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Multi-Mode Replenishment Strategies for Periodic-Review in Two-Echelon Systems Under Seasonal Demand

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Abstract:

Purpose: This paper investigates the impact of special replenishing modes within a 2-echelon inventory system under seasonal demand. Within this system, a periodic-review base-stock policy is employed. Two special replenishing modes are considered: emergency and transshipment in addition to a regular mode. The regular mode serves as the primary replenishment method, while emergency and transshipment modes, characterized by shorter lead times and higher costs, are reserved to prevent stockouts. It specifically examines the differences in outcomes between static and dynamic ordering policies for these special modes.

Design/methodology/approach: Methodologies to determine static and dynamic policies of two special replenishments: emergency and transshipment are proposed. Both emergency and transshipment replenishments are based on (R, s, S). A simulation method was used to evaluate the proposed policies.

Findings: Special modes can be used to maintain service level while utilizing a lower safety stock, thereby reducing overall holding cost. The 60% higher frequency of emergency orders under static policies compared to dynamic policies leads to a lower number of transshipment orders. For short cycles with high per-period demand variability, the gap between static and dynamic policies shrinks, making static policies a viable, less-complex alternative. Levels of demand fluctuations between periods impact a policy choice. While dynamic policies may not provide a distinct advantage over static policies in low-fluctuation scenarios, they can yield cost savings in high-fluctuation environments, albeit with increased effort.

Practical implications: The result from this paper can be adopted to a 2-echelon inventory system with multiple replenishing modes under seasonal demand. It can help inventory managers choose the appropriate policy for their situation.

Originality/value: This paper provides managerial insights regarding the circumstances in which static policy or dynamic policy should be applied and explores the relationship between regular and special replenishing modes in various circumstances.

Keywords: multi-echelon inventory, seasonal demand, transshipment, multiple replenishing modes, periodic review

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1. Introduction

Nowadays, Companies around the world not only compete on price and product but also on the reliability of deliveries due to globalization (Mirabelli & Solina, 2022). One way to higher reliability is better inventory management. There is a considerable number of studies dealing with multi-echelon inventory models in the past decades (Kouki, Arts & Babai, 2024). The system involves a lot of decisions such as how many items should be stored at each location and at what time they should be transported from the warehouse to retailers. Multi-echelon inventory optimization plays an important role in a supply chain minimizing total cost while maintaining specified customer service levels but making decisions in these inventory systems is a challenging task (Achkar, Brunaud, Pérez, Musa, Méndez & Grossmann, 2024). Even though a proper inventory policy is applied. Under stochastic demand, a retailer could run out of stock and the demand is considered lost. In this situation, a retailer could use a special replenishing mode with shorter lead time to prevent stockouts. These special modes can be an emergency replenishment which is a replenishing mode with shorter lead time supplied by the warehouse (Minner, 2003) or a transshipment which is a mode where items are requested from other retailers with excessive on-hand stock (Paterson, Kiesmuller, Teunter & Glazebrook, 2011).

This paper focuses on a 2-echelon inventory system with a single warehouse and N non-identical retailers. In normal circumstances, retailers are supplied by a warehouse which is supplied by external suppliers via a regular replenishment. Items can be stored at the warehouse and retailers. This regular replenishment has a fixed lead time. However, when a retailer faces the risk of stockout, items could be supplied via one of the special modes with fixed shorter lead times i.e., emergency replenishment from warehouse and transshipment from another retailer. Unfilled demand is considered as demand loss. This demand loss makes a problem more complicated because it is difficult to estimate the on-hand stock level (Guijarro, Babiloni, Canós-Darós, Canós-Darós & Estellés-Miguel, 2020). The replenishing modes applied in this paper are shown in Figure 1.

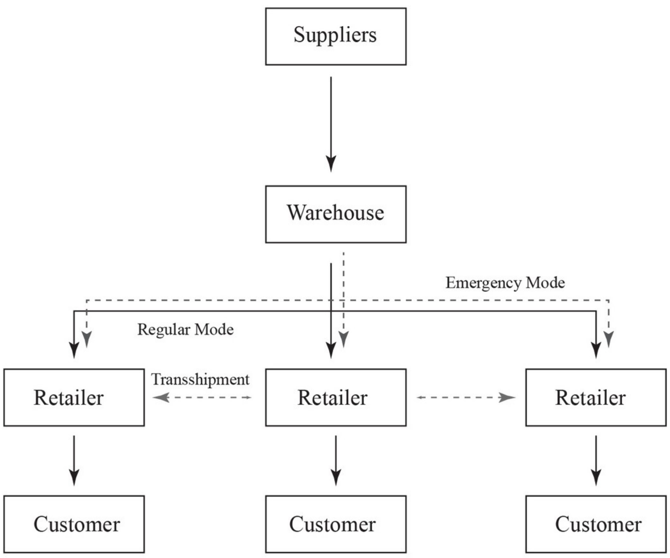

Figure 1. Replenishing modes in the system

Customer demand follows a repeating weekly pattern without any long-term trend. It fluctuates predictably through these cycles, with each day's demand typically distributed normally with unique parameters. We used the algorithm proposed by Sakulsom and Tharmmaphornphilas (2019) to find the ordering policy for a regular replenishment as they work on the same seasonal demand pattern. Although the policies from the algorithm give low total demand loss over the considered time horizon, there are some periods with high lost sales. Therefore, to reduce the number of these periods, special replenishing modes are applied.

This paper proposes methodologies for crafting periodic review-based order policies for special replenishing modes: emergency and transshipment. Both static (invariant across periods) and dynamic (demand-responsive) policies are explored. While static policies provide ease of implementation, dynamic approaches, utilizing periodic demand data, offer potential for cost reduction and minimized special orders, particularly for seasonal products. Nonetheless, a comprehensive investigation of their suitability for each replenishment mode is crucial for optimal performance.

The remainder of this paper is organized as follows. Section 2 reviews the literature related to the system with multiple replenishing modes. Section 3 presents a problem description. Section 4 describes a methodology to determine ordering policies for special modes. Section 5 presents results and discussions. Finally, Section 6 concludes and suggests future research extensions.

2. Literature Review

To mitigate stockout issues, a company can utilize multiple replenishment channels. In addition to its regular mode, it can incorporate other replenishment options with shorter lead times. For instance, a company reliant on sea freight could also employ airfreight as an emergency option, offering faster delivery but at a higher cost.

Emergency replenishing mode is a special replenishment with shorter lead time, but higher cost, used in case of imminent shortage from the higher-echelon location (Tagaras & Vlachos, 2001). Generally, a system with more than one source is considered a dual-supply system, where items are replenished by two distinct sources or from a single source with two different modes (Minner, 2003; Yao & Minner, 2017). The dual-supply problem has been studied in various aspects and applied under two main policies: continuous review and periodic review.

