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Enhancing Quality Control in Lot Reception: A Comparative Analysis of Innovative Attribute Acceptance Sampling Plans

Darwin Peña-González[1], Nestor E. Caicedo-Solano[2], Roberto Torres-Peña[1,2], Ellery Chacuto-López[1,2] Edwan Anderson Ariza[2*]

[1]*Facultad de Educación, Universidad del Magdalena (Colombia)*
[2]*Facultad de Ingeniería, Universidad del Magdalena (Colombia)*

ddpenag@unimagdalena.edu.co, ncaicedo@unimagdalena.edu.co, rtorres@unimagdalena.edu.co, echacuto@unimagdalena.edu.co
[*]*Corresponding author: earizaec@unimagdalena.edu.co*

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Abstract:

Purpose: This study aims to enhance production quality by applying Quality Control (QC) principles through acceptance sampling, specifically analyzing the efficacy of attribute acceptance sampling plans in final lot receptions.

Design/methodology/approach: Through comprehensive review and critical evaluation of various sampling methods found in literature, this paper assesses their efficiency under distinct administrative and operational conditions. It emphasizes the comparison of different attribute acceptance sampling plans by examining variations in parameters and key performance indicators, such as Average Outgoing Quality (AOQ) and inspection time allocation percentage. Furthermore, it proposes a model for Continuous Sampling Plans (CSP) to evaluate these plans' performance in response to operational characteristic variations.

Findings: The analysis reveals that the selected methods significantly aid in decision-making processes for lot acceptance, utilizing non-conforming rates depicted by the Average Quality Level (AQL). This provides a robust framework for improving Quality Control strategies, demonstrating the potential of these methods to optimize production quality through strategic lot acceptance.

Practical implications: This paper outlines a practical approach for industry practitioners to enhance decision-making in lot acceptance, offering a framework to balance Quality Control with operational efficiency effectively.

Originality/value: By comparing a wide range of attribute acceptance sampling plans and introducing a novel CSP model, this research contributes valuable insights into the optimization of QC strategies. Specifically, this work includes the design of a second model to represent the CSP-1 serial sampling plans, allowing for the assessment of various plans to analyze variations in the aforementioned operational characteristics. It offers a unique perspective on enhancing production quality, marking a significant advancement in the field of QC and management.

Keywords: quality control, average outgoing quality, csp sampling plan, operational characteristics curves

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1. Introduction

One of the most expansive areas within statistical Quality Control (QC) is acceptance sampling, which aims to determine the acceptability of a lot without necessitating a complete inspection (Aziz, Hasim & Zain, 2021). This method primarily focuses on the incoming and outgoing control of batches, as well as on the audit control of final products. Acceptance sampling indirectly enhances production quality by fostering higher acceptance levels and discouraging substandard quality through frequent rejections (Bouslah, Gharbi & Pellerin, 2016). Given the diversity of sampling procedures available, selecting a specific method hinge on administrative conditions as well as on sampling efficiency. Therefore, analyzing the performance of various sampling plans to understand their operational characteristics is essential (Netto, Pelissari, Cysneiros, Bonazza & Sanquetta, 2017). For example, the primary objective proposed by Mirabi and Fallahnezhad (2012) is to identify the optimal values for the upper and lower thresholds by employing a Markov process, with the goal of minimizing the total cost associated with a batch acceptance policy. This strategy underscores the role of sampling plans as fundamental components of statistical process control, a suite of quality management tools aimed at improving processes and products (Ortiz-Barrios & Felizzola-Jimenez, 2014).

Current research has significantly advanced sampling plans, reinforcing quality management systems in various industries. Studies from Caicedo and Mahecha to Garcia and Martinez between 2015 and 2022, have integrated Markov models and economic sampling techniques under total quality cost considerations. Models for manufacturing and process control systems have been proposed, highlighting the effectiveness of different strategies to improve product quality and process efficiency. Findings include the optimization of product quality verification and regulatory compliance, providing detailed insight into the application of sampling techniques in defect detection and product consistency.

Thus, in the realm of attribute sampling plans, the focus has progressively shifted toward developing and refining methodologies that enhance the efficiency and accuracy of acceptance decisions. Modern research has built on the foundational work in this field, integrating advanced statistical techniques and computational tools to improve the design and application of sampling plans. Contemporary studies emphasize the practical implementation of these methods in various industrial contexts, ensuring that they are adaptable to different operational environments and quality control requirements. This evolution has led to the creation of more sophisticated models that can dynamically adjust to changes in production quality and operational conditions, thereby providing a more robust framework for lot acceptance decisions. Such advancements underscore the importance of continuous innovation in sampling plans, ensuring that they remain effective tools for maintaining high standards of quality control in manufacturing processes.

Among these considerations, the behavior of the acceptance probability in response to variations in the quality of the material under inspection stands out as a critical aspect of attribute batch acceptance sampling plans (Hlioui, Gharbi & Hajji, 2015; Wang & Lo, 2015; Thomas, 2023). This study performs a comparative analysis of innovative attribute acceptance sampling plans. The study illustrates a model will enable adjustments in plan parameters, facilitating the generation of comparative analyses on the behavior of acceptance probabilities within each plan. Moreover, for attribute sampling plans, understanding the average outgoing quality values, the percentage of time under 100% inspection, and fraction inspection, among other operational characteristics, is crucial. Accordingly, this work includes the design of a second model to represent the CSP-1 serial sampling plans, allowing for the assessment of various plans to analyze variations in the aforementioned operational characteristics.

