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Analysis of Optimization Models Under Different Approaches to Deal with Uncertainty Regarding Pre-Disaster Planning in Food Bank Supply Chains

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Abstract:

Purpose: Pre-positioning is a crucial choice in pre-disaster humanitarian logistics planning that consists of deciding in advance how much aid and where should it be located to enable effective and prompt operations in the case of an emergency. To support managers making such decisions, we propose four mathematical formulations that, considering the uncertainty on the demand to satisfy, seek to optimize aid prepositioning (before the event) and further distribution (after the event) in order to minimize unmet demand (MUD). The purpose of this paper is to evaluate and compare the performance of these formulations on a real case to discuss when and why should each approach be applied.

Design/methodology/approach: The two first formulations adopt the cardinality-constrained (CC) approach to handle uncertainty. These formulations differ in their objective functions, the first formulation's objective seeks to MUD, whilst the second incorporates equity in the way that demand is satisfied. The two remaining formulations are scenario-based (SB) and as in the previous two formulations, seek to MUD with and without equity considerations, respectively.

Findings: Applying our formulations to a case study, we compare the differences between the solutions produced by the proposed formulations and the solutions that would have been produced without uncertainty (perfect information) to have a better understanding of their performance and their behavior. A discussion of the strengths and weaknesses of each model is provided to help managers choose the model that best suits their needs.

Originality/value: The formulations are applied to a case study where a food bank is faced with the arrival of a hurricane in Mexico. As far as our knowledge, it is the first work in literature to deal with humanitarian logistics under a cardinality-constrained approach.

Keywords: demand uncertainty, cardinality-constraints, optimization models, humanitarian logistics, minimize unmet demand, equity

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1. Introduction

Natural catastrophes (including hurricanes, earthquakes, droughts, and floods) have affected countries all across the world more and more in the past years, and predictions indicate that this will continue increasing (EM-DAT - Emergency Events Database, 2011). The problems caused by natural disasters are countless, and humanitarian organizations have the responsibility to deal with these problems. Given the urgency, unpredictability, and complexity of these issues in the global supply chain that is driven by humanitarian entities, improvements in supply chain management and logistics have a direct impact on the capacity of humanitarian organizations to respond to disasters and enhance their overall effectiveness. (Leiras, de Brito Jr, Queiroz-Peres, Rejane-Bertazzo & Tsugunobu-Yoshida-Yoshizaki, 2014).

The process of planning, executing, and overseeing the efficient transportation and storage of goods and commodities, together with related information, from the point of origin to the place of consumption in order to meet the needs of the recipients is known as humanitarian logistics (Thomas & Mizushima, 2005). There are several issues that separate humanitarian logistics challenges from those of business logistics. Some of these challenges are demand's unpredictable nature with regard to its time, location, nature, and scale; the abrupt appearance of demand in significant quantities following a disaster; risks related to delivery punctuality; and a shortage of supply, personnel, technology, transportation capacity, and financial resources (Kovács & Spens, 2009).

One of the most important challenges in humanitarian logistics is uncertainty. The unpredictable nature of disasters makes the planning and execution of relief operations a complex and dynamic process. Indeed, uncertainty is inherent in most humanitarian contexts and cannot be disregarded since it could significantly affect the viability and quality of the problem solution. (Carello & Lanzarone, 2014). Failure to anticipate and manage uncertainty can lead to delayed response times or inadequate resource allocation. This, in turn, can result in an insufficient response to the needs of affected populations. Uncertainty can arise from many sources, including changes in the demand, fluctuations in supply available, and disruptions to transportation roads. There are various approaches to deal with this uncertainty.

This paper explores the use of the CC approach to deal with uncertainty in the context of resource prepositioning and allocation during the prelude to a natural disaster and compares it to the well-known SB approach. CC limit the number of elements in a specific set, thus they can be considered as structural constraints since they limit the possible structure of the database (Thalheim, 1992). The CC approach can provide valuable insights into the trade-offs and risks associated with different decisions and help decision-makers to allocate resources effectively in the face of uncertainty. Two versions of this approach are formulated, one seeking the minimization of the unsatisfied demand, and another striving for an equitable distribution of aid. The approaches are applied to a case study inspired by Hurricane Odile and the results are reported and analyzed to assess the value of the CC approach, comparing results against the SB models.

The remainder of this paper is organized as follows: A brief assessment of related works' literature is provided in Section 2, focusing on uncertainty in humanitarian logistics and applications of the CC approach in optimization models. Section 3 presents the problem description and proposes four mathematical formulations. Section 4 presents a case study. Sections 5 and 6 document the numerical results produced by the various models, and additional experiments where the case study is solved under perfect information to better assess these results. Finally, the paper is concluded with recommendations for further research and conclusions.

2. Related Work

The aim of this section is twofold. It first presents papers that have dealt explicitly with uncertainty in the context of food bank supply chains. Then, it reviews works devoted to the CC approach and their application to diverse optimization problems.

2.1. Uncertainty in Humanitarian Logistics

One of the main obstacles for food banks making operational decisions is the unpredictability of donations and demand. (Rivera, Smith & Ruiz, 2023). One of the primary problems when using optimization models to construct a humanitarian relief chain is uncertainty in the necessary data. In large-scale emergencies in particular, data could not be readily available or easily communicated. (Tofighi, Torabi & Mansouri, 2016).

Different approaches to deal with uncertainty in humanitarian logistics can be found in the literature. They differ in the way to model uncertainty, the way to manage it, and the parameters considered to be uncertain (Liberatore, Pizarro, de Blas, Ortuño & Vitoriano, 2013).

Parameters considered to be uncertain may affect different aspects of the problem. Some related works consider uncertainty in parameters such as uncertain road conditions, which is represented by road availability (Gao, Jin, Zheng & Cui, 2021), time needed to transfer injured people to other destinations (Hoseininezhad, Makui & Tavakkoli-Moghaddam, 2021), storage capacity of storage and distribution centers (Vahdani, Veysmoradi, Noori & Mansour, 2018), and hurricane's effects on each facility's supply levels, amount of donations received, and anticipated demand for the area that it serves (Marthak, Pérez & Méndez-Mediavilla, 2021).

Besides the parameters considered, uncertainty can be modeled by different approaches. One of the most common approaches involves defining probabilistic scenarios. Mete and Zabinsky (2010), and Chan, Kumar and Choy (2007) model uncertainty through scenarios that define distinct parameters such as the amount of demand, the location of affected areas, network routing reliability, and a particular mode of transportation. Mahtab, Azeem, Ali, Paul and Fathollahi-Fard (2022) present a model that maximizes the prepositioning of relief distribution points and the fortification of road segments to guarantee that the greatest number of affected individuals may effectively receive assistance.

Instead of the use of scenarios, another approach is to directly consider uncertainty in different parameters under fuzzy approaches. Fuzzy set theory considers some elements that are essential for dealing with economic, social and technological situations: the uncertainty in data, and the modeler or manager capacity to provide additional information (Canós & Liern, 2008). This method, called fuzzy logic, permits the processing of several different truth values using a single variable. It looks for solutions to issues with imperfect data and heuristics so that precise conclusions can be reached. The fuzzy logic does not add difficulty to traditional mathematics, and it is closer to human thought. Fuzzy Theory allows avoiding the requirements of rigidity which could do a model not to make sense and it provides us with ignoring solutions that could be useful (Canós, Casasús, Crespo, Lara & Pérez, 2011). Shaw, Das and Roy (2022) proposes a mixed-integer non-linear mathematical model related to resource management. Uncertainty is considered in multiple parameters, such as in the model presented by Sheikholeslami and Zarrinpoor (2023), where their method accounts for three sources of uncertainty: demand, costs, and the covering area of facilities. For a detailed explanation of the mathematics supporting Fuzzy Theory, we refer the interested reader to Canós et al. (2011).

Nevertheless, most of contributions to the field of humanitarian logistics research have been on demand uncertainty. Seraji, Tavakkoli-Moghaddam, Asian and Kaur (2022) examine the effects of varying levels of demand uncertainty on the combined emergency response and humanitarian logistics problems. The model is solved for each scenario, with three levels of demand uncertainty at 0.25, 0.5, and 0.75. Balcik, Iravani and Smilowitz (2014) establish two goals: avoiding waste and maximizing equity. They provided empirical evidence that addressing the issue in order to minimize waste results in almost negligible waste and fair distribution of food. Guijarro, Babiloni, Canós-Darós, Canós-Darós and Estellés (2020) applies fuzzy set techniques for the calculation of the on-hand stock levels at order delivery in the lost sales context, based on the uncertainty that real demand introduces. For a deeper analysis of how authors in literature have dealt with uncertainty, refer to Liberatore et al. (2013), Rahman, Majchrzak and Comes (2019), Hezam and Nayeem (2020), and Dönmez, Kara, Karsu and Saldanha-da-Gama (2021).

To the best of our knowledge, uncertainty regarding humanitarian logistics has never been modeled before using a CC approach. This approach has been used successfully in different optimization models to deal with uncertainty. Its advantages are pointed out in the next section, justifying the reason why we considered using this approach for a humanitarian logistics problem.