Moinzadeh and Nahmias (1988) developed a heuristic algorithm for a system with two supply modes under continuous review, applying a (Q1, Q2, R1, R2) policy. Under this policy, an order of Q1 was placed when on-hand inventory reached the R1 reorder point, and an order of Q2, with a shorter lead time, was placed when on-hand inventory reached the R2 reorder point. They used a simulation to validate the algorithm and studied the difference in operating costs between systems with and without a special supply mode. Zhou and Yang (2016) proposed a heuristic to find a policy for two replenishing modes under continuous review, where both modes used batch orders. Chiu, Chiu, Lin and Chang (2019) studied a system with multiple products and two replenishing modes: in-house and outsourcing. They developed a model to minimize the total cost based on the portion of outsourcing, cycle length, and defect rate.

For products with selling seasons where the retailer had a limited time frame to order an emergency shipment, Poormoaied and Hosseini (2021) studied how to apply emergency shipments to the system using a newsvendor model to maximize expected profit. Poormoaied and Demirci (2021) studied a continuous-review stochastic inventory problem with emergency orders where the supplier had uncertain available and unavailable periods. They suggested approaches by analyzing the model for different lead time scenarios. Zhao, Wang and Wu (2022) studied the effect of emergency shipments as a combination of the effect from customers' predicted price based on historical price data. Gao, Qu, Jiang and Hou (2024) studied emergency order strategies in a closed-loop supply chain where new products and remanufactured products were considered by customers. Backlogged demands can be served via emergency orders within certain proportion based on the cost of emergency order and the cost of out-of-stock loss.

For periodic review systems, various aspects of constraints, such as time to place emergency orders or order size, have been studied. Chiang and Gutierrez (1996) proposed a model with two replenishing modes under periodic review. At each review period, either a regular order or an emergency order was placed to bring the inventory level up to an expected level. Chiang (2003) extended the model with different variable costs between a regular mode and an emergency mode. Chand, Li and Xu (2016) proposed a similar model, but allowed the buyer to choose between two delivery modes at the beginning of the period. They assumed unmet demand was backordered, and a backlogging cost varied with the length of backlogging time. Therefore, the buyer had to trade off delivery cost and backlogging cost. Chiang and Gutierrez (1998) allowed multiple emergency orders within a review period. Regular orders and emergency orders were placed periodically, but emergency orders had a smaller review interval. Chiang (2001) analyzed a special case of the same problem with a one-period difference between lead times of a regular mode and an emergency mode. Bylka (2005) proposed a model similar to Chiang and Gutierrez (1998) but included an inventory capacity constraint and a limited backlogging constraint. Tagaras and Vlachos (2001) proposed a model for an emergency mode where an emergency order would be placed as late as possible to make the items arrive right before the end of the period. The emergency order was placed to raise on-hand level up to the threshold level. When the on-hand level was less than the threshold level, an emergency order was placed to raise on-hand up to the threshold level.

When the on-hand level was less than the threshold level, an emergency order was placed to raise on-hand up to the threshold level. Huang, Zeng and Xu (2018) proposed a system where regular and emergency orders were supplied by the capacitated suppliers. Regular orders were triggered before the demand was realized but emergency orders were triggered after demand realization. The quantity of an emergency order depended on remaining capacity of suppliers. Johansen and Thorstenson (2014) proposed a Markov decision model for a system where regular orders were controlled with a reorder point and a fixed order quantity and emergency orders were controlled with reorder and order-up-to points. Both regular and emergency orders had constant lead time. Then Johansen (2019) extended the model by assuming stochastic lead time for regular orders. Johansen (2019) also explored the impact of using emergency order in a periodic-review inventory system by studying many combinations of using normal order with emergency order and proposed a control policy that had slightly higher cost but was more practical. Akbalik and Papine (2018) studied a single item incapacitated lot sizing problem with multi-mode replenishment and batch deliveries. They prove that this type of problem is NP-hard and the multi-mode replenishment is only a special case of the single mode problem. An algorithm was also proposed for the problem. Avci and Selim (2018) solved an inventory problem where any stockout or delay were prevented with faster last-minute emergency order called premium freights. A multi-objective simulation-based optimization approach was developed to minimize total holding cost and premium freight ratios simultaneously. Chen, Zhao, Fransoo and Li (2019) studied a dual mode system under a chance credit constraint where customers were allowed to occasionally exceed the credit limit. They developed a simulation-optimization algorithm to determine the inventory policy and they also studied the impact of the chance credit constraint on the performance of the system. Rosales, Magazine and Rao (2020) explored the replenishing policies for a hospital inventory problem where item could be replenished via an urgent option with higher cost. They also applied joint replenishment when items were ordered for this urgent option to provide cost benefits. A simulation-based algorithm was used to test proposed policies. Poormoaied, Atan and van Woensel (2022) studied a periodic-review retailer who used a quantity-based policy for emergency order where it was triggered when the inventory level was below a certain level. An algorithm was proposed to determine policies for both regular and emergency replenishment to minimize total expected cost.

All previous papers studying inventory systems only considered arborescent distribution systems. (An arborescent system resembles a tree, where each location receives items only from one higher location.) However, this paper also considers lateral transshipment, which relaxes the system for more flexibility but leads to more complex decisions. To allow lateral transshipments, locations at the same level must pool their inventories (Paterson et al., 2011). There are two types of pool policies: complete pooling and partial pooling. With complete pooling, items can be freely transshipped without conditions. Conversely, with partial pooling, items are reserved for local future demand and transshipped only when excess stock exists. Another classification of transshipment orders concerns their timing. Predetermined events used to redistribute inventories before demand observation are proactive transshipments. Reactive transshipments occur in response to stockouts or potential stockouts. Studies of transshipment orders involve both single-echelon and multi-echelon structures.

Robinson (1990) developed a heuristic for multi-location, multi-period problems with transshipments. Optimal ordering policies were determined for two special cases: two non-identical locations and any number of identical locations. Olsson (2015) studied a single-echelon system with two identical locations and positive transshipment lead times. An ordering policy was developed with a heuristic algorithm, separating the system into two sub-systems, each with one retailer. The positive lead time was managed by tracking residual lead times to decide whether to wait for an upcoming regular order or request a transshipment. Tlili, Moalla and Campagne (2012) studied a 2-echelon system with two identical retailers and transshipments. Demand followed an independent, identical normal distribution. They developed an initial solution with heuristics based on simulation optimization and used simulation for fine-tuning to the optimal solution. Tai and Ching (2014) also studied a 2-echelon system with several identical retailers. An ordering policy, including a transshipment policy, was developed using a Markovian model.