2. Sampling Plans

In the field of attribute sampling plans, the development of methodologies has been pivotal in advancing quality control practices. Guenther (1969) pioneered a systematic search procedure using published tables of binomial, hypergeometric, and Poisson distributions. This approach was foundational in refining the Probability of Acceptance (Pa), which quantifies the likelihood that a lot meets the quality standards based on the sampling results and is a critical measure in assessing the efficiency of a sampling plan. Stephens (1978) provided a closed-form solution for single-sample acceptance plans by applying a normal approximation to the binomial distribution. His contribution helped elucidate the Operating Characteristic (OC) Curve, which plots the probability of lot acceptance against defect levels, thereby offering quality managers a powerful tool to visualize and assess the impact of quality variations on acceptance rates. Hailey (1980) introduced a computer program to generate simple sampling plans, facilitating the application of statistical theories in practical settings. This software could dynamically adjust to varying Acceptable Quality Levels (AQL), setting benchmarks for the maximum allowable percentage of defective units, and thus directly influencing the strictness of quality assurance processes. Hald (1981) provided a comprehensive overview of statistical theories that support sampling inspection, including crucial metrics like the Lot Tolerance Percent Defective (LTPD), which defines the poorest quality tolerable within a specified confidence interval, essentially setting a threshold beyond which lots are rejected to prevent poor quality goods from reaching the market. The comparative analyses by Kao (1971) and Hamaker (1979) emphasized the practical differences in sample sizes required between variable and attribute sampling plans. Their findings underscore the importance of understanding the Fraction Defective (p'), or the proportion of defective items in a lot, which is vital for calibrating sampling strategies to ensure quality while maintaining operational efficiency.

Sampling plans serve as a vital QC mechanism for the acceptance and/or rejection of production lots (Geetha & Mathew, 2023). These plans specify the number of product units for inspection in each lot and establish criteria for determining lot acceptability (acceptance and rejection thresholds) (Lamers-Kok, Panella, Georgoudaki, Li, Özkazanç, Kučerová et al., 2022). Acceptance sampling inspection is a critical quality control mechanism designed to ensure that producers deliver quality products that meet or exceed predetermined standards, thereby guaranteeing that consumers receive products of acceptable quality. Key constructs in this study include deterministic seasonal patterns and stochastic trends within data series. Deterministic seasonal patterns refer to predictable and repeating fluctuations that occur at regular intervals due to seasonal factors, while stochastic trends indicate random and unpredictable changes over time. These constructs are important because they highlight the variability (the extent to which data points differ from each other) and predictability (the ability to foresee future data points based on past patterns) in production processes. Understanding these patterns and trends is crucial for developing robust sampling methods that can effectively monitor and control product quality. Research, such as Franco Cardona, Velasquez-Heneo and Olaya-Morales (2008), reveals deterministic seasonal patterns and stochastic trends within data series, underscoring the need for quality control mechanisms that account for both predictable and random variations in production processes. Additionally, nonlinear regression models, as demonstrated by Warren (1994), have the capacity to approximate any continuous function within a defined compact domain, further emphasizing the importance of robust sampling methods to account for such variations. The core objective of acceptance sampling inspection is to ensure the producer delivers quality at or above the predetermined standard, thereby guaranteeing the consumer receives products of acceptable quality. Producers may adopt these sampling procedures to affirm their quality levels meet consumer expectations, as evidenced in the approaches of Carlsson (1989), Boucher and Jafari (1991), and Al-Sultan (1994). Moreover, challenges associated with linear drift in production processes have been explored by Rahim and Banerjee (1988) and Al-Sultan and Pulak (1997). Lauer (1978) has delved into the acceptance probabilities for sampling inspections using attributes with a Beta prior distribution for single sampling plans, while Rajagopal, Loganathan and Vijayaraghavan (2009) investigated the selection of Bayesian Single sampling plans with Beta distribution as the prior.

Recent research has significantly advanced sampling plans, enhancing quality management systems across various industries. Caicedo and Mahecha demonstrated Markov states considering decision-making changes based on Military Standard tables (Caicedo & Mahecha, 2015). Brown and White (2015) proposed an integrated model for manufacturing systems combining quantity and quality, introducing a Markov model with continuous-time and discrete part flow for a single-stage system. Yun-Cheng, Cheng and Yi-An (2019) compared the economical design of quality with traditional single sampling plans under total quality cost considerations. Anderson and Garcia conducted a comprehensive comparison of various sampling plans for quality improvement, offering insights into the effectiveness and suitability of different sampling strategies for enhancing product quality and process efficiency (Anderson & García, 2018). Taylor and Martinez (2019) provided a detailed overview of sampling plans for process control, outlining various sampling techniques and their applications in monitoring and maintaining process stability and quality. Clark and Lewis (2020) explored sampling plans tailored for quality management, examining their implementation in optimizing QC processes and ensuring compliance with quality standards. Miller and Rodriguez (2021) surveyed sampling plans in quality assurance, evaluating the effectiveness of different sampling methodologies in product quality verification and regulatory compliance. Garcia and Martinez (2022) performed a comparative analysis of sampling plans focusing on quality control applications, assessing the performance of various sampling techniques in defect detection, product consistency, and overall QC process improvement.

Ang, Han and Jiang (2021) proposed an optimized sampling plan for mechanical product quality inspection using the Taguchi method, enhancing mechanical engineering QC processes' efficiency and effectiveness. Marques, Maciel, Costa and Santos (2024) and Aslam (2020) explored the optimal design of a sequential sampling plan with random sampling, contributing to statistical methods for designing efficient sampling plans, especially beneficial in scenarios favoring sequential sampling. Chen, Li, Zhang and Chen (2016) and Noughabi (2022) optimized sampling plans for quality inspection in power transformers, offering insights into developing tailored sampling strategies to improve power distribution systems' reliability and performance. Marques et al. (2024) introduced an economic design of a single sampling plan for electronic components' reliability testing under exponential distribution, providing cost-effective reliability testing strategies and ensuring electronic devices' quality and performance in various applications. These studies collectively underscore the progress in sampling plan optimization and their application in quality inspection and reliability testing, contributing to robust quality management systems that enhance product quality, reliability, and customer satisfaction.