2.2. CC Approach in Optimization Models

The quantity of items in a mathematical set is referred to as its cardinality. Consequently, a constraint that limits the number of members in a set is known as a cardinality constraint (Liddle, Embley & Woodfield, 1993). Cardinality constraints provide a structured and explicit way to handle uncertainty. However, their effectiveness depends on the specific problem and the appropriate selection of the constraint parameters.

As explained by Bertsimas and Sim (2004), for each constraint, cardinality provides complete control over the level of conservatism. This method ensures that the solution is practicable if at most Γ_i uncertain coefficients change, protecting against deterministically violations of constraint i when only a predetermined number Γ_i of the coefficients change.

Some advantages of CC approach, as mentioned by Addis, Carello, Grosso, Lanzarone, Mattia & Tànfani **(2015)** are:

a) The approach assumes that parameters reside on an interval and that a subset of them take the maximum value rather than the nominal one. This results in a fairly simple geometry for the convex sets. This implies that knowledge of the probability density functions of the uncertain parameters is only partially necessary.

b) It does not rely on a detailed description of the uncertainty, making it less vulnerable to estimation errors in the data and associated probability distributions.

c) It grants a robust solution with a manageable amount of computing work.

Other advantages of this approach are:

d) Flexibility in specifying the required number of elements or relationships. By defining ranges instead of fixed values, the models can adjust uncertainty and adapt to changing situations. This flexibility enables the system to handle a wide range of scenarios effectively.

e) Provides valuable decision support and risk management in uncertain situations. By explicitly modeling and enforcing constraints, they help to identify risks associated with different cardinality scenarios. This can lead to more informed decision-making and better mitigation strategies in the face of uncertainty.

We have performed a literature search form 2010 up the end of 2023, on the databases *Scopus and Web of Science*, with the objective of identifying the applications of CC in optimization models. Based on our search, there are no scientific contributions reporting the application of CC to a humanitarian logistic problem. Even so, this approach has been well used in various applications in optimization models to deal with uncertainty.

Most of the work regarding CC in the literature is related to portfolio optimization and knapsack problems, although the method has also been applied to supply chain design and healthcare. Ağralı, Geunes and Taşkın (2012) offers a general model that uses a cardinality constraint on the number of supply facilities that may serve a client to take into account a supply chain situation in which many disabled facilities provide a single product to a set of clients. Carello and Lanzarone (2014) creates a CC robust assignment model that addresses the nurse-to-patient assignment problem in home care services by utilizing the capabilities of a mathematical programming model without the need to create scenarios. Zhang (2019) provides a multiperiod mean absolute deviation uncertain portfolio selection model that takes into account cardinality limitations, transaction costs, and risk management to assist investors achieve both optimal return and effective risk management.

Table 1 summarizes our findings. The first column (Title) corresponds to the title of the paper, the second column (Distribution) classifies whether distribution is considered continuous or discrete, the third column (Stochastic variable) relates the variables where uncertainty is considered, and the last column (Type of problem) identifies the type of problem that is addressed.

For a more detailed description of the mathematical application of CC, refer to Liddle et al. (1993) and Bertsimas and Sim (2004).

3. Problem Description and Formulations

This section presents an optimization model to assign the available relief assets among a set of storage centers (food banks) that will further distribute them to municipalities in such a way that the limited budget is respected. More precisely, the model is employed to identify the nodes that ought to serve as storage centers and the capacity of each storage center's flow for handling donations (food). Notice that storage centers are used to consolidate and prepare aid before distribution rather than as long-term storage facilities. Nonetheless, they must have enough logistic capacity to receive the incoming flow of aid, as well as handle, consolidate, and prepare aid to be delivered. For the sake of simplicity, we will refer to this ability to handle logistic flow as capacity. The model's objective is to minimize the maximum proportion of missing units among the affected nodes by transporting donations from storage centers to them. However, this model will be further extended and modified to 1) guarantee a fair distribution of the relief (equity), and 2) deal with uncertainty under different approaches.

Reference	Dist	Stochastic variable	Type of problem
Gibson, Ohlmann and Fry (2010)	D	Availability of items in the future	Knapsack
Boyko, Turko, Boginski, Jeffcoat, Uryasev, Zrazhevsky et al. (2011)	D	Number of sites and number of sensors	Multi-sensor scheduling
Ağralı et al. (2012)	C	Demand	Facility location (Serving customers)
Fogarasi and Levendovszky (2013)	C	Uncertainty around the mean value	Portfolio Optimization
Roman, Mitra and Zverovich (2013)	C	Stochastic dominance	Portfolio Optimization
Scozzari, Tardella, Paterlini and Krink (2013)	C	Stochastic search heuristics	Index Tracking
Carello and Lanzarone (2014)	C	Demand	Home care provider
Addis et al. (2015)	C	1. Duration of surgery 2. Demand	1. Operating room planning 2. Nurse-to-patient assignment
Abdi and Fukasawa (2016)	D	Stochastic right-hand-sides	Knapsack
Zhang (2019)	D	Expected value and Absolute deviation	Portfolio Optimization

Table 1. Applications of CC in optimization models

When addressing humanitarian logistics problems, it is essential to consider some parameters and variables that influence the efficiency and effectiveness of relief operations. Rivera, Smith, Ogazon and Ruiz (2023) synthetize the parameters, decisions, and objective functions commonly considered in literature regarding humanitarian logistics.

For instance, the transportation cost per weight, the cost of handling donations, and the cost of enabling storage centers are crucial for determining the overall logistics cost. Additionally, the total donations available and the capacity in of storage centers dictate the resource limits and distribution potential. The demand in the affected nodes represents the need that must be met, while the limit on the number of storage centers that can be opened ensures that the logistics plan remains manageable and realistic.

Decision variables are critical for the strategic planning of humanitarian logistics. One such variable is whether a node is used as a storage center, usually represented as a binary variable that takes the value 1 if the node is enabled and 0 otherwise. Another decision variable is the number of units stored in each storage center, which directly impacts how resources are allocated and managed across the network. The shortage in demand nodes is a critical variable indicating the unmet demand, which humanitarian efforts aim to minimize. Additionally, the units sent from storage centers to demand nodes represent the flow of resources through the network, reflecting how supplies are distributed. Optimizing these intermediate variables is essential for creating a responsive and efficient logistics system that can adapt to changing conditions and maximize the impact of the available resources.

By carefully considering these parameters and variables, humanitarian logistics can be significantly improved, ensuring timely and effective aid delivery to those in need. These decisions need to be optimized to ensure that the logistics operations are both cost-effective and capable of meeting the demands at various nodes while adhering to the given constraints.

Before proceeding, the following assumptions are established:

- The budget, reaction activities, and readiness strategy are all under the authority of a single decision-maker.
- A node is contemplated as a geographical territory (e.g., a municipality or a town) in which a demand for resources can occur.
- Transportation routes will be selected to satisfy cost constraints.
- It is assumed that all the food to distribute is already available before distribution, limited by donations and capacity constraints.

The distribution within the network is carried out considering point-to-point routes, starting at origin nodes (storage centers) and ending at the affected demand zones (municipalities).

Different objectives are considered in the literature related to relief distribution and food bank operations. Gralla, Goentzel and Fine (2014) states that food banks are concerned in endorsing two main goals: (1) effectiveness, which means providing as much assistance as possible to those in need incurring the lowest costs or resource usage, and (2) equity, that is, seeking an allocation of relief that is fair (equitable) with respect to the needs of each municipality (Eisenhandler & Tzur, 2019). When effectiveness is pursued, the main goal is to distribute as much food as possible to individuals in need by minimizing costs, response time, or unmet demand. This objective aims to maximize the overall impact of the aid, for example by targeting the most critical needs or by maximizing the reach of the aid. In this case, distribution is done optimally but without taking into account humanitarian considerations such as equity, suffering, etc.

On the other hand, equity is about distributing or using resources in a way that ensures victims have equal access to relief assistance. (Gutjahr & Nolz, 2016). Different methods have been proposed to consider equity, such as the measures of the mean absolute deviation, min-max, coverage, and the Gini coefficient (Zhu, Gong, Xu & Gu, 2019). Rivera, Smith and Ruiz (2023) addressed equity by minimizing the maximum amount of unfulfilled relief needs, while Ransikarbum and Mason (2016) suggested a max-min strategy, which aims to maximize the lowest proportion of satisfied demand in the distribution of relief. Velasquez, Mayorga and Cruz (2019) considered equity similarly but defines acceptable inequality as the bounds between the maximum and minimum fraction of prepositioned demand among all locations. Liu, Zhang and Zhang (2021) addressed equity with coverage distance constraints while Erbeyoğlu and Bilge (2020) used coverage time constraints. Mostajabdaveh, Gutjahr and Sibel-Salman (2019) included in the objective the Gini mean absolute difference of distances and Zhang, Liu, Yu and Shen (2021) minimize the absolute difference in shortage in the objective.