Bhatnagar and Lin (2019) studied a multi-location system using a joint transshipment and production policy that determines when a location should produce or perform transshipments. Two heuristics were proposed to determine these policies. Abbasi, Babaei, Hosseinifard, Smith-Miles and Dehghani (2020) proposed an approach to solve a large-scale problem using machine learning models. Their approach was applied to decide on transshipment of blood units in a hospital network, where the model reduced costs by about 29% compared to the current policy. Dehghani, Abbasi and Oliveira (2021) also proposed a model for blood inventory among hospitals using preventive transshipments to avoid shortages. For a manufacturing system, Dhahri, Gharbi and Ouhimmou (2022) proposed a transshipment policy for a system with two unreliable locations to minimize total costs (holding, backlog, and transshipment). The policy parameters were determined by a simulation-based optimization approach. Wang and Minner (2024) developed a deep reinforcement learning algorithm to solve a problem of online retail with multiple sources where customers were served by one of many distribution centers. They also investigate that in which situation transshipment between sources would lead to the lower cost. Zhou, Guo, Yu & Zhang (2024) developed a multi-agent deep reinforcement learning algorithm for a two-echelon inventory system. The system serves spare parts to wind farms scattering different areas. Local warehouses are replenished by a central warehouse and emergency transshipment is also considered.

In this paper, we study two special replenishing modes in addition to the regular replenishment. The emergency mode places orders similar to Chiang and Gutierrez (1998) and Chiang (2001). In this mode, the inventory position is reviewed periodically, and both regular and emergency orders can be placed within each period. Emergency orders have a smaller review interval. If the warehouse cannot fulfill an emergency order, the retailer will request a transshipment from another retailer. This transshipment utilizes a partial pooling concept, where items are only transshipped when they are in excess. Under seasonal demand, we studied the difference between using dynamic and static policies for these two special replenishing modes.

3. Problem Statement

This paper investigates a two-echelon inventory system with a central warehouse supplying non-identical retailers facing seasonal demand. It is assumed that the demand is seasonal without trend. Additionally, each period within a fixed cycle experiences demand following a normal distribution with unique mean and standard deviation, consistent across cycles. All locations utilize a periodic-review base-stock policy, ordering inventory with a fixed lead time. Retailers receive stock from the warehouse, which itself can store items and orders from external suppliers. Unfulfilled demand during any period is considered lost. We consider the service level as a fill rate - the proportion of demand served from on-hand inventory (Nahmias, 2009). Therefore, in each period, the ratio of demand served from on-hand inventory to period's demand must not be lower than expected service level.

While safety stock mitigates demand uncertainties, occasional spikes can still lead to stockouts and service level deficiencies. To address this, the system employs two additional replenishment modes:

1. Emergency Replenishment: Retailers can use this mode to expedite orders from the central warehouse. Despite the shorter lead time, emergency orders incur higher costs. However, warehouse inventory limitations may occasionally prevent fulfillment of these requests.
2. Transshipment Replenishment: When emergency replenishment fails due to warehouse depletion, retailers can activate transshipment. This mechanism facilitates peer-to-peer inventory transfers amongst retailers on the same echelon, utilizing surplus stock in one location to alleviate shortages in another. While transshipment boasts improved lead times over emergency replenishment, the cost structure reflects the potential stockout risk incurred by the supplying retailer. Therefore, emergency mode remains the preferred option unless warehouse constraints dictate otherwise.

All replenishing modes operate on periodic review basis using reorder point and order-up-to point or (R, s, S) where review intervals for all modes are given.

An example of the inventory movement in the system having 3 replenishing modes can be shown in Figure 2. The system contains a central warehouse and 2 retailers. To keep it simple, every location has 1-period lead time and 1-period review interval for a regular replenishing mode and zero lead time and continuous review for special modes. With a 1-period review interval, a regular mode reviews inventory position at the end of every period. If an inventory position reaches a reorder point during any period, an order will be placed at the end of that period. On the other hand, with continuous review for special replenishing modes, whenever an inventory position reaches a reorder point, an order is immediately placed. A reorder point and an order-up-to point for a regular mode of each location are shown in Figure 2.

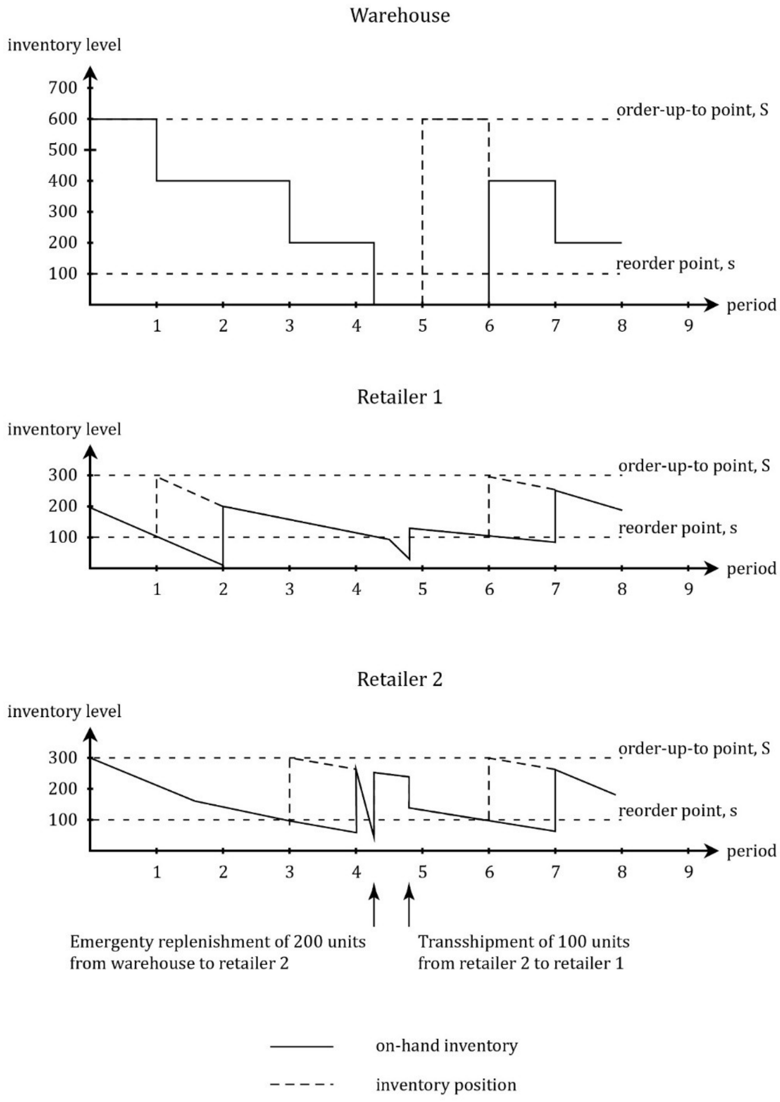

Figure 2. Inventory movement in a system with 3 replenishing modes

An order is placed as the inventory level reaches a reorder point. At the beginning of period 1, retailer 1 reaches a reorder point and places an order of 200 units. The inventory position immediately rises to 300 units and the inventory level at the warehouse drops from 600 to 400 units. Afterwards, the order arrives at the beginning of period 2.