2.1. Acceptance Sampling by Attributes

Acceptance sampling inspection attributes quality responsibility to the producer, who must ensure product quality as specified to avoid issues and additional costs with unacceptable lots. Sampling plans are categorized into single, double, multiple, and sequential types. Their suitability is determined by comparing administrative challenges and the average sample size required by the available plans. Typically, the average sample size for multiple sampling is less than that for double plans, which, in turn, are smaller than those required for single sampling. However, the complexity of administering these plans varies inversely with their straightforwardness.

Acceptance sampling is an inspection methodology used to decide whether to accept or reject a product or service (Curram & Schilling, 1983; Saranya, Vijayaraghavan & Sharma, 2022; Polman, Haan, Veldhuijzen, Heideman, Vet, Meijer et al., 2019). It involves procedures that base these decisions on the inspection outcomes of sampled items. This type of sampling is applicable to a variety of contexts, including finished products, components, raw materials, operations, materials in process, and stored materials, among others. Acceptance sampling procedures are utilized when testing reveals non-conformance or deviation from the functional attributes of products. They can also be applied to various characterizing variables, assessing the degree to which product quality levels align with specifications. The primary objective of these procedures is to classify a lot as accepted or rejected, contingent upon the requisite quality levels (Duarte & Saravia, 2008).

Sampling presents a more advantageous option compared to 100% inspection in several scenarios: when product quality information is unavailable; when the lot comprises a large number of items, necessitating inspection with a considerable probability of inspection error; when automated inspection is not feasible; when ensuring product reliability is essential; even if the manufacturing process capacity of the lot is satisfactory; and in instances where the seller has historically provided excellent quality levels, prompting a desire to reduce inspection frequency, despite the process capacity being insufficient to forego inspection. The benefits of this sampling approach include cost-efficiency, as it requires fewer inspections, and minimizes damage from handling during inspection. It enhances the inspection task by shifting from part-by-part decisions to a lot-by-lot basis, proving useful for destructive testing, and focusing more on rejecting lots rather than returning nonconforming units. However, the disadvantages include the risk of rejecting conforming lots or accepting nonconforming lots, increased time spent on planning and documentation, reduced product information, and limited assurance that the entire lot meets specifications.

2.2. Simple Sampling Plans

Single sampling plans determine the acceptance or rejection of a lot based on the inspection results of a single sample comprising n items from the lot. The advantages of simple sampling plans include straightforward administration. However, due to the static sample size, they do not exploit the cost-saving potential associated with reduced inspection when the incoming quality is significantly high or low. This approach maintains accuracy when the sample size constitutes at most one-tenth of the lot size, but its reliability diminishes if n is too small or if the defective fraction, p', is too large (Vaughn, 1974). According to the Poisson distribution, the probability of acceptance, Pa, when implementing single sampling plans for a lot of size n and a specified defective fraction, p', is given by:

$$Pa = \sum_{r=0}^{c} \frac{(np')^r e^{-np'}}{r!} \tag{1}$$

If the random variable p' can be modeled using a continuous statistical distribution, then Equation (1) can be transformed into Equation (2), incorporating the integration over p':

$$Pa = \sum_{r=0}^{c} \int_0^1 \frac{(np')^r e^{-np'}}{r!} dp' \tag{2}$$

2.3. Double Sampling Plans

In double sampling plans, an initial, smaller sample is extracted from the submitted lot, leading to one of three possible decisions: accept the lot, reject the lot, or take another sample. Should a second sample be necessitated, the acceptance or rejection of the lot will be determined based on the outcomes of this subsequent analysis. One of the advantages of double sampling plans is their ability to reduce the overall sample size required when the incoming quality is either exceptionally high or low. This efficiency is due to the potential for the lot to be either accepted or rejected based on the assessment of the first sample alone (Montgomery, 1996).

To calculate the Pa for a given lot of size n with a fraction p' of defective items, an approximation method utilizing the Poisson distribution can be applied. This method involves the following Equation (3):

$$Pa = \sum_{r=0}^{c_1} \frac{(n_1p')^r e^{-n_1p'}}{r!} + \sum_{r=c_1}^{c_2} \left[\frac{(n_1p')^r e^{-n_1p'}}{r!} \sum_{i=0}^{c_2-r} \frac{(n_2p')^r e^{-n_2p'}}{r!} \right] \tag{3}$$

In this equation, n_1 and n_2 represent the sizes of the first and second samples, respectively, while c_1 and c_2 denote the acceptance numbers for these respective samples. The equation integrates the cumulative probability of acceptance across two scenarios: firstly, direct acceptance based on the first sample, and secondly, conditional acceptance predicated on the outcomes of both the first and second samples. This probabilistic model, grounded in the Poisson distribution, offers a nuanced approach for determining the likelihood of lot acceptance within the framework of double sampling plans.

2.4. Multiple Sampling Plans

In multiple sampling inspections, the approach aligns closely with that of double sampling, with the primary distinction being the requirement for more than two successive samples to arrive at a decision.