The choice between an equity and a non-equity objective in humanitarian logistics can also have important ethical and practical implications. An equity objective may lead to suboptimal outcomes such as inefficiencies in the allocation of resources or a failure to address the most critical needs. A non-equity objective may result in more efficient outcomes, but it may also lead to an unfair distribution of aid and to unequal outcomes for different beneficiaries. The diversity of perspectives and preferences of humanitarian organizations motivated us to consider both objectives. In the following, we will propose an optimization model considering effectiveness by minimizing unmet demand (MUD), which will later be modified into an equity objective based on minimizing the maximum proportion of relief needs. Both objectives will be approached considering uncertainty in demand with a CC approach and SB approach.

3.1. Deterministic Formulations Minimizing the Unmet Demand Without and with Equity

Several sets need to be defined. The set of nodes B reflects the available storage facilities. The set of demand nodes D symbolizes the municipalities impacted by the disaster. The set of arcs A represent the arcs connecting storage centers B with demand nodes D. As per the model parameters, c_{ba} c_{bd} is the transportation cost per weight (in tons) sent from node b to node d and covers the cost from fuel, road fees, etc. Parameter e_b is the cost of handling a unit in in node b and includes the cost of inventory management. Parameter Q is the total amount of donations that are available for distribution. Parameter K_b is the capacity at node b. Parameter F_d represents the demand at node d, or in other words, the total amount of aid that is required at that node. Parameter BC_b is the cost of enabling node b as a storage center and includes the cost of rent, storage, etc. at that place. Parameter P sets the available budget (all monetary units are in Mexican pesos). Finally, parameter ϱ limits the number of storage centers that can be enabled. Finally, let us now introduce the following variables. Variable x_b takes the value 1 if node b is used as a storage center, and zero otherwise. Variable q_b represent the total units that will be stored (*tons*) in node b. Variable r_{bd} represent the units sent from node b to node d, and variable y_d (MUD) represents the shortage in node d.

Notation

Sets

B	nodes with storage capacity
D	demand nodes
A	set of arcs within B and D

Parameters

c_{bd}	transportation cost per weight (*tons*) sent from node b to node d
e_b	cost of handling a unit in node b
Q	total donations available
K_b	capacity in node b
F_d	demand in node d
BC_b	cost of enabling node b as a storage center
P	total budget
ϱ	limit of storage centers that can be opened

Decision variables

x_b	takes value 1 if node b is used as a storage center and 0 otherwise
q_b	units stored in node b (*tons*)

Intermediate variables

y_d	shortage in node d (*tons*)
r_{bd}	units sent from node b to node d (*tons*)

The formulation is structured as follows:

$$minimize \qquad \sum_{d \in D} y_d \tag{1}$$

Subject to:

$$\sum_{b \in B} \sum_{d \in D} r_{bd} - \sum_{b \in B} q_b \leq 0 \tag{2}$$

$$q_b - x_b K_b \leq 0 \qquad \forall b \in B \tag{3}$$

$$\sum_{b\in B} q_b - Q \le 0 \tag{4}$$

$$\sum_{d\in D} F_d - \sum_{b\in B}\sum_{d\in D} r_{bd} \le \sum_{d\in D} y_d \tag{5}$$

$$\sum_{b\in B}\sum_{d\in D} c_{bd} r_{bd} + \sum_{b\in B} BC_b x_b + \sum_{b\in B} q_b e_b \le P \tag{6}$$

$$\sum_{b\in B} x_b = \rho \tag{7}$$

$$x_b \in \{0,1\} \qquad \forall b \in B \tag{8}$$

$$q_b, y_d \ge 0 \qquad \forall d \in D \tag{9}$$

$$r_{bd} \ge 0 \qquad \forall b \in B, d \in D \tag{10}$$

The objective function (1) seeks to MUD among all areas. Constraints (2) ensure that the products sent from a storage center to demand nodes cannot be greater than the units stored in storage centers B. Constraints (3) guarantee that the units stored in node b are less or equal to the maximum capacity of storage center B as long as it is available, otherwise it is 0. Constraints (4) safeguards that the total units assigned to every storage center B are less or equal to the total donations available. Constraints (5) determine the missing units throughout all locations as a result of the discrepancy between the quantity of units received in that node and the demand. Constraint (6) calculates the cost of transportation and storage centers establishment and restricts it to a budget P. Constraints (7) limits the number of available banks to ϱ. Finally, the declaration of binary and integer variables is presented (8-10).

To adapt this model to seek the minimization of unmet demand with equity, we must change objective function (1) and constraints (5) of the deterministic model so that we will minimize the largest proportion of missing resources. To this end, a new variable z representing the largest proportion of missing resources among all areas is defined, so that objective function (1) becomes:

$$\textit{minimize } z \tag{11}$$

and constraints (5) and (9) will be modified as:

$$\frac{F_d - \sum_{b\in B} r_{bd}}{F_d} \le z \qquad \forall d \in D \tag{12}$$

$$z \ge 0 \tag{13}$$

Now we have our deterministic model under two different objectives, which will be modified to consider uncertainty in demand under a CC approach.

3.2. Approaches to Deal with Uncertainty

In many humanitarian logistics problems, the focus is not just on the allocation of resources but also on the number of resources that must be assigned. Due to the uncertainty surrounding these types of problems, it is difficult to decide the amount that must be assigned to demand nodes, since demand can vary significantly. The CC approach allows for the modeling of this type of constraint, making it an ideal tool for addressing these types of problems.

For our problem, we will introduce parameter Γ, which will limit the number of nodes affected by the disaster. The cardinality approach assumes that Γ nodes can be affected by the disaster. Our model tries to identify the worst combination of Γ demand nodes and finds an optimal solution for such a case. Parameter Γ allows us to approach uncertainty over the number of affected areas without necessarily knowing which, but guaranteeing that for Γ affected nodes, there will not be a worse scenario than the one solved.

The next subsections explain how the CC approach founded on the work of Bertsimas and Sim (2004) is applied to the two previous formulations to handle uncertainty in demand.

3.2.1. CC Robust Formulations

We first present a robust counterpart of the deterministic model under the objective of MUD, together with the additional parameters and decision variables. We consider demands F_d as uncertain, according to the CC formulation. Indeed, all uncertain demands $\widetilde{F_d}\widetilde{F_d}$ in the demand matrix $F = \{\widetilde{F_d} | d \in D\}$ $F = \{\widetilde{F_d} | d \in D\}$ are assumed to be independent random variables, each one characterized by a nominal value $\overline{F_d}\overline{F_d}$ and a maximum variation $\widehat{F_d}\widehat{F_d}$, i.e., $\widetilde{F_d}$ $\widetilde{F_d} \in [\overline{F_d} - \widehat{F_d}, \overline{F_d} + \widehat{F_d}][\overline{F_d} - \widehat{F_d}, \overline{F_d} + \widehat{F_d}]$. Moreover, the probability distribution of each F_d is assumed to be symmetric around the nominal value $\overline{F_d}\overline{F_d}$.

We apply the CC approach in all parts of the model where parameters F_d appear, i.e., constraints (5). First, we rewrite constraints (5) in an equivalent way. For this purpose, we define a new non-negative continuous variable τ, and we replace (5) with:

$$\sum_{d \in D} \tau_d = \sum_{d \in D} F_d - \sum_{b \in B} \sum_{d \in D} r_{bd} \tag{14}$$

and

$$\tau_d \leq y_d \qquad \forall d \in D \tag{15}$$

Then we apply the CC approach to constraints (14). In our formulation, we uniquely define the standardized deviation of each demand $\widetilde{F_d}\widetilde{F_d}$ from its nominal value $\overline{F_d}\overline{F_d}$ as:

$$a_d = \frac{\widetilde{F_d} - \overline{F_d}}{\widehat{F_d}} \qquad \forall d \in D \tag{16}$$

With $a_d \in [-1,1]$, were $a_d = 0$ means that demand $\widetilde{F_d}\widetilde{F_d}$ takes its nominal value $\overline{F_d}\overline{F_d}$, while $a_d = 1$ and $a_d = -1$ mean that $\widetilde{F_d}\widetilde{F_d}$ takes its maximum and minimum value, respectively. Accordingly, (14) is rewritten as:

$$\sum_{d \in D} \tau_d = \sum_{d \in D} (F_d + a_d \widehat{F_d}) - \sum_{b \in B} r_{bd} \qquad \forall d \in D \tag{17}$$

where, in the most conservative case, $a_d = 1\forall d$. However, it is improbable that every demand coefficient will take on its worst value at the same time. Thus, exploiting the CC idea, we limit the number of demands that assume the worst value $\overline{F_d} + \widehat{F}_d\overline{F_d} + \widehat{F_d}$, while at the same time the others remain at their nominal value $\overline{F}_d\overline{F_d}$. In other words, we limit the number of demands that simultaneously go to their maximum value for each node $d \in D$ by a cardinality parameter Γ which controls the level of robustness of the solution against the cost of the solution. Low values of Γ do not to penalize the objective function but generate solutions that can easily become infeasible. On the contrary, as Γ is increased towards |D|, more conservative solutions are produced. We now obtain the maximum value of the following problem to obtain our worst case considering the values of the variations in the demand:

$$\max \sum_{d \in D} \left((F_d + a_d \widehat{F_d}) - \sum_{b \in B} r_{bd} \right) \tag{18}$$