In period 4, retailer 2 receives an order that is placed in period 3 at the beginning of the period and demand spike consumes items. The retailer is at risk of stockouts, so it requests an emergency order from the warehouse. The order depletes the inventory at the warehouse. Therefore, in the same period, when retailer 1 requests an emergency order afterward, the warehouse cannot satisfy an order. Consequently, retailer 1 must request a transshipment order from retailer 2 near the end of period 4. Details of the policies will be discussed in the methodology section.

Besides applying special replenishing modes as shown in Figure 2, there are alternatives for these special modes. Due to seasonal demand pattern, applying different policies for each demand period can minimize cost; however, its complexity grows with demand volatility (Tunc, Kilic, Tarim & Eksioglu, 2011). This necessitates a trade-off between operational simplicity and cost optimization. Addressing this, two approaches emerge for special replenishment modes: static and dynamic.

- Static policies: Employing the same ordering rule across all periods, these offer ease of implementation and predictable routines. However, they might underestimate the peak demand of each seasonal demand period.
- Dynamic policies: Tailoring order rules to each period within a cycle, these potentially achieve lower costs by adapting to demand fluctuations. However, they require a more complex design and ongoing adjustments.

Both approaches rely on the fundamental reorder point and order-up-to-level mechanism for inventory control.

We investigate how these emergency and transshipment replenishing modes can reduce stockouts and how they affect the system. Moreover, static policy and dynamic policy are explored that which policy is preferred in different situations.

4. Methodology

The policy of the regular mode is determined with the methodology proposed by Sakulsom and Tharmmaphornphilas (2019). Initial reorder point and order-up-to point for each location are determined with heuristic algorithm and safety stock levels are determined with simulation to find the final policy which satisfies the expected service level on training instances. However, under stochastic demand, a retailer could run out of stock which leads to demand loss. Special replenishing modes would be used to prevent shortage. In this paper, it is assumed that reorder point and order-up-to point for the regular mode are given. The methodologies to specify inventory policies for emergency and transshipment modes are developed.

Special replenishing modes operate on a faster timescale than regular ones, utilizing shorter lead times and review intervals. To facilitate this, each period is subdivided into smaller sub-periods. We assume demand within each sub-period to be a proportional fraction of the entire period's demand. Inventory positions are reviewed at the end of every sub-period for potential special orders. If placed at the end of sub-period j, such orders arrive at the end of j + leadtime and are immediately available. However, a key constraint exists: special orders can only prevent stockouts within the period they are placed. Orders arriving in the next period are deemed ineffective and therefore unplaced. This means, for example, with a lead time of one sub-period, special orders placed during the last sub-period of any period would arrive in the next period's first sub-period. Since they would not prevent stockouts in the original period, these orders are excluded. The details of the special modes are as follows.

4.1. Emergency Mode

Under the emergency replenishment mode, equation (1) defines the reorder point by factoring in anticipated demand until the period's end, thereby guaranteeing the desired service level. Since an emergency mode is triggered when a location faces risk of stockout, a dynamic policy adapts to seasonal fluctuations by having a unique reorder point for each period in the demand's cycle. In contrast to a dynamic policy, a static policy employs a single, fixed reorder point throughout the cycle. To ensure adequate inventory levels and avert stockouts in every period, this reorder point is set to match the highest value calculated for any individual period within the cycle.

$$s = \hat{x}_{R1+L1} + z\sigma_{R1+L1} \quad (1)$$

where,

$R1$ = review interval of an emergency mode (sub-period)

$L1$ = lead time of an emergency mode (sub-period)
$\hat{x}_t$ = expected demand in the duration t (unit)
σ_t = standard deviation of demand over the duration t
z = the z-value corresponding to the expected service level

While emergency replenishment aims to prevent stockouts, it should not disrupt the optimal order timing established for the regular replenishment mode. This is because, as Sakulsom and Tharmmaphornphilas (2018) observed, ordering the same quantity in different periods under seasonal demand can lead to varying holding costs. Therefore, the regular order is strategically placed within the demand cycle to minimize such costs. The order-up-to point for emergency orders is also designed with this in mind. Placing an excessively large emergency order can potentially delay the regular order, potentially incurring higher holding costs due to the mismatch in timing. Consequently, emergency mode policies carefully balance the goal of eliminating unsatisfied demand with maintaining the smooth operation of the regular replenishment cycle.

From a regular replenishing mode, we can calculate expected on-hand levels of each period. For example, in Table 1, for a 4-period demand cycle, a regular order is expected in period 1 of each cycle. The on-hand levels shown in the table are the levels at the end of periods. These numbers repeat cycle after cycle. Therefore, expected on-hand levels at the end of periods 1, 2, 3 and 4 are 1,480, 5,000, 3,800 and 2,360 respectively. Please note that the expected on-hand levels in Table 1 are calculated based on the expected demand without considering demand's deviation. For a dynamic policy, if retailer's on-hand level reaches emergency reorder point in any sub-period, an emergency order is placed to raise on-hand level to the expected on-hand level of that period. Then, a regular mode can continue with the same ordering pattern.

For a static policy, the order-up-to point is set to the lowest expected on-hand level that surpasses the emergency reorder point, which is 1,480 in this example. The lowest level is chosen to ensure that stock replenishment without disrupting the well-defined rhythm of regular orders.

