulation.

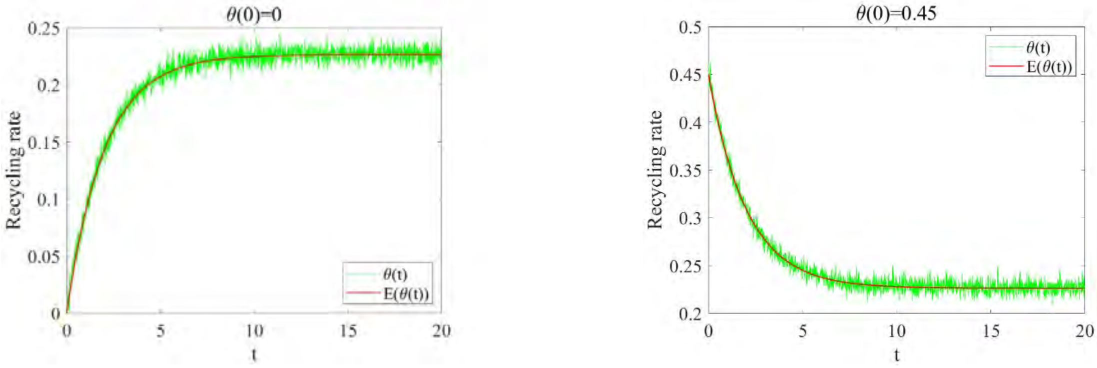


Figure 1. Evolution path of recovery rate under different initial values

Figure 1 shows the evolutionary path of recovery under random interference. In expected terms, returns can increase or decrease over time. The expected rate of return converges to a stable state over time, regardless of the initial rate of return. The recovery rate always hovers around the expected value due to random perturbations.

4.3.2. Comparison of the Impact of Outsourcing Prices

The discussion is about the effect of outsourcing prices on recovery rate, recovery effort level, product recyclable design level, wholesale price, retail price, and profit of each member under the distributed decision of third-party outsourcing. The premise of the discussion is that the expected value of the recovery rate is adopted, and the initial value of the recovery rate is chosen to be 0, that is, $\theta(0) = 0$.

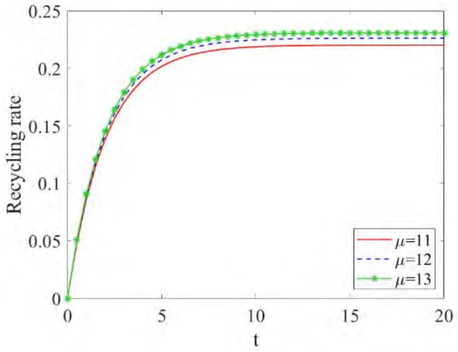
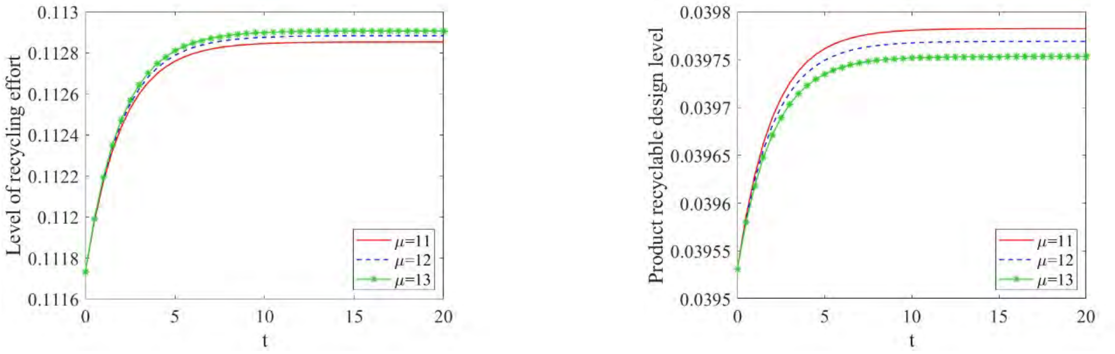


Figure 2. Evolution of expected recovery rate under different outsourcing prices

Figure 2 shows the evolution of the recovery rate, which is expected to stabilize gradually over time. As the outsourcing price decreases, the recovery rate increases, and it takes longer for the recovery rate to stabilize.

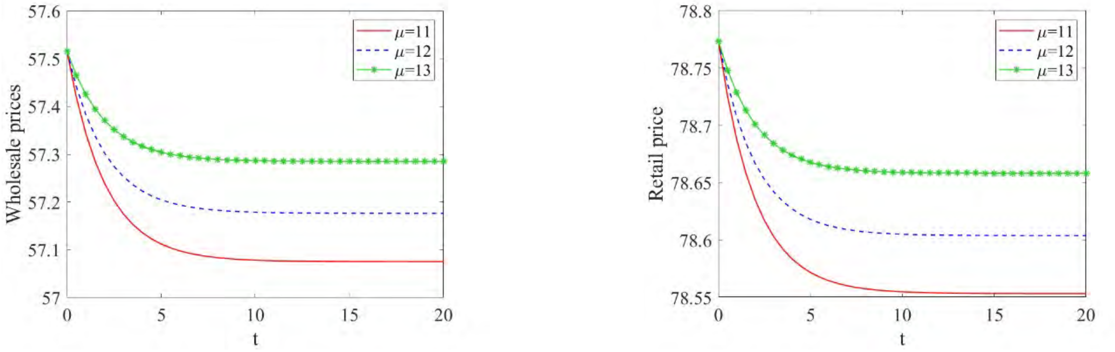
Figure 3 (a) and (b) illustrate the evolution processes of the recycling effort level and product recyclable design level under different outsourcing prices. As the outsourcing price increases, TPR's income also increases, leading to a proportional increase in TPR's investment in recycling. Therefore, the recycling effort is directly proportional to the outsourcing price. Conversely, as the outsourcing price increases, the cost of OEM also rises. To improve profits, OEM reduces its investment in recyclable product design. Consequently, the level of recyclable product design is inversely proportional to the outsourcing price.