1) Sequential Sampling Plans: These plans are particularly advantageous when tests are destructive or expensive. Sequential sampling plans can significantly reduce sample sizes while maintaining robust protection levels.
2) Variable Sampling Plans: Unlike attribute sampling plans, which classify products as conforming or non-conforming, variable sampling plans utilize the actual measurements of sampled products to inform decision-making. These plans are inherently more complex to manage than attribute sampling plans and demand a higher level of administrative expertise. Variable sampling plans can offer protection comparable to that of an attribute sampling plan but with a reduced sample size. There are several types of variable sampling plans in practice, including those where the standard deviation (α) is known, unknown, and unknown but estimated using the range. When compared to an attribute sampling

plan, the required sample size for the aforementioned variable plans can be quantified as a percentage. Variable sampling plans enable the assessment of a process's performance relative to nominal or specified limits. While attribute plans make binary accept/reject decisions for a lot, variable plans provide nuanced insights into process performance. However, a limitation of variable plans is the assumption of a normally distributed population. Contrary to attribute sampling plans, separate characteristics within the same items will exhibit different means and variances, necessitating distinct sampling plans for each characteristic. Consequently, variable plans are more intricate to manage and incur higher measurement costs than attribute plans.

3) MIL-STD-414: The most prevalent variable sampling plan, MIL-STD-414, encompasses strategies for known variability, unknown variability (standard deviation method), and unknown variability (range method). Utilizing these methodologies, MIL-STD-414 facilitates testing against single or dual specification limits, estimation of process averages, and assessment of the source population's dispersion.

3. Serial or Continuous Sampling Plans, CSP

These plans are specifically designed for continuous production processes to address the challenges associated with applying lot-by-lot sampling plans in such environments. Continuous sampling plans feature alternating sequences of sampling and 100% inspection. Typically, these plans commence with a 100% inspection phase. Once a predetermined number of consecutive units are identified as defect-free, the process transitions to sampling inspection. This sampling inspection persists until a specified number of defective units are discovered, prompting a return to 100% inspection. Serial sampling plans are rectification-based inspection strategies, wherein each identified defective unit is either reworked or replaced with satisfactory units. This approach enhances product quality, achieving an average outgoing quality that is lower than or equivalent to the original process's defect fraction. Selecting a serial sampling plan necessitates the determination of an Average Outgoing Quality Limit (AOQL) that the plan aims to achieve. Various types of serial sampling plans exist, explained as follows:

3.1. CSP-1 Plan

This plan initiates with a 100% inspection of all units. Once *i* consecutive units are found to be defect-free, the 100% inspection phase is concluded, and only a fraction *f* of the units undergo inspection. These sample units are selected randomly, one at a time, from the ongoing production. Should any sampled unit be found defective, the process reverts to 100% inspection.

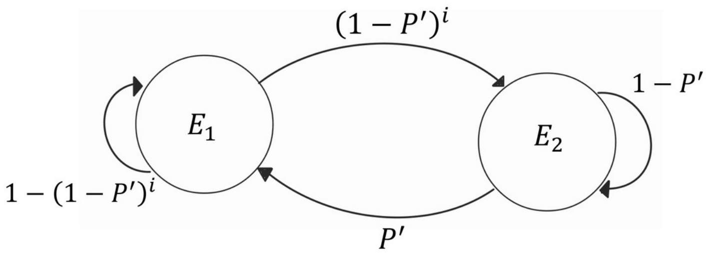

Figure 1. CSP-1 plan network

The parameters are defined as follows:

P' Probability of process non-conformity

E_1: State of 100% inspection

E_2: State of fractional inspection

Based on this model, it is possible to construct a transition matrix as observed in Table 1.

Markov States	E_1	E_2
E_1	$1-(1-p')^i$	$(1-p')^i$
E_2	p'	$(1-p')$

Table 1. Transition matrix for CSP-1 plan network

The CSP-1 encompasses two states: one representing a 100% inspection of units and the other a fractional inspection. A Markovian analysis of these states is performed to ascertain the long-term probabilities of maintaining one type of inspection over the other. With the transition matrix, the long-term inspection type probabilities can be determined. The steady-state equations are:

$$\begin{aligned} x_1^* &= 1-(1-p')^i x_1^* + P' x_2^* \\ x_2^* &= (1-p')^i x_1^* + (1-P')x_2^* \\ x_1^* + x_2^* &= 1 \end{aligned} \tag{4}$$

x_1^*: Probability of being in State 1(E_1)

x_2^*: Probability of being in State 2 (E_2)

From these equations, the stationary probabilities of remaining in 100% inspection (x_1^*) or in fractional inspection (x_2^*) can be determined, as observed in Equation (5):

$$\begin{aligned} x_1^* &= \frac{P'}{P' + (1-P')^i} \\ x_2^* &= \frac{(1-P')^i}{P' + (1-P')^i} \end{aligned} \tag{5}$$

Thus, the AOQL for the CSP-1 plan can be calculated according to the Equation (6), where Q represents the Lot size:

$$AOQL = \frac{QP' - QP'x_1^* - QP'fx_2^*}{Q} \tag{6}$$

The numerator of the expression represents the nonconforming units overlooked by the plan, summarizing the total nonconforming units allowed by the plan (QP') minus the nonconforming units detected during both 100% and fractional inspections ($QP' x_1^* + QP' f x_1^*$).

3.2. CSP-2 Plan

Under this type of plan (Figure 2), 100% inspection is reestablished when two nonconforming units are detected within a sequence of k sample units.