Subject to:

$$\sum_{d \epsilon D} a_d \leq \Gamma \tag{19}$$

$$a_d \in \{0,1\} \qquad \forall d \in D \tag{20}$$

Due to the structure of the equations of our deterministic model, and the sub-problem obtained with the CC approach, we can apply the duality theorem (Bertsimas & Sim, 2004) to solve both problems simultaneously. By strong duality the (18)-(20) problem can be rewritten as a minimization problem to align with the objective of problem (1)-(10). To this end, the following parameters must be introduced:

- $\widehat{F_d}$ maximum variation in demand F_d
- Γ cardinality, or number of demand nodes for which demand reaches their highest (worst) demand

And the following variables:

- u dual variable
- π_d dual variable to determine the affected nodes
- τ_d substitute variable for y_d to use the cardinality approach
- a_d if takes value 1 if demand node d is affected, and 0 otherwise

So that problem (18)-(20) becomes:

$$\min_{d \in D} \ \Gamma u_1 + \sum_{d \epsilon D} \pi_d \tag{21}$$

Subject to:

$$u_1 + \pi_d \geq \widehat{F_d} \qquad \forall d \in D \tag{22}$$

$$u_1 \geq 0 \tag{23}$$

$$\pi_d \geq 0 \qquad \forall d \in D \tag{24}$$

Now, we must substitute the equations obtained using the duality theorem into the main problem. First, constraints (5) will be modified as:

$$\sum_{d\in D} F_d + \Gamma u + \sum_{d\in D} \pi_d - \sum_{b\in B}\sum_{d\in D} r_{bd} = \sum_{d\in D} \tau_d \tag{25}$$

And the following constraints will be added:

$$\tau_d \leq y_d \qquad \forall d \in D \tag{26}$$

$$u + \pi_d \geq \widehat{F_d} \qquad \forall d \in D \tag{27}$$

$$(F_d + a_d\widehat{F_d}) - \sum_{b\in B} r_{bd} \leq \tau_d \qquad \forall d \in D \tag{28}$$

$$\pi_d - a_d \geq 0 \qquad \forall d \in D \tag{29}$$

$$\sum_{d\in D} a_d = \Gamma \tag{30}$$

$$a_d \in \{0,1\} \qquad \forall d \in D \tag{31}$$

$$u \geq 0 \tag{32}$$

$$\pi_d, \tau_d, \geq 0 \qquad \forall d \in D \tag{33}$$

$$\theta_b, \tau_b \geq 0 \qquad \forall d \in D \tag{34}$$

The complete formulation for the CC model MUD is given in Appendix B.

Now we consider the adaptation of the deterministic model that minimizes the equity objective given by Equations (11)-(13), (2)-(4), (6)-(8), and (10). As in the previous case, we consider demands F_d as uncertain, and we apply the CC approach in all parts of the model where parameters F_d appear, i.e., constraints (12). First, we rewrite constraints (12) in an equivalent way. For this purpose, we define a new non-negative continuous variable τ, and we replace (12) with:

$$\tau_d = \frac{F_d - \sum_{b\in B} r_{bd}}{F_d} \qquad \forall d \in D \tag{35}$$

$$\tau_d \leq z \qquad \forall d \in D \tag{36}$$

Then we apply the CC approach to constraints (35) to obtain:

$$\tau_d = \frac{(F_d + a_d\widehat{F_d}) - \sum_{b\in B} r_{bd}}{(F_d + a_d\widehat{F_d})} \qquad \forall d \in D \tag{37}$$

We now obtain the maximum value of the following problem to obtain our worst case, considering the values of the variation in the demand:

$$\max_{d \in D} \frac{\left(F_d + a_d \widehat{F_d}\right) - \sum_{b \in B} r_{bd}}{\left(F_d + a_d \widehat{F_d}\right)} \tag{38}$$

Subject to:

$$\sum_{d \in D} a_d = \Gamma \tag{39}$$

$$a_d \in \{0,1\} \qquad \forall d \in D \tag{40}$$

Due to the structure of the sub-problem, since there is not a strong duality between both problems (Bertsimas & Sim, 2004) we cannot apply a duality theorem to solve both problems simultaneously. We will proceed by solving two problems iteratively, as illustrated in Figure 1. The main problem, which includes Equations (11)-(13), (2)-(4), (6)-(8), and (10) will be solved to obtain values for r_{bd}. We will then use these values to solve the sub-problem (38) to (40) and obtain values for a_d. After this, Equation (12) will have to be modified in order to consider the values of a_d obtained in the sub-problem:

$$\frac{\left(F_d + a_d \widehat{F_d}\right) - \sum_{b \in B} r_{bd}}{\left(F_d + a_d \widehat{F_d}\right)} \leq z \qquad d \in D$$

and iterations will continue until the solution converges (until the sub-problem repeats the same values for a_d in any iteration). Appendix B includes the whole formulation for the main problem and the sub-problem.

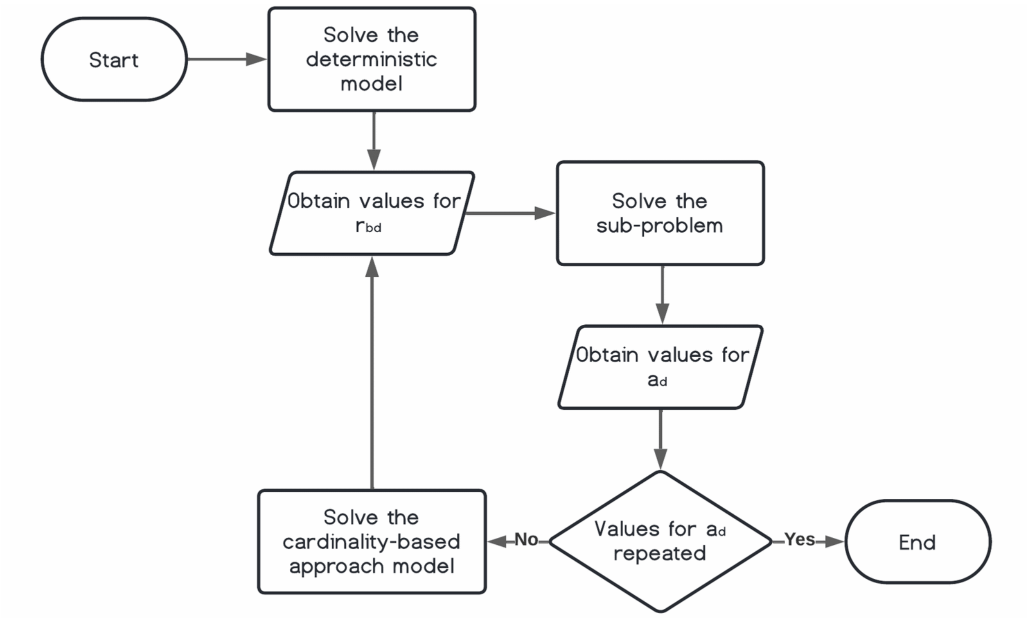

Figure 1. Flowchart to solve the CC model with equity.

In chapter 5 we compare the results obtained with this model against the other approaches applied to a case study.

4. Case Study

In order to demonstrate how different considerations of uncertainty raise distinct challenges for managers and lead to possible different outcomes, the proposed models will be used to solve a real case. For our case study, we will consider Hurricane Odile in state of Baja California Sur (BCS), Mexico, a tropical cyclone that made landfall on the

Baja California Peninsula in 2014 with the highest intensity was Hurricane Odile (Dirección de Logística Nacional, 2015).

We were able to gather sufficient information from the logistics planning manager of BAMX, which stands for "Bancos de Alimentos de Mexico," in order to model this scenario. A map of Baja California Sur and the bordering states was utilized to assess distance-safe food banks and potential demand nodes impacted by the disaster to construct the node network. The nodes that ought to serve as storage centers and the volume of donations that each storage center should be able to manage were identified using the suggested models.

Figure 2 illustrates the node network considered. Available food banks to be used are represented by yellow nodes and the possible demand nodes are represented by gray nodes. It is important to mention that storage centers can also be affected by the disaster, in which case they will be considered as a demand node with donations available (in case it is enabled as a storage center).

Table A.1 in the Appendix lists the municipalities in the network and their total population. This parameter is used to estimate the values of daily demand in tons of product and final demand in a node when a disaster strikes as shown in Table A.2, which is also reported in the Appendix.

Normally, food banks operate on a daily basis, receiving donations of food from individuals, enterprises, and other sources, which they then distribute to those in need. The amount of food they deliver to each municipality is proportional to the population in need at the corresponding location. However, when a disaster strikes, the demand for food in certain areas can increase dramatically. In these situations, food banks may need to quickly mobilize additional resources to meet the increased demand.