(a)

(b)

Figure 3. (a) Evolution process of recycling effort level under different outsourcing prices; (b) Evolution process of product recyclable design level under different outsourcing prices



(a)

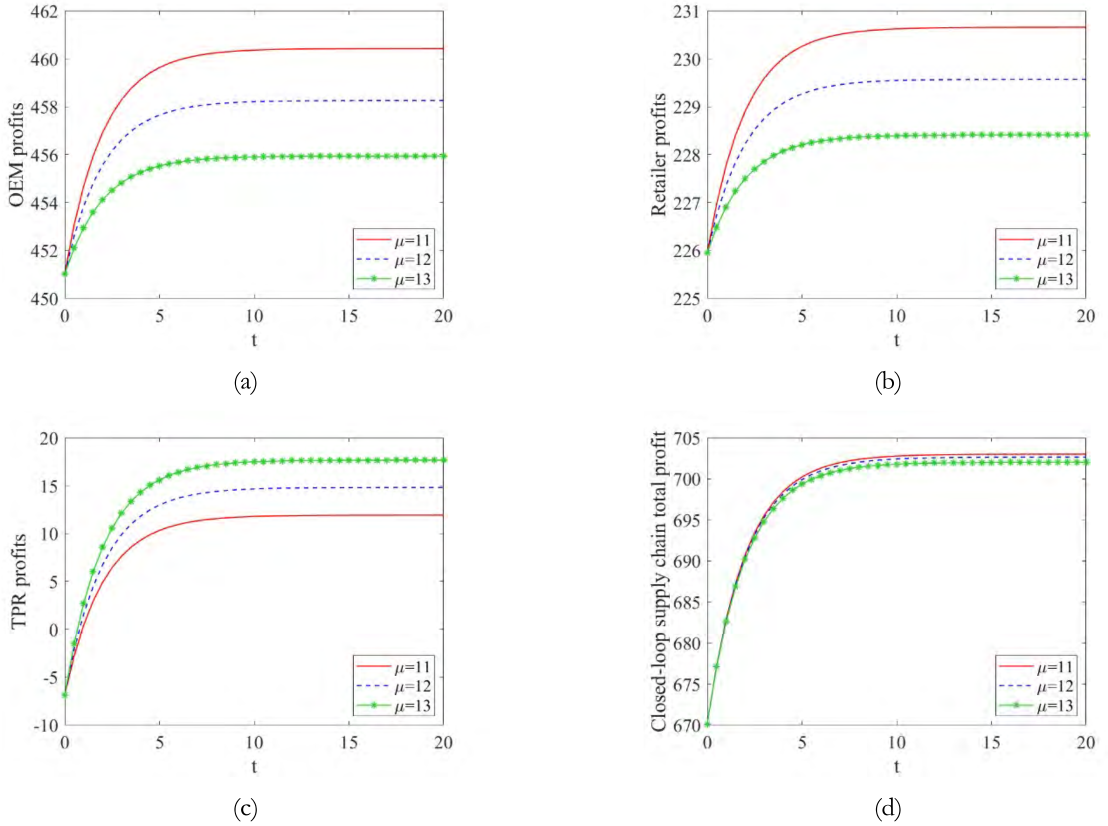
(b)

Figure 4. (a) Evolution process of OEM wholesale prices under different outsourcing prices; (b) Evolution process of retail prices of retailers under different outsourcing prices

Figure 5 (a) and (b) illustrate the evolution of OEM and retailer profits under different outsourcing prices. As the outsourcing price decreases, the cost borne by OEMs and retailers decreases, leading to increased profits for both parties. It can also be observed that as the recovery rate increases, the profits of OEMs and retailers also increase. This is because a higher recovery rate results in cost savings for OEMs, which in turn boosts their profits.

Figure 5 (c) shows the evolution process of TPR profits under different outsourcing prices. As the outsourcing price increases, TPR's revenue also increases, leading to higher profits for TPR. Additionally, as the recovery rate increases, the amount of work carried out by TPR increases, leading to increased profits.

Figure 1 (d) illustrates the evolution process of the total profit of the closed-loop supply chain under decentralized decision-making at different outsourcing prices. It can be seen that as the outsourcing price decreases, the total profit of the closed-loop supply chain under decentralized decision-making increases. Over time, as the recovery rate increases, the total profit of the closed-loop supply chain also increases. This is because a higher recovery rate results in cost savings for all participants in the supply chain, leading to increased profits for everyone involved.



(a) (b) (c) (d)

Figure 5. (a) Evolution process of OEM profit under different outsourcing prices; (b) Evolution process of retailer profit under different outsourcing prices; (c) Evolution process of TPR profit under different outsourcing prices; (d) evolution process of total closed-loop supply chain profit under different outsourcing prices

5. Coordination of Contracts

In this section, I first present the centralised benchmark in which a single decision maker governs the forward and reverse flows of the closed-loop supply chain. I then introduce a cost-sharing contract that aligns decentralised incentives and mitigates the free-rider behaviour identified in Section 4.