Reorder	1,480
Order-up-to	5,480

		Cycle 1				Cycle 2			
Period	**0**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
Demand		880	480	1,200	1,440	880	480	1,200	1,440
On-hand	2,360	1,480	5,000	3,800	2,360	1,480	5,000	3,800	2,360
Order		4,000				4,000			

Table 1. An example of regular-mode ordering policy

4.2. Transshipment Mode

When emergency replenishment from the central warehouse falls short, retailers share surplus inventory to avert stockouts. This peer-to-peer network is activated and called transshipment. While transshipment offers a flexible solution, it respects the delicate balance of inventory levels across the network. To avoid creating stockouts elsewhere, transshipment requests are carefully calibrated to secure only the minimum necessary quantity to prevent immediate shortfalls at the requesting retailer. Therefore, an order-up-to point for a transshipment mode, S_T, is calculated with equation (2). Equation (2) is like equation (1) which is used to calculate a reorder point for an emergency mode. However, they differ in how they address review interval and lead time. To prevent stockouts, a location should hold items at least equal to S_T. Therefore, a reorder point is $S_T - 1$ which means that the transshipment mode operates as (R, S_T).

$$S_T = \hat{x}_{R2+L2} + z\sigma_{R2+L2} \quad (2)$$

where,

$R2$ = review interval of a transshipment mode (sub-period)
$L2$ = lead time of a transshipment mode (sub-period)

Using the transshipment mode, we also need to decide which retailer should supply the order. We apply a partial pooling concept that items are reserved for the local future demand (Paterson et al., 2011) to prevent stockout. The items will be transshipped from a retailer when they are excessive. Based on a partial pooling concept, the potential retailers are the locations where on-hand items are more than their reorder points after sending a transshipment order. We use the following ratio in equation (3) to find the potential retailers. The potential retailers are the ones with a ratio higher than 1. Then, the transshipping retailer is the potential retailer with the highest ratio.

$$r_i = \frac{I_i - T}{s_i} \tag{3}$$

where,

T = a considered transshipment order quantity (unit)
r_i = ratio of retailer i
I_i = on-hand items of retailer i (unit)
s_i = a reorder point of retailer i (unit)

4.3. An example of Special Modes

An example of emergency and transshipment modes is shown in Table 2. In the example, standard deviation of demand is assumed to be 5% of the period's demand. Each period is divided into 4 sub-periods. The lead time for an emergency mode is 2 sub-period and the lead time for a transshipment is 1 sub-period. A review interval for special modes is 1 sub-period. The ordering policy for a regular mode, demand pattern and expected on-hand are the same as in Table 1. For each period, average sub-period demand is the same. As demand of period 1 is 880 units and standard deviation is 880 × 5% so sub-period's demand in period 1 is $\frac{880}{4} = 220$ and its standard deviation is $\sqrt{\frac{1}{4}}(880 \times 5\%) = 22$.

Ordering policies for special replenishing modes are shown in Table 3. For an emergency mode, order-up-to points for the dynamic policy are set to the expected on-hand level from a regular replenishment shown in Table 1. Under the dynamic policy, reorder points are calculated every sub-period (review interval) and consider a two-sub-period lead time. For example, in period 1, average demand during lead time and review period is $\hat{x}_{R1+L1} = \hat{x}_{1+2} = 660$ and standard deviation is $\sigma_{R1+L1} = \sqrt{1+2}(22) = 38.11$ and a reorder point satisfying 95% service level is $s = \hat{x}_{R1+L1} + z\sigma_{R1+L1} = 660 + 1.64 \times 38.11 = 722.54$. Then, a reorder point is rounded up to 723. A static policy applies a single, fixed reorder point and order-up-to point across all periods, selecting these values strategically: the highest reorder point and lowest order-up-to point derived from the dynamic policy's calculations. For a transshipment mode, order-up-to points for the dynamic policy are determined based on equation (2) using 1 sub-period lead time and 1 sub-period review interval. Therefore, its order-up-to points are lower than the reorder points of an emergency mode. Reorder points are order-up-to minus 1. A static policy for transshipment mode applies a single fixed order-up-to point, which is the highest order-up-to point from the dynamic policy.

To ensure alignment with actual inventory levels, special replenishment modes initiate orders only after previously placed orders have been received within a sub-period. These modes base their decisions on a comprehensive assessment of the "inventory position," which encompasses both the on-hand inventory physically present at the retailer and any outstanding orders already in transit through special modes. If the inventory position dips below or aligns with the emergency reorder point, but remains above the transshipment reorder point, an emergency order is triggered to replenish stock up to the emergency order-up-to point. However, if the warehouse lacks sufficient stock to fulfill the entire emergency order, no items are shipped at all. If the inventory position declines further, reaching a level at or below both the emergency and transshipment reorder points, emergency replenishment takes precedence. Only if an emergency order is not placed within that sub-period does transshipment become a viable option. Once an emergency order is successfully placed, it elevates the inventory position above both reorder points, effectively negating the need for transshipment within that sub-period.

Number of Sub-periods	4		**SD/Average Demand**	5%
Lead Time (Emergency)	2		**Expected Service Level**	95%
Lead Time (Transshipment)	1		**z (Expected Service Level)**	1.64
Review Interval	1			

Periods	1	2	3	4
Period Demand (units)	880	480	1,200	1,440
On-Hand (units)	1,480	5,000	3,800	2,360
Sub-period Demand (units)	220	120	300	360
SD (sub-period)	22	12	30	36
SD (Emergency)	38.11	20.78	51.96	62.35
SD (Transshipment)	31.11	16.97	42.43	50.91

Table 2. An example of parameters for special replenishing modes

Periods	Dynamic Policy				Static Policy
	1	2	3	4	
Order-up-to (Emergency)	1,480	5,000	3,800	2,360	1,480
Reorder (Emergency)	723	395	986	1,183	1,183
Order-up-to (Transshipment)	492	268	670	804	804
Reorder (Transshipment)	491	267	669	803	803

Table 3. An example of ordering policies for special replenishing modes

4.4. Result Verification Process

To evaluate the impact of special replenishing modes and types of policies on inventory systems, a simulation method is used. Test instances were generated based on assumptions of demand such as normal distribution and seasonal fluctuation without trend. These in stances were different in terms of parameters including number of periods per cycle, demand standard deviation and number of retailers.

4.4.1. Test Instances

It is assumed that the demand is seasonal without trend within a fixed cycle of periods and each period experiences demand following a normal distribution with unique mean and standard deviation. In the experiment, four settings were used: two with four-period cycles and two with seven-period cycles. Examples of parameter settings are displayed in Table 4. Each retailer has a unique demand pattern, as shown by the average demand per period. Ordering costs for regular replenishment modes vary across settings. Retailers with higher ordering costs tend to place orders less frequently than those with lower costs. For instance, settings 1 and 2 share the same demand pattern but differ in ordering costs. Setting 2, with its lower ordering cost, results in a higher expected number of orders per cycle. A similar pattern is observed in settings 3 and 4, where the lower ordering cost in setting 4 leads to a higher expected order frequency.