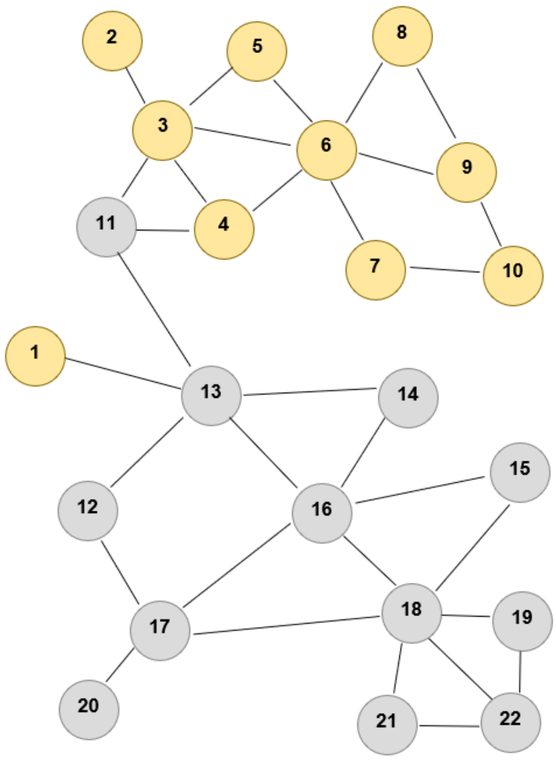

Figure 2. Node network considered.

4.1. Potential Scenarios for the Hurricane Strike

Before the Hurricane strike, managers analyzed meteorological maps to try to anticipate its trajectory and build possible scenarios of damages and population needs. Figure 3 depicts Hurricane Odile two days before it struck land.

Figure 3. Hurricane Odile's before land impact.

According to the most likely trajectories of Odile, managers envisaged five possible scenarios:

- Scenario 1: The Hurricane affects all the peninsula of BCS, so 10 of the nodes should need relief.
- Scenario 2: Only the six municipalities that are closest to the ocean should require aid as the Hurricane mostly affects the tip of the BCS peninsula.
- Scenario 3: The Hurricane passes over the Gulf of Baja California, between the states, impacting Puerto Vallarta and Mazatlán (nodes 6 and 7).
- Scenario 4: The Hurricane follows its path through the Gulf of Baja California but affects a total of four municipalities (the inner states of the state of BCS).
- Scenario 5: The Hurricane only affects the towns where the gulf ends as it moves through the entire Gulf of Baja California.

In each of the five scenarios, a total of 15 different nodes are affected by the disaster (five storage centers and 10 demand nodes). Our models consider the daily demand for every demand node, and a possible increase in demand if such node is affected by the Hurricane, as shown in Table A.2.

4.2. SB Formulations

The deterministic optimization models can be easily modified to handle a SB approach (i.e. Rivera, Smith, Ogazon & Ruiz, 2023). To this end, each scenario corresponds to the different combinations in which demand nodes can be affected. We introduce a set of scenarios S, and a parameter θ_s that gives the probability that scenario s occurs. Parameters F_{ds} refer to the demand at node d in scenario s. Finally, variables y_{ds} give the commodity shortage in node d in scenario s. The objective function (1) becomes:

$$\text{minimize} \sum_{d \in D} \sum_{s \in S} \theta_s y_{ds} \tag{41}$$

and constraints (5) become:

$$\sum_{d \in D} F_{ds} - \sum_{b \in B} \sum_{d \in D} r_{bd} \leq \sum_{d \in D} y_{ds} \qquad s \in S \tag{42}$$

The SB formulation for the unmet demand minimization problem consists therefore in Equations (40)-(41), (2)-(4), and (6)-(10). Appendix B reports the complete formulation.

Similarly, to elaborate the SB counterpart of the missing resources minimization problem with equity, we redefine variable z into z_s, so that it will now depend on the scenario. Objective function (11) becomes:

$$minimize \sum_{s\in S} \theta_s z_s \tag{43}$$

and constraints (12) and (13) are rewritten as:

$$\frac{F_{ds} - \sum_{b\in B} r_{bd}}{F_{ds}} \leq z_s \qquad \forall d \in D, s \in S \tag{44}$$

$$z_s \geq 0 \qquad \forall s \in S \tag{45}$$

The SB formulation for the unmet demand minimization problem with equity consists therefore in Equations (43)-(45), (2)-(4), (6)-(8), and (10). Appendix B reports the complete formulation.

5. Numerical Results

The goal of the following numerical experiments is to analyze how different objective functions under different approaches lead to diverse solutions. Furthermore, since it is not possible to compare the results produced by the considered models in a straightforward manner due to the different ways they handle uncertainty and therefore managerial risk, a qualitative discussion from a managerial perspective will follow the numerical results.

5.1. Solutions Produced by the Proposed Models

As stated previously, the models will be solved with uncertainty in demand, considering daily and post-disaster demand, as outlined in Table A.2. The results obtained in this section will then be evaluated in a real case and five additional scenarios for further comparisons.

For both CC formulations, we will deal with uncertainty by considering a level of cardinality ranging from 0 to 15, where the level of cardinality represents the number of nodes affected by the disaster. For each value of cardinality, the models will identify the nodes that would lead to the worst possible outcome (depending on the level of cardinality) and solve the model considering them. As was explained before, increasing the value of Γ assumes that more nodes will be affected by the hurricane, so the total demand will also increase although the supply will remain constant.

For both SB formulations, five possible scenarios (explained in section 4) will be considered to solve the model. This will give us a total of 34 solutions, for which variables x_b and q_b represent the different decisions that could be made depending on the approach. These solutions will be evaluated in the real case.

5.1.1. Results Produced by the CC Formulations

Tables 2a and 2b show the results obtained for each value of the cardinality parameter Γ by the unmet demand minimization formulation (MUD), and the EF. Columns under header x_b list the food banks selected by each formulation. Columns under header %min report the minimum percentage of unmet demand among all zones, while header %max report the largest percentage of unmet demand among all zones. Finally, the objective function produced by each formulation and the associated total cost are reported by columns OF and TC, respectively.

Γ	x_b	q_b	%min	%max	OF	TC
0	2-3-5-6-7	8-44-8-12-10	0	0	0	96 075
1	3-4-5-6-7	51-55-8-44-42	0	9.09	1	119 965
2	3-4-5-6-7	51-55-8-44-42	0	100	107	111 865
3	4-5-6-7-10	55-8-44-54-39	0	100	203	74 206
4	2-4-5-6-7	50-44-8-44-54	0	100	275	60 037
5	2-5-6-7-9	50-41-44-54-1	0	100	346	51 119
6	2-5-6-7-9	50-51-44-54-1	0	100	407	51 119
7	2-3-5-7-9	50-54-51-45-0	0	100	462	49 721
8	2-3-5-7-9	50-54-51-45-0	0	100	489	49 721
9	2-3-5-7-9	50-54-51-45-0	0	100	516	49 721
10	2-3-5-7-9	50-54-51-45-0	0	100	543	49 721
11	2-3-5-7-9	50-54-51-45-0	0	100	565	49 721
12	2-3-5-7-9	50-54-51-45-0	0	100	579	49 721
13	2-3-5-7-9	50-54-51-45-0	0	100	592	49 721
14	2-3-5-7-9	50-54-51-45-0	0	100	604	49 721
15	2-3-5-7-9	50-54-51-45-0	0	100	612	49 721

Table 2a. Results produced for the considered values of cardinality by MUD

Let us focus first on the unmet demand. Notice first that the total missing resources increase with the level of cardinality, since increasing Γ by one implies that an additional node is affected and therefore the total demand increases although the supply remains unchanged. Both formulations reached identical values of unmet demand (OF), but they distribute the available supply in very different manners, as showed by columns %max. Indeed, for values of $\Gamma \geq 2$, the MUD formulation assigns no unit to at least one demand zone.

Γ	It	x_b	q_b	%min	%max	OF	TC
0	0	2-3-5-6-7	8-44-8-12-10	0	0	0	96 075
1	3	3-4-5-6-7	51.74-55-7.96-44-41.30	0.4975	0.4975	1	120 575
2	3	3-4-5-6-7	54-55-5.21-44-41.79	34.85	34.85	107	173 676
3	3	3-4-5-6-7	54-45.42-3.97-44-52.61	50.37	50.37	203	136 364
4	3	2-3-4-6-7	33.68-54-23.68-44-44.64	57.89	57.89	275	121 204
5	3	2-3-5-6-7	30.25-54-28.94-44-42.81	63.37	63.37	346	106 477
6	3	2-3-5-6-7	41.04-54-26.03-44-34.93	67.05	67.05	407	133 661
7	3	2-3-4-5-7	24.17-54-55-23.87-42.96	69.78	69.78	462	134 340
8	4	2-3-4-5-7	23.22-54-55-22.93-44.84	70.97	70.97	489	141 984
9	4	2-3-4-5-7	22.35-54-55-22.07-46.58	72.07	72.07	516	150 335
10	5	2-3-4-5-7	21.53-54-55-21.27-48.20	73.08	73.08	543	158 078
11	6	2-3-4-5-7	20.92-54-55-20.65-49.43	73.86	73.86	565	163 840
12	5	2-3-4-5-7	20.54-54-55-20.28-50.18	74.32	74.32	579	167 157
13	3	2-3-4-5-7	20.20-54-55-19.95-50.85	74.75	74.75	592	169 476
14	3	2-3-4-5-7	19.90-54-55-19.65-51.45	75.12	75.12	604	172 221
15	1	2-3-4-5-7	19.70-54-55-19.46-51.84	75.37	75.37	612	173 760

Table 2b. Results produced for the considered values of cardinality with equity

In contrasts, the largest percentages of unmet demand rise up to 75.37% in the case of the formulation with equity, so all the demand zones receive at least some relief. This difference is clearly illustrated for $\Gamma = 1$, where the difference between demand and supply is equal to 1 *ton*. In this case, MUD decides that the node having the highest distribution cost will receive 1 *ton* less than its demand, while the model with equity fulfills 99.5025% of need at every demand node.