5.1. Centralized Decisions

In the centralised benchmark, a single decision maker jointly determines pricing, recyclable design, and recycling efforts to maximise the total channel profit. To clarify the economic structure of this objective, Equation (19) decomposes total profit into four addends: $(p(t) - c_n)D(t)$ captures the margin from products manufactured with raw materials; $(c_n - \gamma_1 - c_r)\theta(t)D(t)$ reflects the net gain from remanufacturing relative to virgin production; and the terms $-C_h$ and $-C_A$ represent system-wide investment costs in recyclable design and recycling effort. This decomposition highlights how the centralised planner internalises forward- and reverse-channel incentives.

$$\max_{p,A,h} J^c = E\left(\int_0^\infty e^{-rt}\big((p(t) - c_n)D(t) + (c_n - \gamma_1 - c_r)\theta(t)D(t) - C_h - C_A\big)dt\right) \tag{19}$$

When $k > B_5$, solving the associated HJB equation yields the planner's feedback strategies:

$$\begin{cases} p^* = \dfrac{dk_1 + m\beta(2s_1\theta + s_2) + (bc_nk_1 - c_nm^2)(1-\theta) + (bk_1 - m^2)\theta(\gamma_1 + c_r)}{-m^2 + 2bk_1} \\ h^* = \dfrac{dm + 2b\beta(2s_1\theta + s_2) + bmc_n(\theta - 1) - bm\theta(\gamma_1 + c_r)}{-m^2 + 2bk_1} \\ A^* = \dfrac{\alpha(2s_1\theta + s_2)}{k} \end{cases} \tag{20}$$

$$\begin{cases} s_1 = \dfrac{-(4m_5 - 2km_1r) - \sqrt{(4m_5 - 2km_1r)^2 - 16m_4bkk_1m_2^2}}{8m_4} \\ s_2 = \dfrac{(2k\sigma^2m_1 + 4km\beta m_3)s_1 + 2bkk_1m_2m_3}{2km_1r - 2m_5 - 4m_4s_1} \\ s_3 = \dfrac{m_4s_2^2 + 2km\beta m_3s_2 + kk_1m_3}{2km_1r} \end{cases} \tag{21}$$

Where,

$B_5 = (4km^2\delta A_3 + 2kbm\beta m_2A_4 + {m_2}^2A_7 + m_2\sqrt{m^2A_9 + 4k^2b^2\beta^2A_8})/4(-b\alpha^2{m_2}^2 + 2bk\delta A_3)$,
$B_6 = (4km^2\delta A_3 + 2kbm\beta m_2A_4 + {m_2}^2A_7 - m_2\sqrt{m^2A_9 + 4k^2b^2\beta^2A_8})/4(-b\alpha^2{m_2}^2 + 2bk\delta A_3)$.
$A_7 = 2bk\beta^2 - \alpha^2m^2$, $A_8 = (\beta m_2 + mr)^2 + 4\beta mnm_2$, $A_9 = (4bk\beta(\alpha^2m_2((\beta m_2(2b-1) +$
$mA_4)) + 4\beta k\delta A_3) + \alpha^4m^2{m_2}^2)$.

The centralised equilibrium exhibits lower retail prices and higher recycling effort relative to decentralised decisions. This outcome arises because the double marginalisation in both the forward channel (OEM–retailer) and the reverse channel (OEM–TPR) disappears once decisions are coordinated by a single planner.

5.2. Coordination of Contracts

Under decentralised decisions, the OEM under-invests in recyclable product design because the benefits of higher recovery are partly captured by the TPR. To mitigate this free-rider problem, I introduce a cost-sharing contract in which the TPR bears a share λ(0 < λ < 1) of the OEM's design cost. The profit functions under the contract are given.

$$\begin{cases} maxJ_M^{sc} = E\left(\displaystyle\int_0^\infty e^{-rt}((\omega(t) - c_n)D(t) + (c_n - \mu)\theta(t)D(t) - (1-\lambda)C_h)dt\right) \\ maxJ_R^{sc} = E(\displaystyle\int_0^\infty e^{-rt}\left((p(t) - \omega(t))D(t)\right)dt) \\ maxJ_T^{sc} = E(\displaystyle\int_0^\infty e^{-rt}((\mu - \gamma - c_r)\theta(t)D(t) - C_A - \lambda C_h)dt) \end{cases} \tag{22}$$

When $k_1 > B_8$, $k > B_{10}$, the optimal feedback control strategy of the coordination mechanism can be obtained.

$$\begin{cases} \omega^* = \dfrac{-2k_1(1-\lambda)(d + bc_n - b\theta c_n + b\theta\mu) + m^2(c_n - \theta c_n + \theta\mu) - 2m\beta(2x_1\theta + x_2)}{m^2 - 4bk_1 + 4\lambda bk_1} \\ p^* = \dfrac{-k_1(1-\lambda)(3d + bc_n - b\theta c_n + b\theta\mu) + m^2(c_n - \theta c_n + \theta\mu) - 3m\beta(2x_1\theta + x_2)}{m^2 - 4bk_1 + 4\lambda bk_1} \\ h^* = \dfrac{m(-d + bc_n - b\theta c_n + b\theta\mu) - 4b\beta(2x_1\theta + x_2)}{m^2 - 4bk_1 + 4\lambda bk_1} \\ A^* = \dfrac{\alpha(2y_1\theta + y_2)}{k} \end{cases} \tag{23}$$

$$\begin{cases} x_1 = \dfrac{-((4z_8 - 2kr)z_2 - 4\beta bkmz_1) - \sqrt{((4z_8 - 2kr)z_2 - 4\beta bkmz_1)^2 + 64\beta^2k^2b^3k_1z_1^2z_4}}{-32\beta^2bk} \\ x_2 = -\dfrac{(2\alpha^2y_2z_2 + 2\beta kmz_3 + \sigma^2kz_2)x_1 + bkk_1z_1z_3z_4}{(z_8 - kr)z_2 - \beta bkmz_1 - 8\beta^2bkx_1} \\ x_3 = \dfrac{-4\beta^2bkx_2^2 + 2(\alpha^2y_2z_2 + \beta kmz_3)x_2 + kk_1z_3^2z_4}{r} \end{cases} \tag{24}$$