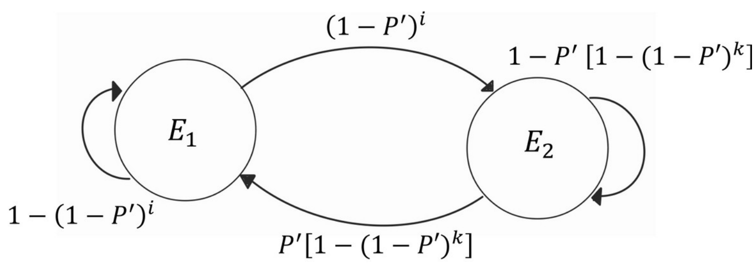

Figure 2. CSP-2 plan network

From Figure 2, the transition matrix is derived as observed in Table 2.

Markov States	E_1	E_2
E_1	$1-(1-p')^i$	$(1-p')^i$
E_2	$p'[1-(1-p')^k]$	$1-p'[1-(1-p')^k]$
	$x_1^*(1-(1-p')^i + x_2^*\{p'[1-(1-p')^k]\}$	$x_1^*(1-p')^i + x_2^*\{1-p'[1-(1-p')^k]\}$

Table 2. Transition matrix for CSP-2 plan network

The probabilities of remaining in 100% inspection and transitioning to or remaining in fractional inspection are given by:

$$X_1^* = X_1^*(1-(1-P')^i + X_2^*\{P'[1-(1-P')^k]$$
$$X_2^* = X_1^*(1-P')^i + X_2^*\{1-P'[1-(1-P')^k]\}$$
$$X_1^* + X_2^* = 1 \tag{7}$$
$$X_1^* = 1 - X_2^*$$

From this system, it can be deduced that:

$$X_2^* = \frac{(1-P')^i}{(1-P')^i + P'[1-(1-P')^k]} \tag{8}$$

Substituting Equation (8) in the Equation (7) yields:

$$X_1^* = 1 - \frac{(1-P')^i}{(1-P')^i + P'[1-(1-P')^k]} = \frac{P'[1-(1-P')^k]}{(1-P')^i + P'[1-(1-P')^k]} \quad X_1^*$$
$$= \frac{P'[1-(1-P')^k]}{(1-P')^i + P'[1-(1-P')^k]} \tag{9}$$

Furthermore, the inspected units, IU, are defined as:

$$IU = QX_1^* + QX_2^* f \tag{10}$$

Replacing Equation (8) and (9) in Equation (10), and expressing it as a function of P', the number of non-conforming units ($\#NCU$) detected by the inspection can be calculated as:

$$\#NUC = QP'(IU)$$
$$\#UNCD = QP'(QX_1^* + QX_2^* f)$$
$$\text{AOQL} = \frac{QP' - \#NUC}{Q}$$
$$\text{AOQL} = \frac{QP' - \#NUC}{Q}$$
$$\text{AOQL} = \frac{QP' - QP'\frac{[1-(1-P')^k] + f(1-P')^i}{(1-P')^i + P'[1-(1-P')^k]}}{Q} \tag{11}$$
$$\text{AOQL} = P'\left[\frac{(1-P')^i + P'[1-(1-P')^k] - P'[1-(1-P')^k] + f(1-P')^i}{(1-P')^i + P'[1-(1-P')^k]}\right]$$
$$\text{AOQL} = P'\left[\frac{(1-P')^i + f(1-P')^i}{(1-P')^i + P'^{[1-(1-P')^k]}}\right] = \left[\frac{(1-P')^i(1+f)}{(1-P')^i + P'^{[1-(1-P')^k]}}\right]$$

If $k = i$ we have:

$$\text{AOQL} = \left[\frac{(1-P')^i(1+f)}{(1-P')^i + P'[1-(1-P')^k]}\right] = P'\left[\frac{(1-P')^i(1+f)}{(1-P')^i(1-P')+P}\right] \quad (12)$$

This Equation (12) provides a method to calculate the *AOQL*, factoring in the detected non-conforming units and the efficiencies of switching between 100.

3.3. CSP-3 Plan

The CSP-3 Plan, while closely resembling the CSP-2 Plan, is engineered to afford additional protection against irregular production patterns (Figure 3). It stipulates that upon the discovery of a defective unit during sample inspection, the subsequent four units must undergo immediate inspection. Should any of these four units be found defective, 100% inspection is promptly reinstated. In the absence of any defective units among these four, the plan reverts to the CSP-2 protocol. This adjustment to the CSP-2 Plan is aimed at safeguarding against abrupt declines in quality.

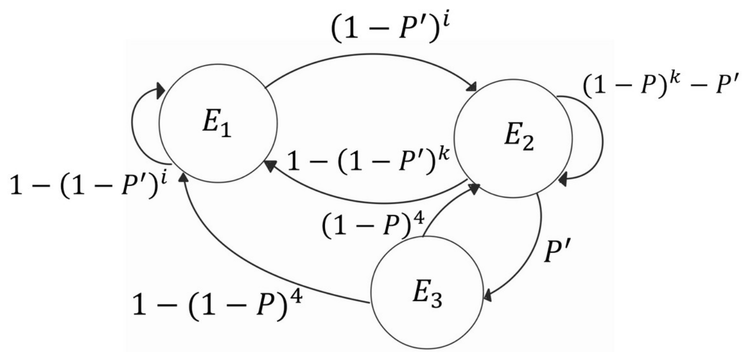

Figure 3. CSP-3 plan network.

where,

E_3: State of 100% inspection of four consecutive units, initiated upon detecting a non-conforming product in 2, E_2.