For the rest of the considered values of cardinality parameter Γ, MUD shows the same behavior: it first fulfills the whole demand of nodes with low distribution costs before starting to send relief to nodes with higher distribution cost. In fact, foodbanks can be affected by the disaster, hence the MUD model selects these foodbanks as storage centers when they are affected to achieve a lower cost in distribution. Foodbanks can relieve their own demand with distribution cost 0 since donations does not have to be moved.

As we can see in Table 2a, the total cost increases in the first 2 levels of Γ, and then starts decreasing until $\Gamma = 7$, where our model found its optimal solution. At this point, the last warehouse that results affected by the disaster is considered, so all donations available are already assigned to satisfy demand with distribution cost 0. Since supply is less than demand, in the solution some nodes receive 100% of their needs while others receive nothing, while the model with equity distributes donations as fairly as possible among demand nodes.

If we look now at the networks decisions made by the formulations, their choices are quite different although we can observe that the storage centers selected by MUD vary as the demand increases (i.e., as Γ increases). The formulation with equity produced more consistent solutions, and in particular, selected the same subset of storage centers (2, 3, 4, 5, and 7) for $\Gamma \geq 7$.

Therefore, and as one might expect, MUD distributes the available relief in a more efficient manner (lower cost), while the model with equity achieves a fairer distribution, but incurs systematically higher distribution costs. One might nonetheless be surprised by the behavior of the total cost incurred by the solutions produced by the models. For the model of MUD, the total cost increases from $\Gamma = 0$ to $\Gamma = 1$, because the total transported relief increases, and then the cost decreases until $\Gamma = 7$, although the delivered relief remains the same. The reason is that as more and more nodes are affected, their demand increases. Since some of these nodes have low distribution costs, MUD increases the relief sent to them, thus decreasing the quantities sent to the nodes having higher distribution costs. By doing so, it distributes the same amount of relief, but incurs a lower cost. For $\Gamma = 7$ and higher, each new node considered affected has higher distribution cost than the ones already considered, so the solution does not change. The formulation with equity, on the other hand, must readjust the distributed quantities from one case to the other according to the considered set of affected nodes. For this reason, the total cost it produces shows a more complex pattern.

5.1.2. Results Produced by SB Formulations

The SB models assume that each of the 5 proposed scenarios will happen with equal probability 1/5. Therefore, the formulation seeks a solution that, on average, minimizes its objective value over the 5 scenarios. Each scenario includes a different subset of nodes affected, their demand, which is referred to as post-disaster demand, is higher than the regular one.

Table 3 reports the results produced by the SB counterparts of the MUD and EF. Columns 2 and 3 show the selected storage centers and the amounts of relief sent to them, whilst columns 4 and 5 indicate the minimum and maximum percentages of unmet demand among the demand zones. Finally, columns 6 and 7 give the objective function (in total unmet demand) and the total cost of the solution, respectively.

Model	x_b	q_b	OF	TC
MUD	2-3-4-5-6	39-54-55-8-44	161.8	255 595
Equity	2-3-4-5-6	42.98-54-55-4.02-44	67.93% (176.2)	312 820

Table 3. Results produced by MUD and EF to the 5 demand scenarios

5.2. Evaluation of the Results Applied on the Real Case

Now we will evaluate the solutions produced by the previous models in the case of the real scenario. Recall that all the presented models work under uncertainty, suggesting values of decision variables x_b and q_b that must be implemented before the disaster strikes. However, the decisions concerning the distribution of donations (represented by variables r_{bd}) are taken after the strike once the affected nodes are known. Therefore, in our analysis, we will use the values produced by the models for variables x_b and q_b, and we will perform a post-optimization to elect the best values of variables r_{bd} according to the real after-strike demand and optimizing the total cost. Table A.2 in appendix A illustrates the daily demand and the post-disaster demand for the real case scenario. To have a point of comparison of the effectiveness of our models, we will begin our analysis by solving the real case considering perfect information (i.e., the outcome of the strike is known a priori).

It is assumed that managers operating with perfect knowledge are aware of the disaster's location and the nodes that will be impacted in advance, so they can make optimal decisions. Although it is not possible to know the disaster's outcome in advance, this unrealistic approach allows us to estimate the price of information, or in other words, how uncertainty deteriorates the quality of our decisions. The results obtained considering perfect information are reported in Table 4.

Model	x_b	q_b	%min	%max	OF	TC
MUD	2-3-4-5-6	39-54-55-8-44	0	100	110	296 020
Equity	2-3-4-5-6	41.84-54-55-5.16-44	35.48	35.48	110	325 458

Table 4. Results produced with "perfect information"

For both models under perfect information, we have the same selection of storage centers, but donations are distributed slightly differently in order to achieve equity, incurring a notably higher cost for this model due to distribution costs.

Now we proceed to evaluate the solutions given by our models (Table 2a, Table 2b, and Table 3) in the real case. Table 5 illustrates the objective and total cost obtained by evaluating each of the solutions of our models in the real case. The first row shows the results obtained with perfect information MUD (PID), rows 2-8 show the results with CC approach MUD (CD), where the number in parentheses represents the level of cardinality, and row 9 shows the results of the SB model MUD (SD). In row 10 are the results obtained with perfect information with equity (PIE), rows 11-24 contain the results with the CC approach with equity (CE), and finally row 25 shows the results of the SB approach with equity (SE).

Regarding the level of Γ considered, the total costs have significant fluctuations from $\Gamma = 0$ to $\Gamma = 7$. After this point, the solutions for the objective of MUD are the same, and for the objective of EF, fluctuations in the total costs are minimal. Since this case considers a total of $\Gamma = 15$ demand nodes, it is notable that although more than half of the demand nodes are affected by the disaster, this does not change the solutions considerably.

For any real case, it is unlikely that more than half of the demand nodes will be affected by the disaster, so we can conclude that, for this particular case, considering $\Gamma = 7$ would be the best option.

As expected, solutions obtained with MUD for every case incurred a lower cost than the solutions obtained with EF. Both formulations distribute the available supplies, obtaining the same missing amount in tons of product, but with different total costs. The columns of *%min* and *%max* help us understand the main difference between both objectives.

The MUD formulation seeks efficiency so it will minimize distribution costs but may leave some demand nodes with 0 relief. On the other hand, an equitable solution ensures that every demand node receives help proportionally to its demand but may incur higher distribution costs. When budget is limited, MUD will allow maximizing the distributed supplies. But if the budget is not an issue, or the main issue, an equitable solution is a better option since donations are distributed fairly.

Model	%min	%max	OF	TC
PID	0	100	110	296 020
CD (1)	0	100	110	306 290
CD (2)	0	100	110	306 290
CD (3)	0	100	110	312 410
CD (4)	0	100	110	314 220
CD (5)	0	100	110	328 650
CD (6)	0	100	110	326 720
CD (7-15)	0	100	110	311 840
SD	0	100	110	296 020
PIE	35.48	35.48	110	326 345
CE (1)	35.48	35.48	110	334 525
CE (2)	35.48	35.48	110	332 940
CE (3)	35.48	35.48	110	338 120
CE (4)	35.48	35.48	110	342 554
CE (5)	35.48	35.48	110	363 050
CE (6)	35.48	35.48	110	360 420
CE (7)	35.48	35.48	110	347 635
CE (8)	35.48	35.48	110	346 985
CE (9)	35.48	35.48	110	346 130
CE (10)	35.48	35.48	110	345 875
CE (11)	35.48	35.48	110	344 760
CE (12)	35.48	35.48	110	344 890
CE (13)	35.48	35.48	110	343 950
CE (14-15)	35.48	35.48	110	344 020
SE	35.48	35.48	110	326 679

Table 5. Evaluation of the results in the real case

Besides the objective, the choice to apply the CC or SB approaches to consider uncertainty in demand has significant differences. CC formulations can distribute all donations, achieving the same missing resources in tons of product considering perfect information, but incur a higher cost. On the other hand, for this particular case, the SB formulations display exceptional performance. For the objective of MUD, the SB formulation had the same performance as with perfect information, and for the objective of EF, the cost was just $334 higher, achieving almost the same solution as with perfect information. This can happen when the estimation of scenarios to consider is outstanding, but sometimes it is not possible to adequately predict possible outcomes that can occur due to the uncertainty surrounding natural disasters. The effectiveness of this approach depends on the accuracy of the selected scenarios. When there is too much uncertainty to decide the possible outcomes, the CC approach can work correctly with no dependance on the precision of the scenarios considered.

To have a better point of comparison between both approaches, next section evaluates each model under different possible outcomes that could have occurred.