$$\begin{cases} y_1 = \dfrac{(4\beta bkz_2z_{11} + 2kz_2^2z_{12}) - \sqrt{(4\beta bkz_2z_{11} + 2kz_2^2z_{12})^2 - 16\alpha^2z_2^2b^2kk_1z_{16}}}{8a^2z_2^2} \\ y_2 = \dfrac{(\beta kz_2(2z_{10} - bmz_1) + \sigma^2kz_2^2)y_1 + (b^2kk_1z_{17} + \beta bkz_{20} + (\gamma + c_r)bkk_1z_2z_3z_4)}{8\beta^2bkz_2x_1 - 2\alpha^2y_1z_2^2 + kz_{21}z_2^2} \\ y_3 = \dfrac{2\beta kz_2y_2z_{10} + \alpha^2y_2^2z_2^2 - \lambda kk_1m^2z_3^2 + 8\beta b\lambda kk_1z_9}{2kz_2^2r} \end{cases} \tag{25}$$

Where,

$$\begin{array}{l} B_7 = \Big(-Z_5 - \big({Z_5}^2 - 4(8\alpha^2b^3mZ_1 + 16\delta kz_{21}b^2{z_4}^2)Z_4\big)^{1/2}\Big)/(16\alpha^2b^3mZ_1 + 32\delta kz_{21}b^2{z_4}^2), \\ B_8 = \Big(-Z_5 + \big({Z_5}^2 - 4(8\alpha^2b^3mZ_1 + 16\delta kz_{21}b^2{z_4}^2)Z_4\big)^{1/2}\Big)/(16\alpha^2b^3mZ_1 + 32\delta kz_{21}b^2{z_4}^2)\ , \\ B_9 = \Big(2\alpha^2y_1z_2Z_8 - \big(2\alpha^2y_1z_2{Z_8}^2 - 48\alpha^4{y_1}^2{z_2}^2((Z_7 + \beta bmz_1)Z_6 - 4\beta^2b^3k_1z_4{z_1}^2)\big)^{1/2}\Big)/(2((Z_7 + \\ \beta bmz_1)Z_6 - 4\beta^2b^3k_1z_4{z_1}^2))\ ,\ B_{10} = \Big(2\alpha^2y_1z_2Z_8 + \big(2\alpha^2y_1z_2{Z_8}^2 - 48\alpha^4{y_1}^2{z_2}^2((Z_7 + \beta bmz_1)Z_6 - \\ 4\beta^2b^3k_1z_4{z_1}^2)\big)^{1/2}\Big)/(2((Z_7 + \beta bmz_1)Z_6 - 4\beta^2b^3k_1z_4{z_1}^2)),\ z_1 = c_n - \mu,\ z_2 = 4bk_1z_4 + m^2,\ z_3 = \\ bc_n - d,\ z_4 = \lambda - 1,\ z_5 = \gamma + c_r - \mu,\ z_6 = c_n - 1,\ z_7 = \lambda z_1 - \mu z_4,\ z_8 = 2\alpha^2y_1 - k\delta,\ z_9 = \\ -2\beta bx_2^2 + mz_3x_2,\ z_{10} = mz_3 - 4\beta bx_2,\ z_{11} = 8\beta x_1 + mz_1,\ z_{12} = 2\delta + r,\ z_{13} = 2z_2z_4z_5 + mz_1, \\ z_{14} = z_2z_5 - 4b\lambda k_1z_1,\ z_{16} = -mz_1z_{13} - 64\lambda\beta^2x_1^2 + 4\beta bkmx_1z_{14},\ z_{17} = -4\mu k_1z_3z_4^2 + m^2z_6z_7, \\ z_{18} = 2z_3x_1 - bz_1x_2,\ z_{19} = -32\beta b\lambda k_1x_1 + mz_2z_5,\ z_{20} = x_2z_{19} + 4k_1m\lambda z_{18},\ z_{21} = \delta + r,\ Z_1 = \\ z_1z_5{z_4}^2 - 2b\beta kx_1z_4z_5 + 2b\beta\lambda kx_1z_1\ ,\ Z_2 = 4\beta bx_1z_{21} + bmz_1z_{21} + 4\beta b\delta x_1\ ,\ Z_3 = 2z_4z_5m^3z_1 + \\ m^2{z_1}^2 + 64\lambda\beta^2{x_1}^2 - 4bk\beta x_1z_5m^3 + 16b^2km\beta x_1\lambda z_1\ Z_5 = 8\beta bkz_4Z_2 + 8bk\delta m^2z_4z_{21} + \\ \alpha^2b^2Z_3\ ,\ Z_4 = 64\beta^4b^2k{x_1}^2 + 16\beta^3b^2mkz_1x_1 + \delta kz_{21}m^4 + (8\beta^2bkx_1z_{12} + 2\beta bmkz_1z_{21})m^2\ ,\ Z_6 = \\ z_{21}z_2 - \beta bmz_1, Z_7 = z_2(3\delta + 2r), Z_8 = z_{21}z_2 + Z_7 + 2Z_6. \end{array}$$

5.3. TPR Bears the Impact of Different Proportions of the OEM's Product Recoverable Design Investment Costs

In this part, the evolution process of recovery rate, retail price, recovery effort level, product recyclable design level, and total profit of the closed-loop supply chain over time is interpreted under the centralized decision of TPR outsourcing. Also, under the coordination mechanism of the decentralized decision of TPR outsourcing, when TPR undertakes different proportions of OEM's investment cost of product recyclable design, Evolution process of recycling rate, wholesale price, retail price, recycling effort level, product recyclable design level, OEM profit and total profit of closed-loop supply chain over time. Where, when λ = 0, it is the decentralized decision of the closed-loop supply chain.