From this model, a transition matrix can be derived as showed in Table 3:

Markov States	E_1	E_2	E_3
E_1	$1-(1-p')^i$	$(1-p')^i$	0
E_2	$1-(1-p')^k$	$(1-p')^k - p'$	p'
E_3	$1-(1-p')^4$	$(1-p')^4$	0

Table 3. Transition matrix for CSP-3 plan network

Thus:

$$X_1^* = X_1^*[1-(1-P')^i] + X_2^*[1-(1-P')^k] + X_3^*[1-(1-P')^4] \quad (13)$$

$$X_2^* = X_1^*[(1-P')^i] + X_2^*[(1-P')^k - P'] + X_3^*[(1-P')^4] \quad (14)$$

$$X_3^* = X_2^* P' \quad (15)$$

with,

X_3^*: Probability that an item is in state 3 (E_3), in steady state

It follows that:

$$X_1^* = 1 - X_2^* - X_2^* P' = 1 - X_2^*(1 + P')\ (14) \tag{16}$$

$$X_2^* = \frac{(1 - P')^i}{\begin{bmatrix} 1 - (1 - P')^i + P'(1 - P')^i - (1 - P')^k + P' - \\ (1 - P')^4 P' \end{bmatrix}} \tag{17}$$

$$X_3^* = \frac{P'(1 - P')^i}{\begin{bmatrix} 1 - (1 - P')^i + P'(1 - P')^i - (1 - P')^k + P' - \\ (1 - P')^4 P' \end{bmatrix}} \tag{18}$$

4. Simple Sampling Plans for Acceptance by Attributes

This component of the study explores the variation in the probability of lot acceptance using different Simple Attribute Sampling Plans, defined by the parameters *n* (sample size) and *c* (maximum allowable defects). The analysis focuses on:

i. *Parameter Definition (n, c)* : Establishment of the parameters characterizing each sampling plan. These parameters are adjustable, allowing exploration of different configurations of the sampling plan. For *n* (samples) and *c* (number for acceptation or no-acceptation of lots).

ii. *Evaluation of Acceptance Probability*: The probability that a lot will be accepted under various nonconformity fractions p', ranging from 0 to 1, is calculated. This probability is visualized through an approximation to the Operating Characteristic Curve.

iii. *Inspection Cycle and Counting:*

- *Lot Counters:* The variables *Accept* (accepted lots) and *NoAccept* (rejected lots) are reset to zero after inspecting 100 lots, to begin a new count in each evaluation cycle.
- *Internal Inspection Cycle:* An internal cycle is developed for the inspection of each lot, where a *rep* counter (representing the number of lots inspected for each nonconformity fraction p' is incremented with each new lot. A sample of size *n* is taken from the lot, and the original entity pauses while the sample is inspected.

iv. Sample Inspection and Decision:

- *Defect Recording:* A variable *CantNoConforms* records the number of defective products identified in the sample.
- *Evaluation and Decision:* Once the sample inspection is complete, it is evaluated whether the number of defects exceeds the maximum allowed c. Based on this result, it is determined whether the lot is accepted or rejected.

v. *Visualization and Analysis:*

- *Operation Curve:* Figure 4 shows the Operation Curve of the Plan, which illustrates how the Acceptance Probability evolves as the lot's nonconformity fraction increases, allowing evaluation of the plan's performance in various applicable scenarios.

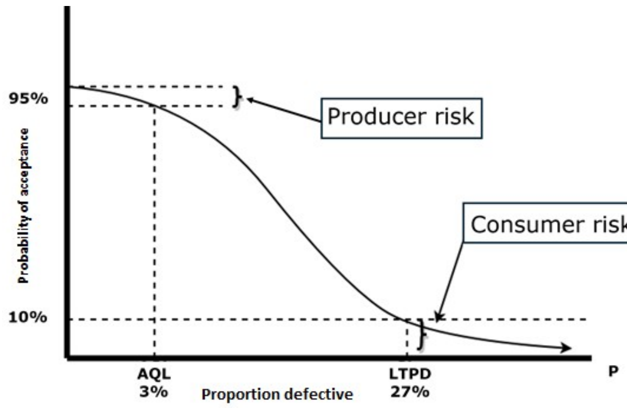

Figure 4. Operation characteristic curve for sampling plans

5. Analysis of Results for Simple Acceptance Sampling by Attributes

The Simple Sampling Plans for Acceptance by Attributes, as previously discussed, underwent experimental manipulation to elucidate the variability in the Probability of Acceptance and/or Rejection within these plans. Consequently, adjustments were made to the values of *n* and *c* within the model to facilitate a comparative analysis of the plans' behavior. Simulations were conducted for fixed values of *n* with varying *c*, and for increasing values of *n* with proportional adjustments in *c*. The Operation Curves generated from each simulated plan served as the primary analytical tool. It is pertinent to note the existence of two types of Operating Curves (OC).

Type B OCs arise from sampling randomly selected lots from a process operating randomly with an average nonconformity fraction p'. Consequently, the lots may exhibit varying defective fractions, aligning with a binomial probability distribution. This sampling method effectively mirrors direct process sampling. The Probability of Acceptance, as dictated by the binomial distribution, can be theoretically calculated using the binomial formula or one of its approximations, such as the Poisson distribution. In this context, Pr(Accept) is interpreted as the proportion of processed lots anticipated to be accepted by the process. The Poisson distribution assumes that nonconformities occur independently at a constant rate, which holds true in numerous instances.