5.3. Evaluation of the Results in Each of the 5 Scenarios

For a better understanding of the behavior of our formulations, different scenarios must be solved for further conclusions. For this reason, we will now consider that the strike led to other outcomes. In particular, we will assume each of the 5 proposed scenarios as real outcomes of the strike.

We will begin our analysis by solving each of the five scenarios considering perfect information. The results obtained under this analysis are shown in table 6 (table A.3 in appendix A illustrates the daily demand and post-disaster demand for every scenario). We then solved each of the 5 scenarios with our four models to compare the results. The columns of %min and %max **as well as the column of OF** are not included since all the solutions are equivalent in distribution to the one obtained with perfect information in Table 6.

Table 7 illustrates the total cost obtained by evaluating the solutions of our models in each of the 5 scenarios. Regarding the total cost, we obtained the same expected behavior, where for all scenarios, the total costs incurred with MUD result significantly lower than the costs obtained with the objective of EF. For scenarios 1, 2, and 4 (S1, S2, and S4) we have a behavior like what happened in the real case. The SB formulations had a very similar performance than the evaluations done with perfect information (the same for the objective of MUD). This happens since the outcome predicted for these scenarios was like what happened in the real case, where no foodbanks were affected by the disaster.

On the other hand, for scenarios 3 and 5 (S3 and S5), where foodbanks were affected by the disaster, neither formulation was close to the cost achieved with perfect information, but CC approaches had a notably better performance than the evaluations obtained with SB approaches. In particular, for scenario 3, all the solutions with CC incurred a lower cost than with SB, and for scenario 5, we have the same behavior after $\Gamma = 5$, once the model considers the affected food banks in the CC formulation.

This gives us valuable insight into the differences obtained with different approaches to deal with uncertainty. For SB formulations, we have a high dependency on the prediction of scenarios, where a bad selection of scenarios will lead to worst solutions. The CC formulations protect against the worst scenarios that can happen, giving good solutions independently of what happens. In CC formulations, the objective function is not heavily penalized independently of the outcome, which is known as the price of robustness (Bertsimas & Sim, 2004).

Model	Sc	x_b	q_b	%min	%max	OF	TC
MUD	1	2-3-4-5-6	39-54-55-8-44	0	100	161	257 720
	2	2-3-4-5-6	39-54-55-8-44	0	100	36	293 895
	3	4-5-6-7-9	55-8-44-54-39	0	100	74	74 205
	4	2-3-4-5-6	39-54-55-8-44	0	100	6	258 145
	5	2-3-4-5-7	50-54-35-51-10	0	100	57	72 521
Equity	1	2-3-4-5-6	42.57-54-55-4.43-44	44.60	44.60	161	309 252
	2	2-3-4-5-6	40.22-54-55-6.78-44	15.25	15.25	36	306 874
	3	3-4-5-6-7	41.16-55-5.84-44-54	27.01	27.01	74	105 412
	4	2-3-4-5-6	39.23-54-55-7.77-44	2.91	2.91	6	261 664
	5	2-3-4-5-7	50-54-32.97-51-12.03	22.18	22.18	57	101 437

Table 6. Results for each scenario under perfect information

Model	TC (S1)	TC(S2)	TC (S3)	TC (S4)	TC (S5)
PID	257 720	293 895	74 205	258 145	72 521
CD (1)	266 740	302 915	100 750	267 165	162 950
CD (2)	266 740	302 915	100 750	267 165	162 950
CD (3)	273 920	310 095	99 110	274 345	172 230
CD (4)	275 010	311 185	132 030	275 435	142 000
CD (5)	287 620	323 795	148 250	288 045	124 980
CD (6)	285 070	321 245	157 750	285 495	115 480
CD (7-15)	272 380	308 555	139 150	272 805	121 330
SD	257 720	293 895	160 720	258 145	126 890
PIE	309 252	306 874	105 412	261 664	101 437
CE (1)	317 330	316 430	124 350	273 500	202 910
CE (2)	316 620	316 350	124 180	274 630	203 840
CE (3)	322 190	324 680	120 260	280 840	209 990
CE (4)	325 210	324 970	156 670	281 950	181 050
CE (5)	347 640	339 150	176 440	294 490	161 120
CE (6)	345 380	336 610	183 760	291 990	151 530
CE (7)	332 050	323 530	165 160	279 210	163 660
CE (8)	331 450	323 340	162 270	278 780	165 780
CE (9)	330 570	323 030	159 430	278 340	167 830
CE (10)	329 960	323 250	156 680	277 810	170 140
CE (11)	329 320	321 830	155 430	277 120	171 250
CE (12)	329 150	321 910	153 860	277 320	172 490
CE (13)	328 430	321 160	152 550	276 860	173 510
CE (14)	328 730	321 320	150 610	276 970	175 090
CE (15)	328 730	321 320	150 610	276 970	175 090
SE	309 693	307 034	182 574	261 922	178 427

Table 7. Evaluation of the results in the 5 scenarios

5.4. Comparison of the Results Produced by CC and SB Formulations

Each of the proposed approaches handles uncertainty in a different manner, so the outcomes they produced are also quite different. The CC approach prepares for the worst possible outcome, a perspective that inherently makes it more conservative. This approach limits the number of non-zero variables in the solution, thereby directly managing the exposure to uncertainty according to the user beliefs or experience. By focusing on worst-case scenarios, this method ensures a robust solution that can withstand significant adverse conditions. The characteristics that ensure that the CC approach can handle uncertainty properly, particularly in risk-averse environments, are the following:

- *Risk Mitigation*: By preparing for the worst, the CC approach effectively mitigates risk. This is particularly valuable in volatile environments where the consequences of adverse outcomes can be severe.
- *Robustness*: Solutions are typically more robust, ensuring that performance remains acceptable even under unfavorable conditions.
- *Conservatism*: The conservative nature of this approach often leads to solutions that may not be optimal under average conditions but offer protection against extreme scenarios.

The disadvantage of this approach is when the actual situation does not correspond to a worst-case scenario, since the conservative solutions produced by the CC can result in poor performance (i.e., potentially higher costs or lower distribution of donations).

In contrast, the SB approach seeks optimal solutions based on a set of predefined scenarios. This method's effectiveness largely depends on the quality and representativeness of the chosen scenarios. It aims to provide solutions that perform well across these scenarios, balancing performance and risk. The features that make this approach suitable for uncertain humanitarian logistics problems are:

- *Optimality*: When the scenarios are well-chosen, this approach can yield optimal or close to optimal solutions that maximize returns or minimize costs effectively across the expected range of conditions.
- *Flexibility*: The SB method is flexible and can adapt to different sets of scenarios, allowing for dynamic adjustment as new information becomes available.

The primary limitation of this approach is its dependence on the accuracy and representativeness of the scenarios. If the scenarios fail to capture critical uncertainties, the solutions may be less effective or even risky.

5.5. Insights

The CC strategy, which emphasizes risk mitigation by bracing for the worst-case scenario, is fundamentally conservative when analyzing the robustness of the solutions. In other words, solutions that come from this method are made to be resilient and stable even in the face of adversity. But frequently, prospective benefits are sacrificed for this conservative viewpoint. Restricting the amount of non-zero variables could lead to missing out on opportunities that could happen in less severe situations. On the other hand, the scenario-based method is better at taking advantage of fortunate circumstances. This method can optimize resource allocation to produce the best results if the scenarios are well-crafted and properly represent probable future states. However, its success is conditional on the quality of the scenarios. Poorly constructed scenarios can lead to suboptimal or even risky decisions.

Regarding the adaptability, this is one of the significant advantages of the scenario-based approach. This method can be continuously updated and refined as new information becomes available, allowing it to reflect changing conditions and emerging trends. This flexibility is crucial in dynamic environments where uncertainties and risks evolve over time. The scenario-based approach can pivot and adjust strategies to align with the latest insights, maintaining optimal performance across a range of potential futures. On the other hand, the cardinality-constrained approach is more rigid. Its conservative nature means it is designed to be robust against worst-case scenarios but may not adapt quickly to new information or changing conditions. This static characteristic can be a limitation in environments where rapid adaptation is necessary for success.

Finally, from an implementation standpoint, the cardinality-constrained approach tends to be simpler. Defining constraints that limit the number of non-zero variables is straightforward, and ensuring compliance with these constraints is relatively easy. This simplicity can be advantageous in terms of ease of use and lower computational requirements. However, the scenario-based approach demands a more complex setup. It requires the careful design and validation of multiple scenarios, each representing different possible future states. This process is resource-intensive and necessitates a deep understanding of the factors influencing the scenarios. Additionally, ongoing scenario management is required to keep the model relevant and effective. Despite this complexity, the potential for achieving highly optimal solutions makes the scenario-based approach attractive in contexts where the investment in scenario planning can be justified by the benefits of more precise and adaptive decision-making.

The choice between these approaches depends on the particular requirements and limitations of the given situation. A strong defense is offered by the CC approach in situations where risk aversion is critical and there is a substantial chance of extreme negative outcomes. By contrast, the SB method presents an optimal solution when reliable scenarios generation is possible.