These four settings were tested across four levels of demand standard deviation: 5%, 10%, 15%, and 20% of the average demand, resulting in a total of 16 instances. While different demand deviations necessitate varying safety stock levels and, consequently, different policies for regular modes, the ordering frequencies remain similar under the same average period demand pattern.

Setting	Number of Retailers	Periods/ Cycle	Average Demand/Period			Expected Number of Orders/Cycle			
			Retailer 1	Retailer 2	Retailer 3	Warehouse	R1	R2	R3
1	2	4	880, 480, 1200, 1440	880, 1840, 2400, 2880		1	1	1	
2			880, 480, 1200, 1440	880, 1840, 2400, 2880		2	3	2	
3	3	7	107, 101, 111, 109, 76, 142, 54	242, 269, 263, 281, 184, 106, 55	458, 344, 396, 452, 295, 611,244	2	1	1	7
4			107, 101, 111, 109, 76, 142, 54	242, 269, 263, 281, 184, 106, 55	458, 344, 396, 452, 295, 611,244	7	7	1	7

Table 4. Parameters of each setting

4.4.2. Simulation Process

With test instances, a simulation method is used to evaluate impact of emergency and transshipment modes on the system under the conditions of applying static and dynamic policies. Demand lost sale is collected from these conditions. Total cost of inventory system and other variables such as number of special and regular orders and service level are collected as well.

All instances were tested under a 95% service level. The regular replenishment mode has a lead time of one period, and its inventory levels are reviewed at the end of each period. For the special replenishment modes, each period is divided into four sub-periods, with inventory levels reviewed at the end of every sub-period. The emergency mode has a lead time of two sub-periods, while the transshipment mode has a lead time of one sub-period. Each policy for special modes was tested on a 10,000-period-horizon instance, corresponding to 40,000 sub-periods. The system makes decisions to place an order based on the algorithm shown in Figure 3.

```
procedure Simulation
I: On-hand inventory
O_m: oncoming order via replenishing mode m
S_m: reorder point of replenishing mode m
R: regular mode, E: Emergency mode, T: Transshipment mode
for each period in horizon do
    warehouse receives and decides to place order
    for each retailer do
        receive regular order
        if I + O_R ≤ S_R do
            place regular order
        for each sub-period in period do
            realize demand
            if I + O_E ≤ S_E do
                if lead time <= remaining sub-period do
                    Emergency order placed
            if I + O_E + O_T ≤ S_T do
                if lead time <= remaining sub-period do
                    if potential retailer exists
                        Transshipment order placed
            if I ≥ demand do
                I ← I – demand, lost ← 0
            else
                I ← 0, lost ← demand – I
                TotalLost ← TotalLost + lost
calculate TotalCost
return TotalLost, TotalCost
```

Figure 3. Algorithm for Simulation

5. Result and Discussion

While increasing safety stock can mitigate unsatisfied demand, it often comes at the expense of higher holding costs. Special replenishing modes offer a promising alternative to achieve this balance. These methods strategically transfer items within a system to meet demand, effectively reducing unsatisfied periods without necessitating additional inventory storage. We investigate that how the special modes can reduce lost sale and how they affect the inventory system. As mentioned, the algorithm used for a regular mode was adopted from Sakulsom and Tharmmaphornphilas (2019). Therefore, the result for a system with no special mode is obtained from this algorithm.

5.1. Reducing Lost Sale and Safety Stock with Special Modes

Initially, the regular replenishment policy was applied to all problems. Although a 95% service level was attained on average, maximum lost sales remained concerning, reaching 14% of demand per period (Figure 4). However, the introduction of special replenishment modes, specifically emergency and transshipment, effectively mitigated lost sales, with the maximum dropping to 0%, regardless of whether static or dynamic policies were used for the special modes. This improvement came with a minor increase in holding costs of around 0.5% to 0.6%.

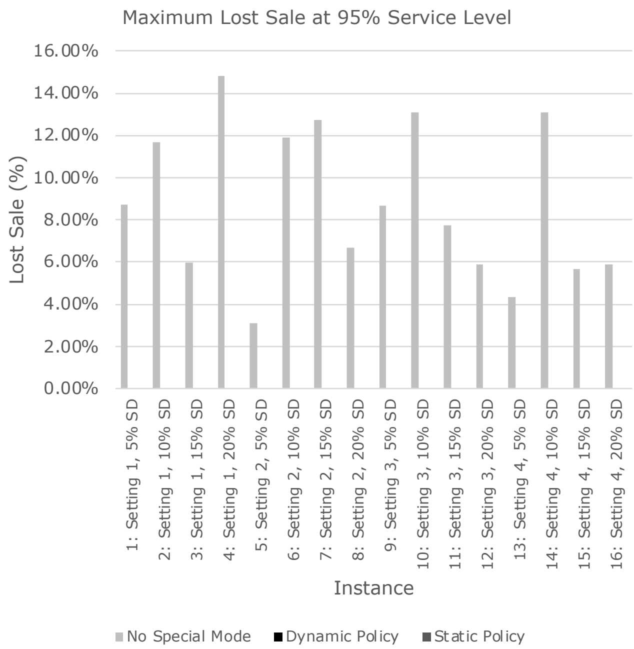

Figure 4. Maximum period's lost sale of the system without special modes

We investigated how different safety stock levels affect the number of special orders. However, as there are two types of policies for special modes (static and dynamic), we observed the number of special orders under each policy. When comparing static and dynamic policies, factors to consider are the ratio of special orders to regular orders, as depicted in Figure 5. The ratio of the number of special orders from a dynamic policy to regular orders is calculated as $\frac{\textit{Number of special orders}_{dynamic}}{\textit{Number of regular orders}} \times 100$, and the ratio of the number of special orders from a static policy to regular orders is calculated as $R_{Static} = \frac{\textit{Number of special orders}_{static}}{\textit{Number of regular orders}} \times 100$.

Figure 5 reveals that both $R_{Dynamic}$ and R_{Static} decline as safety stock increases. This trend holds true for both static and dynamic policies. However, a crucial difference emerges - the dynamic policy consistently triggers fewer special orders compared to the static policy. Therefore, when the system's holding cost outweighs the ordering cost of special modes, strategically reducing safety stock and relying on special orders to address stockouts becomes a compelling option, particularly with a dynamic policy.

Figure 5 also depicts the outcome of decreased safety stock for retailers in the 4-period instances outlined in Table 4. We represent safety stock as a ratio to cycle demand. For instance, a safety stock of 40% for a cycle demand of 1,200 units translates to 480 units. This study employed various expected service levels below 95% (namely, 90%, 85%, and 80%). These lower levels facilitated reduced safety stock, while special mode policies still aimed for a 95% service level. As expected, lower service levels lead to decreased safety stock but increased $R_{Dynamic}$ and R_{Static}.