Conversely, Type A OCs presuppose that all inspected lots contain an identical fraction of defectives, rendering the Probability of Acceptance as the average proportion of lots accepted from an infinite series of identical lots. Under these circumstances, Pr(Accept) is determined by a hypergeometric distribution, reflective of sampling from a finite universe—the lot itself. It may also be approximated using a binomial summation. However, the inherent characteristics of these curve types suggest that, for lots of infinite size, Type A OC is mathematically equivalent to Type B. Therefore, for large batch sizes, no distinction between the two types of Operating Curves is presumed. This research adopts this latter assumption. In practice, the precise quality level of a lot under inspection remains unknown. Were it otherwise, direct judgment of lots without the necessity for inspection would be feasible. The Operating Curve delineates the anticipated performance of the sampling plan under specific conditions, emphasizing the importance of understanding the implications of Operating Curves when selecting *n* and *c* values. Comparing Operating Curves facilitates the evaluation of different simple sampling plans. Ideally, a Sampling Plan would accept lots with a probability of 1 for those whose nonconformity fraction falls below the AQL and reject with equal certainty those lots exceeding this threshold. With these considerations in mind, the analysis focuses on the outcomes obtained with the model.

5.1. Analysis of Results for Constant *n* and Varying *c*

This analysis was conducted with a constant $n = 40$ and varying *c* values (0, 1, 2, and 3). A similar approach was taken for $n = 50$ to amass data under differing conditions yet with analogous characteristics, enabling a generalization of the analysis. The Operating Curves plotted for each simulated plan are showed in Figure 5:

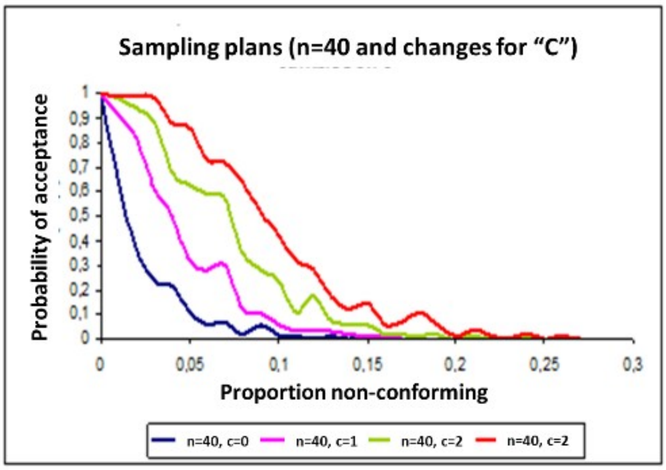

Figure 5. Comparison of sampling plans with constant n=40 and variations in c

The results illustrate that as the allowable number of defective units within the sample, denoted by c, decreases, the Operation Curve of the plan shifts leftward. This shift signifies that the plans increasingly adopt the propensity to reject lots at lower levels of nonconformity fraction p'. This tendency becomes more pronounced as c diminishes, with the curve evolving from a concave to a convex shape at $c = 0$. Analyzing the Probability of Acceptance distribution reveals the impact of c on the rate of change in the Probability of Acceptance:

- For the binomial distribution:

$$\Pr(Accept) = P(x \leq c) = \sum_{x=0}^{c} \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x} \tag{19}$$

- For the Poisson distribution:

$$\Pr(Accept) = P(x \leq c) = \sum_{x=0}^{c} \frac{e^{-np}(np)^{-x}}{x!} \tag{20}$$

This dynamic indicates that the Probability of Acceptance begins to decline sharply, even at minimal values of p', potentially disadvantaging both the supplier and the consumer by rejecting lots of acceptable quality.

5.2. Analysis of Results for Different Sample Sizes *n* and Proportional *c*-Values

In this analysis, simulations were performed for varying n values while maintaining proportional changes in c. The scenarios assessed included $n = 40$, $c = 1$; $n = 80$, $c = 2$; and $n = 160$, $c = 3$, to facilitate generalization of findings. The results, depicted in Figure 6, utilized the binomial distribution for Type B operating curves concerning the producer's risk.

The findings demonstrate that as the sample size increases, the Operating Curve becomes steeper, indicating a sharper slope. This trend approaches the ideal plan curve, where a high Probability of Acceptance is afforded to lots with low fractions of nonconformities, and this probability swiftly diminishes for p' values exceeding the Not Acceptable Condition (NAC).

This phenomenon is elucidated by the statistical distribution of the Probability of Acceptance, as a larger sample size increases the likelihood of detecting fewer than the desired number of nonconformities at low success probabilities. Conversely, for high success probabilities, the chance of finding fewer successful events decreases rapidly. Thus, producers are motivated to ensure very low rates of nonconformity to avoid the risk of complaints, claims, or issues with customers due to high percentages of defective products.

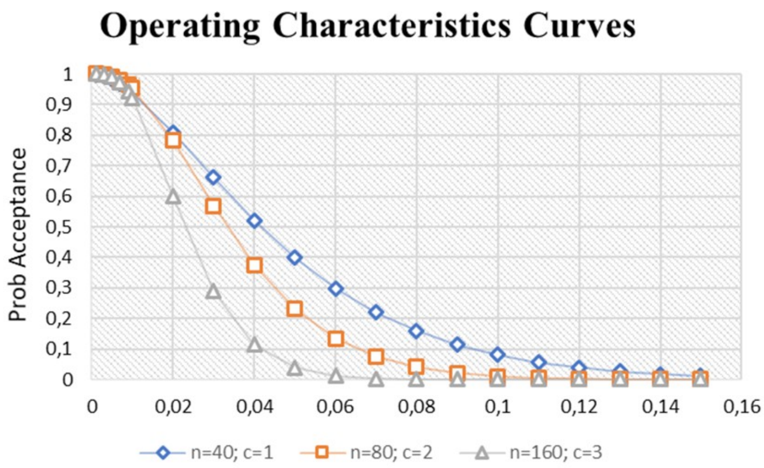

Figure 6. Comparison of sampling plans with variations in *n* and *c*

Accordingly, it is evident that the precision with which a plan discriminates between acceptable and unacceptable lots improves with sample size. However, choosing the appropriate sample size must consider the optimal cost-benefit ratio. The enhanced accuracy of larger samples must be weighed against the increased costs. Although beyond the scope of this text, it is noteworthy that Sampling Plan design methods often establish specific risks for the producer (α) and the consumer (β), where α represents the likelihood of rejecting high-quality lots, and β indicates the chance of accepting poor-quality lots. The definitions of “good” or “bad” quality are contingent upon the AQL and LTPD agreed upon by the consumer and supplier.