6. Conclusions and Future Research

The uncertainty in demand is one of the biggest challenges when dealing with humanitarian logistics. Uncertainty can arise from a number of sources, including changes in demand, fluctuations in donations available and disruptions to transportation roads. In this work, we adopt the CC approach to deal with this uncertainty under two different objective functions, one seeking MUD and a second one incorporating equity in the manner demand is satisfied. Whether to consider an equity or a non-equity objective will depend on the expected results and ethical implications, some of which are mentioned in this work. One difficulty of the CC approach is to decide the level of cardinality, but our experiments showed an interesting behavior, since the same solution was observed to be produced beyond a given cardinality.

We used the proposed approaches and objectives to tackle the case study of hurricane Odile that struck Mexico in 2014, and we compared the solutions they produced to the ones obtained by a SB approach. For this realistic case, the CC models showed good performance, but not as good as the SB models. Indeed, the scenarios proposed by the experts were very accurate (very close to the real strike's outcome) which allowed this approach to produce near-optimal solutions. This confirms the idea that, whenever it's possible to gather good predictions on the possible outcomes, the best course of action is to use them. But in the absence of accurate information, the use of the CC models can be more suitable.

The generalizability of these results is subject to certain limitations. For instance, the two formulations were compared only on a humanitarian operation in Mexico. Additional experiments should be required to confirm the extent to which these results might be generalized to other cases. Furthermore, the scenarios and the values of the post-disaster demand for each scenario were suggested by experts of BAMX. The numerical results and the further analysis in this paper are linked to the quality and the accuracy of these suggestions. Finally, we limited our analysis to only two approaches while, as explained in the theoretical framework, other approaches have been proposed to deal with uncertainty.

To continue with our work, some directions for future research are pointed out. The first recommendation is to consider uncertainty in supply. In our work we focused completely on uncertainty in demand, but in humanitarian logistics the amount of supply available is difficult to predict since it depends on donations. The ability of donators to provide the expected quantity of supplies, the variety of goods offered, and the reception of spontaneous and occasionally even unwanted donations are some of the aspects associated to supply uncertainty.

The second recommendation is to evaluate our models under different situations to have a more solid base to support our findings. In our work, the selected scenarios were too good, giving advantage to this approach over the CC approach. There will be cases where the scenarios can be difficult to predict, hence the solutions of these models will have a bad performance.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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Appendix A

Number	Municipality	State	Population
1	Tijuana	Baja California	1,301,000
2	Hermosillo	Sonora	812,229
3	Los Mochis	Sinaloa	231,977
4	Culiacan	Sinaloa	675,773
5	Chihuahua	Chihuahua	809,232
6	Mazatlan	Sinaloa	502,547
7	Puerto Vallarta	Jalisco	203,342
8	Torreon	Coahuila	608,836
9	Zacatecas	Zacatecas	1,579,000
10	Guadalajara	Jalisco	1,495,000
11	Topolobampo	Sinaloa	6,361
12	Todos los Santos	Baja California Sur	6,485
13	La Paz	Baja California Sur	244,219
14	San Juan	Baja California Sur	5,300
15	Cabo Pulmo	Baja California Sur	5,800
16	Los Barriles	Baja California Sur	1,056
17	Cabo San Lucas	Baja California Sur	81,111
18	San Jose del Cabo	Baja California Sur	93,069
19	El Vado	Baja California Sur	11,600
20	El Caribe	Baja California Sur	49,600
21	Ballenas	Baja California Sur	11,600
22	Vistahermosa	Baja California Sur	11,600

Table A.1. Basic municipality information

Node	Daily Demand	Post-disaster demand
1	0	0
2	8	0
3	6	0
4	0	0
5	8	0
6	12	0
7	10	0
8	0	0
9	0	0
10	0	0
11	0	0
12	0	0
13	11	0
14	2	0
15	2	0
16	2	0
17	7	0
18	3	0
19	3	60
20	2	60
21	3	60
22	3	60

Table A.2. Daily and post-disaster demand for the real case

Node	Daily Demand	Post-Disaster Demand				
		Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
1	0	0	0	0	0	0
2	8	0	0	0	0	72
3	6	0	0	0	0	55
4	0	0	0	0	0	0
5	8	0	0	0	0	71
6	12	0	0	119	0	0
7	10	0	0	96	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	11	106	0	0	106	0
14	2	13	0	0	13	0
15	2	14	0	0	14	0
16	1	8	0	0	8	0
17	7	61	61	0	0	0
18	3	22	22	0	0	0
19	3	27	27	0	0	0
20	2	12	12	0	0	0
21	3	27	27	0	0	0
22	3	27	27	0	0	0

Table A.3. Daily demand and post-disaster demand for each of the 5 scenarios

Appendix B

Formulations

B.1 CC model MUD

$$\text{minimize} \quad \sum_{d\in D} y_d$$

Subject to:

$$\sum_{b\in B}\sum_{d\in D} r_{bd} - \sum_{b\in B} q_b \leq 0$$

$$q_b - x_b K_b \leq 0 \qquad b \in B$$

$$\sum_{b\in B} q_b - Q \leq 0$$

$$\sum_{d\in D} F_d + \Gamma u + \sum_{d\in D} \pi_d - \sum_{b\in B}\sum_{d\in D} r_{bd} = \sum_{d\in D} \tau_d$$

$$\tau_d \leq y_d \qquad d \in D$$

$$u + \pi_d \geq \widehat{F_d} \qquad \forall\ d$$

$$\left(F_d + a_d\widehat{F_d}\right) - \sum_{b \in B} r_{bd} \leq \tau_d \qquad d \in D$$

$$\pi_d - a_d \geq 0 \qquad d \in D$$

$$\sum_{d \in D} a_d = \Gamma_1$$

$$\sum_{b \in B}\sum_{d \in D} c_{bd} r_{bd} + \sum_{b \in B} BC_b x_b + \sum_{b \in B} q_b e_b \leq P$$

$$\sum_{b \in B} x_b = \rho$$

$$x_b, a_d \in \{0,1\}$$

$$u, v \geq 0$$

$$y_d, \pi_d, \tau_d, \geq 0 \qquad \forall\ d$$

$$\theta_b, \tau_b \geq 0 \qquad \forall\ b$$

$$r_{bd} \geq 0 \qquad \forall\ b, d$$

B.2 CC formulation with equity

Main Problem

$$minimize\ z$$

Subject to:

$$\sum_{d \in D} r_{bd} - x_b q_b \leq 0 \qquad b \in B$$

$$\frac{(F_d + a_d\widehat{F_d}) - \sum_{b \in B} r_{bd}}{(F_d + a_d\widehat{F_d})} = \tau_d \qquad d \in D$$

$$\tau_d \leq z$$

$$\sum_{b \in B}\sum_{d \in D} c_{bd} r_{bd} + \sum_{b \in B} BC_b x_b \leq P$$

$$x_b \in \{0,1\}$$

$$z \geq 0$$

$$\tau_d \geq 0 \quad \forall \ d$$

$$r_{bd} \geq 0 \quad \forall \ b, d$$

Sub-problem

$$\max_{d \in D} \frac{\left(F_d + a_d \widehat{F_d}\right) - \sum_{b \in B} r_{bd}}{\left(F_d + a_d \widehat{F_d}\right)}$$

Subject to:

$$\sum_{d \in D} a_d = \Gamma$$

$$a_d \in \{0,1\}$$

B.3 SB formulation MUD

$$\text{minimize} \qquad \sum_{d \in D} \sum_{s \in S} \theta_s y_{ds}$$

Subject to:

$$\sum_{b \in B} \sum_{d \in D} r_{bd} - \sum_{b \in B} q_b \leq 0$$

$$q_b - x_b K_b \leq 0 \qquad b \in B$$

$$\sum_{b \in B} q_b - Q \leq 0$$

$$\sum_{d \in D} F_{ds} - \sum_{b \in B} \sum_{d \in D} r_{bd} \leq \sum_{d \in D} y_{ds} \qquad s \in S$$

$$\sum_{b \in B} \sum_{d \in D} c_{bd} r_{bd} + \sum_{b \in B} BC_b x_b + \sum_{b \in B} q_b e_b \leq P$$

$$\sum_{b \in B} x_b = \rho$$

$$x_b \in \{0,1\}$$

$$r_{bd} \geq 0 \ \forall \ b, d$$

$$y \geq 0$$

B.4 SB formulation with equity

$$\text{minimize} \qquad \sum_{s\in S} \theta_s z_s$$

Subject to:

$$\sum_{d\in D} r_{bd} - q_b \leq 0 \qquad b \in B$$

$$q_b - x_b K_b \leq 0 \qquad b \in B$$

$$\sum_{b\in B} q_b - Q \leq 0$$

$$\frac{F_{ds} - \sum_{b\in B} r_{bd}}{F_{ds}} \leq z_s \qquad d \in D, s \in S$$

$$\sum_{b\in B}\sum_{d\in D} c_{bd} r_{bd} + \sum_{b\in B} BC_b x_b + \sum_{b\in B} q_b e_b \leq P$$

$$\sum_{b\in B} x_b = \rho$$

$$x_b \in \{0,1\}$$

$$r_{bd} \geq 0 \; \forall \, b, d$$

$$z \geq 0$$