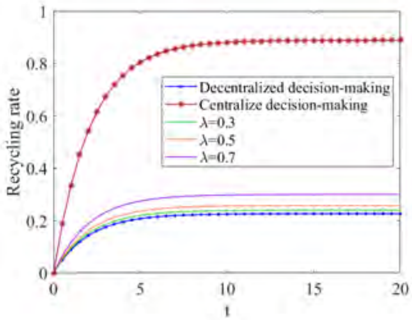
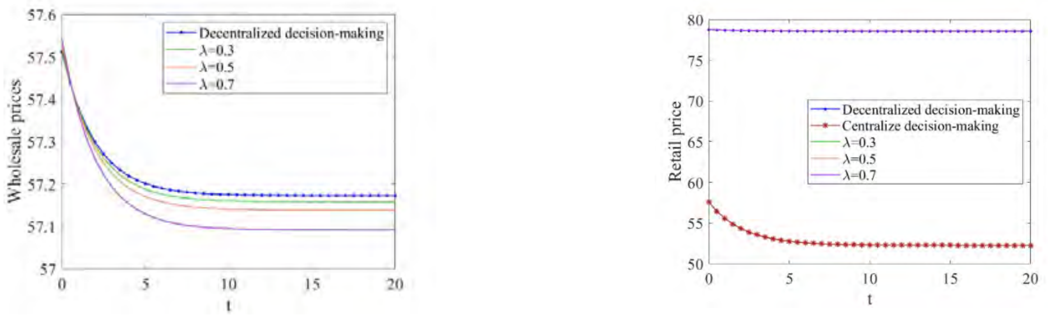


Figure 6. Effect of different bearing ratios on recovery rate

Figure 6 shows the evolution of the recovery rate, indicating that the expected recovery rate of waste products will gradually become stable over time. Fixed the outsourcing price, the recovery rate under centralized decision-making could become the highest, and the recovery rate under decentralized decision-making was the lowest. TPR undertaking a higher proportion of OEM product recoverable design investment cost led to a higher product recovery rate, and longer recovery rate to reach stability.

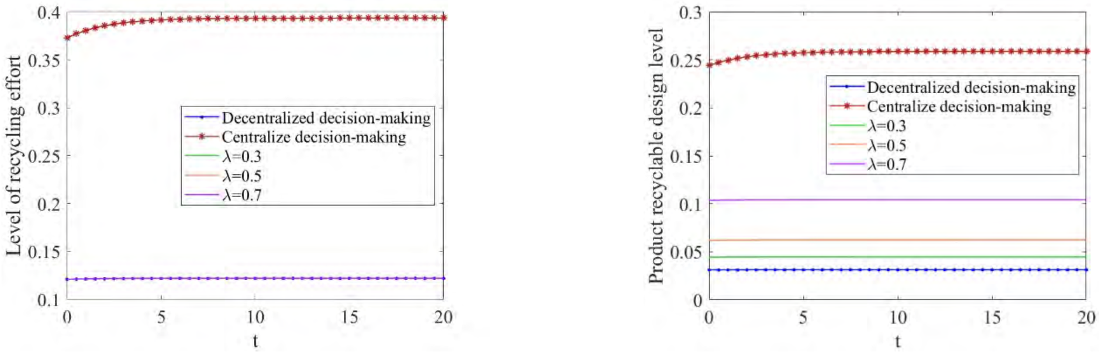


(a) (b)

Figure 7. (a) The impact of different commitment ratios on wholesale prices; (b) the impact of different commitment ratios on retail prices

Figure 7 (a) shows that the wholesale price of products will gradually become stable and lower than the initial value over time. Since no wholesale price decision variable in centralized decision-making, the evolution of wholesale price under decentralized decision-making and coordination mechanism is mainly analyzed. In the stable state, the wholesale price under the distributed decision is the highest, but the initial value of the wholesale price under the distributed decision is the lowest. When TPR undertakes a higher proportion of OEM product recoverable design investment cost, the higher the wholesale price of the product at the initial value, the lower the wholesale price in the stable state, and the longer it takes for the wholesale price to stabilize.

Figure 7 (b) shows that the retail price of a product will gradually become stable over time and lower than the initial value. Since a big difference between retail prices under decentralized decision-making and coordination mechanisms and those under centralized decision-making, the evolution process of retail prices under decentralized decision-making and coordination mechanisms is not obvious, which can be analyzed and compared according to the figure in the previous section. When other conditions are fixed, the retail price under centralized decision-making is the lowest, while the retail price under decentralized decision-making and coordination mechanisms with different bearing ratios is the same. Because of the decentralized model a particular case, the proportion of the recoverable design investment cost of OEM products borne by TPR is zero. Therefore, the ratio of product recoverable design investment cost borne by TPR to OEM does not affect the evolution process of retail price under the closed-loop supply chain model of coordination mechanism.