Furthermore, settings 1 and 2 in Figure 5 represent anticipated order frequencies per cycle. Setting 1 expects only one order per cycle, whereas setting 2 anticipates two or three. This divergence results in differing safety stock levels across the settings. Notably, instances in setting 1 have safety stock ranging from 50% to 60% of cycle demand, while those in setting 2 exhibit a lower range of 30% to 40%.

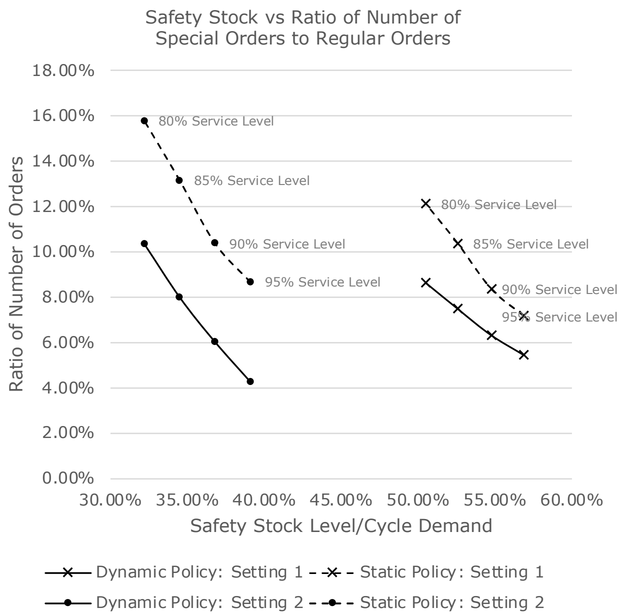

Figure 5. Safety stock levels vs ratio of number of orders

Figure 6 shows average actual service level and holding cost reduction when lowering target service levels from 95% to 90%, 85%, and 80%. Figure 6a presents results from setting 1, which anticipates only one order per cycle, while Figure 6b displays results from setting 2, which expects two or three orders. In Figure 6a, utilizing lower safety stock also contributes to a holding cost reduction of up to 2.0% for setting1-instances (refer to the bar chart on the left vertical axis). In Figure 6b, the holding cost reduction can reach up to 5% for setting2-instances.

However, when safety stock is lowered without special modes, actual service levels can decline to as low as 76% (see the line chart on the right vertical axis). Conversely, with special modes in place, the actual service level can reside within the range of 96% to 100% for setting1-instances and achieve 100% for setting2-instances. Therefore, special modes offer the potential to reduce holding costs while concurrently upholding high actual service levels.

5.2. Comparison Between Emergency and Transshipment Modes

The presence of two distinct special modes, emergency and transshipment, motivates our focus on their relationship with demand deviation. Figure 7 reveals a declining trend in the number of both emergency and transshipment orders as demand deviation increases. This decrease can be attributed to two key factors: higher safety stock levels and the expected number of orders per cycle.

Under conditions of higher demand deviations, retailers naturally maintain larger safety stocks at each location. This larger buffer reduces the risk of stockouts and, consequently, the need for emergency orders. Additionally, instances with a higher expected number of orders per cycle, as defined in Table 4, tend to generate more special orders overall. This relationship is clearly visible in settings 1 and 2, where instances 1 to 4 with a lower expected order frequency exhibit fewer special orders compared to instances 5 to 8 in setting 2 with a higher frequency. Furthermore, we observed that special orders often occur just before a new order arrives, particularly when a retailer's on-hand inventory is low and the risk of stockouts is high. Since instances in setting 2 have more frequent regular orders compared to setting 1, they naturally experience a higher incidence of special orders. This trend also holds true for instances 9 to 16.

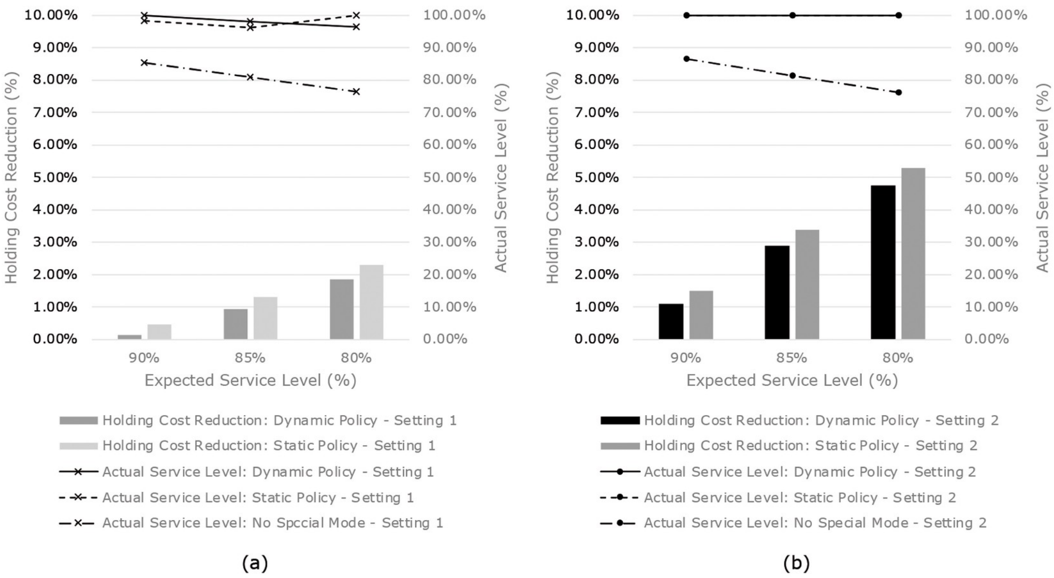

Figure 6. Average actual service level and holding cost reduction at different expected service levels.