6. CSP-1 Continuous Sampling Plans

In this section, a model was designed for the analysis of Continuous Sampling Plans, specifically the CSP-1 type. This model facilitates the examination of the probabilities of being in each phase of the plan —100% Inspection and Fraction Inspection— and also enables the observation of the Average Outgoing Quality’s behavior for varying values of the parameters *i* and *f* characteristic of these plans. The model begins with generating entities that symbolize the products of a continuous production process. For this analysis, a production run of 100 000 units was selected to ease the interpretation of results.

Subsequently, the entity determines whether to proceed with 100% Inspection or Fraction Inspection based on the Plan’s current state. This status is denoted by a variable named “Status,” which assumes a value of 1 for 100% Inspection and 2 for Fractional Inspection. As outlined, CSP-1 Plans initiate with a 100% Inspection phase. Depending on the outcome of this initial decision, the product will follow one of two paths: either continuing with 100% Inspection or transitioning to Fraction Inspection.

7. Discussion

Sampling Plans serve as a pivotal guide in the acceptance and/or rejection of lots. While they prescribe a course of action, their aim is not to ascertain the quality of the lot directly. It is crucial to note that Attribute Acceptance Sampling Plans are not designed for Quality Control, which is the domain of control charts, representing the essence of quality management. Acceptance sampling’s role is primarily to accept or reject lots based on predefined criteria.

The effective implementation of attribute sampling plans requires careful consideration of their design and integration with quality control systems. The parameters of sampling plans, such as sample size and acceptance criteria, must be tailored to the specific context and quality requirements of the industry. This adaptation is crucial since different industries and processes have varying tolerances for defects and variances in production quality.

Integrating sampling plans with quality control systems enables continuous monitoring of the production process, facilitating early identification of issues. This integration ensures that decisions on the acceptance or rejection of lots are based not only on representative samples but also on a deep understanding of the overall production process performance. In summary, proper design and integration of sampling plans are essential to ensure that final products consistently meet the expected quality standards.

The implementation of a sampling plan in the inspection process, using sampling systems such as CSP sampling plans, requires careful consideration of switching rules. These rules are essential as they determine when and how to switch between different levels of inspection, directly affecting the efficiency and effectiveness of the sampling system.

Excluding reduced inspection from the sampling plans available to the producer can deprive them of the benefits of maintaining exceptional quality. Reduced inspection acts as a reward for high-quality standards, allowing the producer to use smaller sample sizes and increase the probabilities of lot acceptance. Therefore, it is crucial to consider this option in the sampling plans adopted for attribute inspection.

The design of different switching rules between classes of inspections can be customized according to the needs of the involved parties. The performance of these rules is reflected in the operating characteristic curves, but this should not be the sole evaluation criterion. It is also important to consider other derived curves, such as the probability of being in normal inspection, reduced inspection, and the probability of switching between inspections.

In the future, these sampling plans could be integrated with process capability indicators, which are crucial for decision-making in product quality. These indicators can determine when to switch from one sampling plan to another based on quality results. This approach offers a promising field for research in statistical quality control, combining acceptance sampling and attribute inspection, thereby enhancing decision-making and quality management in various industries.

8. Conclusions

Through this investigation, we successfully developed a simulation model that generically represents Simple Attribute Acceptance Sampling Plans. This model simulates the lot inspection process under these plans, yielding results for the Probability of Acceptance across various potential defective fractions within the lots. It requires only the specification of the Plan's n and c parameters for analysis. The development of this model facilitated the evaluation of such Plans across different n and c values, thereby enabling the derivation of Operating Curves as a testament to the performance of the evaluated plans.

The derived Operating Curves enabled a comparative analysis across various scenarios, leading to several key conclusions:

- As A decrease in the allowable number of defective units in the sample, c, shifts the Operating Curve leftward, indicating an increased propensity of the Plans to reject lots at lower nonconformity levels, p'. This trend becomes markedly pronounced with further reductions in c, affecting the Probability of Acceptance significantly and potentially disadvantaging both suppliers and consumers by rejecting high-quality lots.
- An increase in sample size renders the Operating Curve steeper, enhancing the Plan's precision in distinguishing between good and bad lots, thereby approximating the ideal plan that accepts lots below the Acceptable Quality Level (AQL) and rejects those exceeding it.

Additionally, a model for CSP-1 Continuous Sampling Plans was also developed, enabling the simulation of a continuous sampling process and providing valuable insights into its operational characteristics.

From the CSP-1 model simulations, various insights were gleaned:

- The percentage of 100% Inspection increases with larger fractions of nonconforming units, reflecting the Plan's protective mechanism against allowing significant quantities of defective products to pass.
- The concept of AOQL emerged, highlighting that for lower and higher p' values, the CSP remains minimal due to frequent 100% inspections for large nonconformity fractions and small p' values, respectively.

- Increasing the proportion of units subjected to sampling inspection, *f*, leads to a rise in 100% Inspection occurrences, enhancing defective unit detection but also improving the Average Outgoing Quality due to more stringent inspections.

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