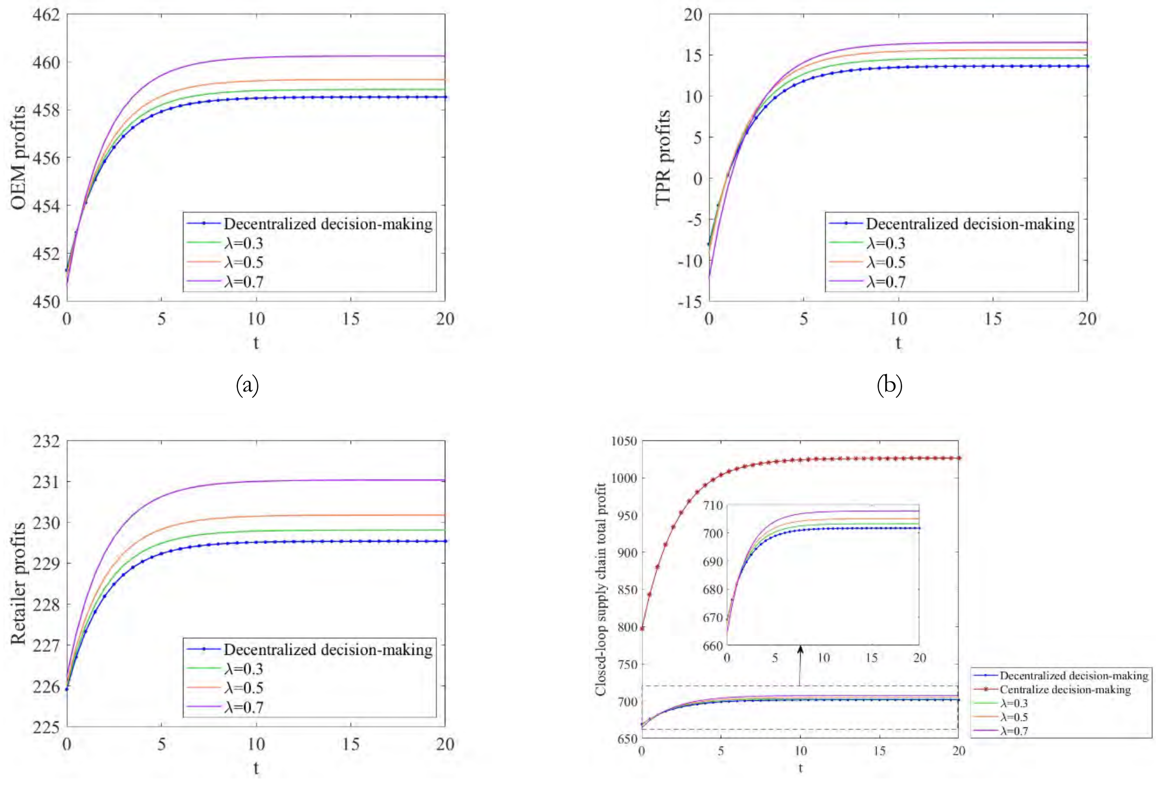


(a) (b)

Figure 8. (a) The impact of different commitment ratios on the level of recycling effort; (b) the impact of different commitment ratios on the level of product recyclable design

Figure 8 (a) shows the evolution process of the recovery effort level, indicating that the recovery effort level of the product will gradually become stable and higher than the initial value over time. Since the recovery effort level under decentralized decision-making and coordination mechanism is quite different from that under centralized decision-making, the evolution process of recovery effort level under decentralized decision-making and coordination mechanism could be more precise, which can be referred to in the previous figure. When other conditions are given, the recovery effort level is the highest under centralized decision-making. In contrast, the evolution process of recovery effort level under decentralized decision-making and coordination mechanisms with different commitment ratios is consistent. Therefore, the proportion of product recoverable design investment cost borne by TPR to OEM does not affect the evolution process of recovery effort level under the closed-loop supply chain model of coordination mechanism.

Figure 8 (b) shows the evolution process of the recyclable product design level, indicating that the product recyclable design level gradually becomes stable and higher than the initial value over time. As the product recyclable design level under decentralized decision-making and coordination mechanism differs significantly from that under centralized decision-making, and the evolution fluctuation of product recyclable design level under decentralized decision-making and coordination mechanism is slight, the evolution process of product recyclable design level under decentralized decision-making and coordination mechanism is not apparent, which can be referred to the previous figure. When other conditions are given, the level of product recyclable design under decentralized decision-making is the lowest, while the level of product recyclable design under centralized decision-making is the highest. The higher the proportion of product recyclable design investment cost that TPR undertakes for OEM, the higher the level of product recyclable design.



(a)

(b)

(c)

(d)

Figure 9. (a) The impact of different commitment ratios on OEM profits; (b) the impact of different commitment ratios on TPR profits; (c) The impact of different commitment ratios on retailers' profits; (d) The impact of different commitment ratios on the total profit of closed-loop supply chain.

Figure 9 (a) and (b) represent the evolution process of OEM profit and TPR profit, respectively, indicating that OEM profit and TPR profit will gradually become stable and higher than the initial value. The evolution process of

OEM profit and TPR profit is relatively consistent. The OEM and TPR profit under the distributed decision is the lowest in the stable state. When the proportion of recoverable design investment cost of OEM borne by TPR is higher in the stable state, the profit of OEM and TPR is higher, and the time for OEM profit and TPR profit to reach stability is longer. The initial value of OEM and TPR profits is the highest under distributed decision-making. The higher the proportion of TPR bearing the recoverable design investment cost of OEM products, the lower the initial value of OEM profit.

Figure 9 (c) shows the evolution process of retailer profit, indicating that retailer profit will gradually become stable and higher than the initial value over time. The retailer's profit is the lowest under distributed decision-making. The higher the proportion of recyclable design investment cost of OEM borne by TPR, the higher the retailer's profit, and the longer it takes for the retailer's profit to reach stability. In other words, when TPR undertakes a higher proportion of the recoverable design investment cost of OEM products, the lower the cost borne by OEM and the lower the wholesale price of products. As a result, the lower the retailer's cost, the higher the retailer's profit.

Figure 9 (d) represents the evolution process of the closed-loop supply chain's total profit, indicating that the closed-loop supply chain's total profit will gradually become stable and higher than the initial value over time. When it reaches a stable state, the total profit of the closed-loop supply chain under centralized decision-making is the highest. In contrast, the total profit of the closed-loop supply chain under decentralized decision-making and coordination mechanisms with different bearing ratios is not different. However, when the proportion of the recoverable design investment cost of OEM products borne by TPR is higher, the total profit of the closed-loop supply chain is higher, and the total profit of the closed-loop supply chain under decentralized decision-making is the lowest. Choosing the appropriate cost-bearing ratio is conducive to improving the profit of closed-loop supply chain members and the total profit. However, under the coordination mechanism, the profit-seeking behaviors of closed-loop supply chain members bring a double marginal effect, so the total profit of a closed-loop supply chain under centralized decision-making is the highest.

6. Conclusions and Implications

This paper studies the effect of product recyclable design on the member dynamic equilibrium strategy of the third-party remanufacturing closed-loop supply chain system. The Itô process describes the characteristics of the stochastic evolution of the recovery rate of the supply chain system. Based on the behavior of OEM investment in product recyclable design, the profit target function of OEM is constructed to pursue profit maximization. According to the composition of the retailer's profit, the profit target function of the retailer pursuing maximum profit is constructed. Based on the behavior of recycling and remanufacturing activities of TPR, the profit target function of TPR is constructed to pursue profit maximization. The optimal decision and profit evolution of a closed-loop supply chain under the centralized decision, decentralized decision, and coordination contract mechanism are discussed, respectively.

In decentralized decision-making, a stochastic differential game Stackelberg model is established for a closed-loop supply chain system with OEM as a leader, retailer, and TPR as a follower. Based on the stochastic differential game theory, retailers' strategies of retail price and TPR recovery effort level response are derived from decentralized decision-making, and the strategies of OEM wholesale price and product recovery design level are obtained. Based on the response function of the retailer and TPR and the strategy of OEM, the HJB partial differential equation, which should be satisfied by the optimal value function of OEM, TPR, and retailer, is constructed. The optimal value functions of retailers, TPR, and OEM are obtained by solving the partial differential equations. Then the dynamic retail price of retailers and the recycling effort equilibrium strategy of TPR, as well as the dynamic wholesale price and product recycling design level strategy of OEM, are obtained. To reveal the evolution characteristics of the stochastic recovery rate, the expected and variance characteristics of the stochastic recovery rate and the stability of the expected and variance of the stochastic recovery rate were analyzed.

Under the distributed decision, the influence of random fluctuation on the recovery rate and the influence of the initial recovery rate on the evolution are analyzed. The results show that, in expected terms, returns can increase or decrease over time. The expected rate of return converges to a stable state over time, regardless of the initial rate of return. The recovery rate always hovers around the expected value due to random perturbations. The influence of different outsourcing prices on the closed-loop supply chain system is analyzed. It can be concluded that the higher the outsourcing price, the higher the wholesale price, retail price, and TPR profit, but the lower the recovery rate, recycling effort level, product recyclable design level, OEM profit, retailer profit, and total closed-loop supply chain profit. Therefore, according to different needs, OEM and TPR must negotiate appropriate outsourcing prices to maximize profits.

Under centralized decision-making, only one decision maker simultaneously determines recyclable product design, retail price, and recycling efforts. Compared with distributed decision-making, the centralized mode has lower retail prices and more extensive collection efforts. Under decentralized decision-making, the profit-seeking behavior brings a double marginal effect, which increases the retail price and decreases the market demand. OEM and TPR expect each other to invest in recyclable product design and recycling efforts to free-ride to gain higher profits. Therefore, the investment motivation of each member of the supply chain will be weakened, and ultimately, the profit will decline. The supply chain performance under decentralized decision-making is significantly lower than that under centralized decision-making.

Under the coordination contract mechanism, based on the decentralized decision model, the cost-sharing contract is adopted to analyze the system. TPR shares a certain proportion of OEM products' recoverable design investment cost. Through numerical examples, TPR bears the influence of the different proportions of OEM product recoverable design investment costs on the closed-loop supply chain. The results show that The higher the proportion of TPR, the higher the recovery rate, product recyclable design level, OEM profit, TPR profit, retailer profit, and total closed-loop supply chain profit; the lower the wholesale price, the retail price and recovery effort level are not affected. The recovery rate of a closed-loop supply chain under a coordination contract mechanism is higher than that under decentralized decision-making but lower than that under centralized decision-making. Therefore, the appropriate sharing ratio of OEM and TPR members of the closed-loop supply chain is conducive to improving the recovery rate of the closed-loop supply chain system, reducing the wholesale price, and improving the total profit of the closed-loop supply chain.

Future research can be considered from the following points:

1. This paper assumes that OEM outsources remanufacturing activities to TPR, and it can be considered that OEM licenses remanufacturing activities patents to TPR's closed-loop supply chain system.
2. This paper takes new and remanufactured products as homogeneous products. It adopts the exact pricing, which can further discuss the optimal decision-making of a closed-loop supply chain when consumers have different payment intentions for the two products.
3. Considering the information asymmetry between OEM and TPR based on this paper, the two sides have fair preferences.

Declaration of Conflicting Interests

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author received no financial support for the research, authorship, and/or publication of this article.

Authors' contributions

The author contributed to all aspects of the study.

Data availability

Data included in the article itself or supplementary material

Use of Artificial Intelligence

Artificial intelligence (AI) tools were used solely for language editing and improvement of manuscript readability. The author takes full responsibility for the content, interpretation, and conclusions of this article.

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Appendix A. Proof.

1) The balanced recovery effort level control strategy A^* should be positive, namely: $A^* = \frac{\alpha(2e_1\theta+e_2)}{k} > 0, \forall\theta \in [0,1]$.

If $e_2 < 0$, when $\theta = 0$ or θ is slightly positive, A^* is negative. Since the equilibrium control strategy A^* is required to be positive, e_2 is also required to be positive, that is $2rkn_2 - 4\alpha^2 e_1 n_2 - 2n_{10} > 0$. According to Equation (13), the equation of e_1 is: $re_1 = \frac{4\alpha^2 n_2 e_1^2 + 4n_{10}e_1 + 2kb^2 k_1 n_1 n_6 + 4mf_1 n_8 n_6}{2kn_2}$.According to the above equation of e_1, two solutions can be obtained: $e_1 = \frac{-(4n_{10}-2kn_2 r) \pm \sqrt{(4n_{10}-2kn_2 r)^2 - 16\alpha^2 n_2(2kb^2 k_1 n_1 n_6 + 4mf_1 n_8 n_6)}}{8\alpha^2 n_2}$.

The following discussion is made since there are two solutions to the e_1 equation.

(a) When $e_1 = \left(-(4n_{10} - 2kn_2r) + \sqrt{(4n_{10} - 2kn_2r)^2 - 16\alpha^2 n_2(2kb^2k_1n_1n_6 + 4mf_1n_8n_6)}\right)/(8\alpha^2 n_2)$. From $2rkn_2 - 4\alpha^2 e_1 n_2 - 2n_{10} > 0$, we can infer that $B_2 < k_1 > B_1$, this is contrary to the theorem because the recovery effort level cost coefficient should not be small.

(b) When $e_1 = \left(-(4n_{10} - 2kn_2r) - \sqrt{(4n_{10} - 2kn_2r)^2 - 16\alpha^2 n_2(2kb^2k_1n_1n_6 + 4mf_1n_8n_6)}\right)/(8\alpha^2 n_2)$. From $2rkn_2 - 4\alpha^2 e_1 n_2 - 2n_{10} > 0$, we can infer that $k_1 > B_1$, this is in line with the reality that the cost factor of the recycling effort level will not be small. Otherwise, the recycling company will recycle all the waste products.

Therefore, we can do this by assuming that $k_1 > B_1$ to rule out larger roots. When $e_1 = \left(-(4n_{10} - 2kn_2r) - \sqrt{(4n_{10} - 2kn_2r)^2 - 16\alpha^2 n_2(2kb^2k_1n_1n_6 + 4mf_1n_8n_6)}\right)/(8\alpha^2 n_2)$, we can verify $e_2 > 0$ and $A^* > 0$, $\forall\theta \in [0,1]$.

$$B_1 = ((4k\delta m^2 A_3 + \alpha^2 bmn_6 A_2 + 2b\beta kA_1A_4) + bA_1\sqrt{A_6{}^2 + 8k\beta\alpha^2 n_6 A_5})/8(2kbA_3 - n_1n_6\alpha^2 b^2),$$
$$B_2 = ((4k\delta m^2 A_3 + \alpha^2 bmn_6 A_2 + 2b\beta kA_1A_4) - bA_1\sqrt{A_6{}^2 + 8k\beta\alpha^2 n_6 A_5})/8(2kbA_3 - n_1n_6\alpha^2 b^2).$$
$$A_1 = mn_1 + 8\beta f_1, A_2 = -mn_1 + 8\beta f_1, A_3 = \delta + r, A_4 = 2\delta + r, A_5 = m\delta + b\beta n_1, A_6 = 2kr\beta + \alpha^2 mn_6.$$

2) The level control strategy h^* of balanced product recyclable design should be positive, that is: $h^* = \frac{4\beta b(2f_1\theta + f_2) + m(d - b(c_n(1-\theta) + \theta\mu))}{-m^2 + 4bk_1} > 0, \forall\theta \in [0,1]$.

Suppose it is true that $f_2 < 0$, h^* is negative when $\theta = 0$ or θ is slightly positive. Since the equilibrium control strategy h^* is required to be positive, f_2 is also required to be positive, namely $kn_2r - n_9 - 8n_7f_1 > 0$. According to Equation (12), the equation for f_1 is: $rf_1 = \frac{16n_7f_1^2 + 4n_9f_1 + kk_1b^2n_1^2}{2kn_2}$. According to the above equation of f_1, two solutions can be obtained: $f_1 = \frac{-(4n_9 - 2kn_2r) \pm \sqrt{(4n_9 - 2kn_2r)^2 - 64n_7kk_1b^2n_1^2}}{32n_7}$.

The following discussion is made since there are two solutions to the f_1 equation.

(a) When $f_1 = (-(4n_9 - 2kn_2r) + \sqrt{(4n_9 - 2kn_2r)^2 - 64n_7kk_1b^2n_1^2})/32n_7$, from $kn_2r - n_9 - 8n_7f_1 > 0$. We can infer that $B_4 < k < B_3$, contrary to the theorem, because the product recyclable design level cost factor should not be small.

(b) When $f_1 = (-(4n_9 - 2kn_2r) - \sqrt{(4n_9 - 2kn_2r)^2 - 64n_7kk_1b^2n_1^2})/32n_7$, from $kn_2r - n_9 - 8n_7f_1 > 0$. We can infer that $k > B_3$, this aligns with the reality that the level cost coefficient of recyclable product design will not be small. Otherwise, the recycling company will recycle all waste products.

$$B_3 = (\alpha^2 e_1(m^2A_4 + 2b(m\beta n_1 - 2k_1A_4)) + \alpha^2 e_1\sqrt{16\beta^2 b^3 k_1 n_1{}^2 + r^2 n_2{}^2})/(A_5{}^2 - 4b\delta k_1A_3 + rmA_5), B_4 = (\alpha^2 e_1(m^2A_4 + 2b(m\beta n_1 - 2k_1A_4)) - \alpha^2 e_1\sqrt{16\beta^2 b^3 k_1 n_1{}^2 + r^2 n_2{}^2})/(A_5{}^2 - 4b\delta k_1A_3 + rmA_5).$$