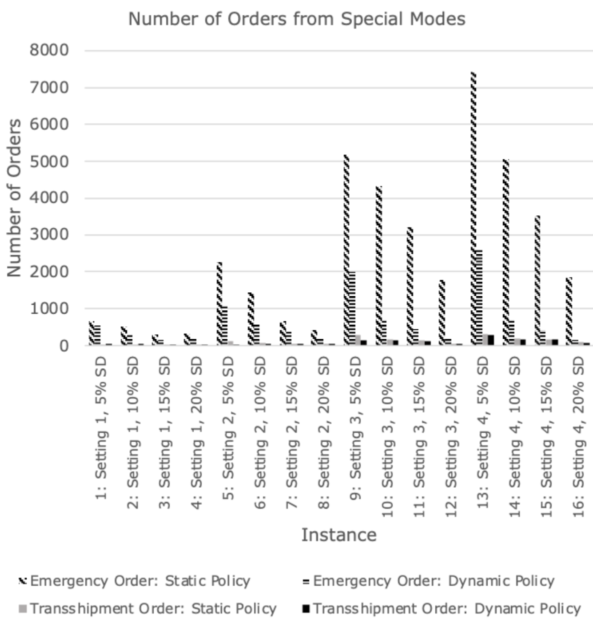

Figure 7. Number of orders from special modes

When comparing static and dynamic policies, the number of emergency orders under static policies is 60% greater than under dynamic policies. The number of transshipment orders is typically lower than the number of emergency orders because the system prioritizes the emergency mode over the transshipment mode. A transshipment order is only triggered under two conditions: either when an emergency order cannot be placed due to the warehouse being unable to fulfill it, or when the emergency order cannot be delivered on time during the last sub-period. To analyze this relationship, we calculate the ratio of the number of transshipment orders to the number of emergency orders as $R_{TE} = \frac{\textit{Number of transshipment orders}}{\textit{Number of emergency orders}} \times 100$.

Figure 8 displays the ratio for each instance. Notably, the average ratio under dynamic policies is 18%, which contrasts significantly with the 5% observed under static policies. This difference arises from a higher prevalence of emergency orders in static policies, leading to a lower ratio compared to dynamic policies. Interestingly, the ratio tends to increase as demand deviation increases in dynamic policies. This can be explained by the fact that, while both emergency and transshipment orders decrease under high demand deviation, the number of emergency orders decreases at a faster rate.

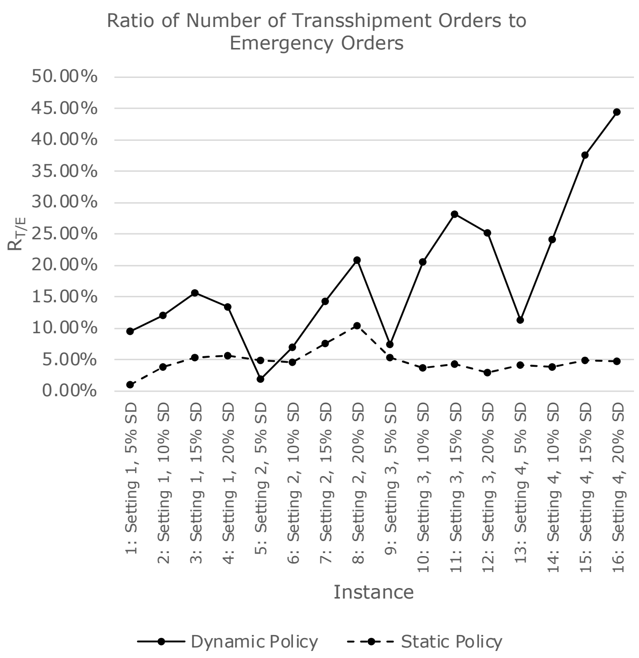

Figure 8. Ratio of number of transshipment orders to emergency orders

5.3. Choosing between Static and Dynamic Policies

5.3.1 Based on Demand Deviation and Length of Cycle

When choosing between a static and dynamic policy, a crucial factor to consider is the ratio of special orders to regular orders, as depicted in Figure 9. The ratios of the number of special orders from both dynamic and static policies to regular orders are calculated using the same methodology described in section 5.1. The differences between static and dynamic policies are more pronounced in 7-period instances compared to 4-period instances. This is because static policies utilize only a single policy for each special mode, regardless of the period within the cycle, whereas dynamic policies employ distinct policies for each period and each special mode. Consequently, the longer the cycle, the greater the divergence between the outcomes of static and dynamic policies.

Therefore, based on the findings presented in Figure 9, under conditions of high demand deviation and short demand cycles, a static policy can potentially serve as a viable substitute for a dynamic policy.

5.3.2. Based on Levels of Demand Fluctuations within a Cycle

Seasonal demand exhibits cyclical fluctuations, following a pattern that repeats itself cycle after cycle. While the average demand within a cycle can remain consistent, the degree of fluctuation may vary. For example, consider a demand cycle with four quarters and a total average demand of 400 units. This demand could be distributed in various ways, such as: a)100 units per quarter (uniform demand), b) 80, 120, 100, and 100 units per quarter (moderate fluctuation), and c) 60, 140, 80, and 120 units per quarter (high fluctuation).

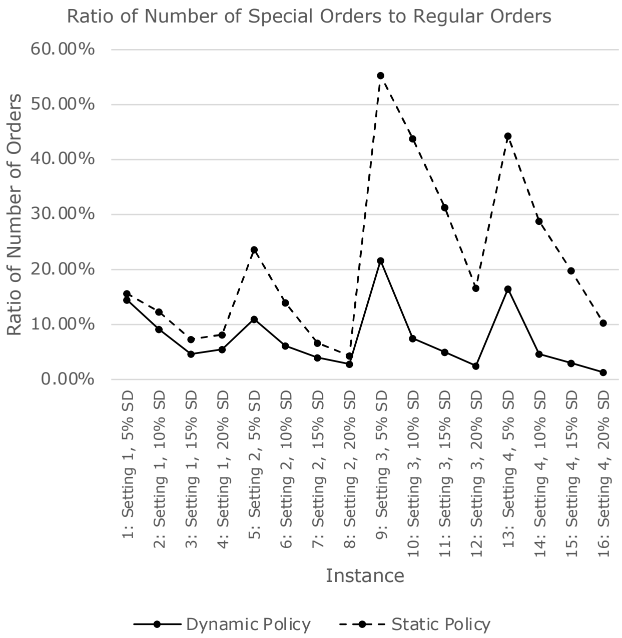

Figure 9. Ratio of number of special orders to regular order

Each of these patterns has the same total cycle demand but differs in the degree of fluctuation around the cycle's average demand. With a total demand of 400 units and four quarters, the average quarterly demand is 100 units. Therefore, pattern a) has no fluctuation, as demand remains constant at 100 units per quarter, while pattern c) exhibits the highest fluctuation, with significant deviations from the average in each quarter.

This study investigates the impact of different levels of demand fluctuation within a 4-period cycle. The higher the fluctuation, the greater the extent to which individual periods deviate from the average cycle demand. To illustrate, if the average quarterly demand is 100 units, a period with demand of 70 or 130 units would represent a 30% fluctuation. Figure 10 showcases 4-period demand patterns for two retailers, with average fluctuations ranging from 0% to 30%.

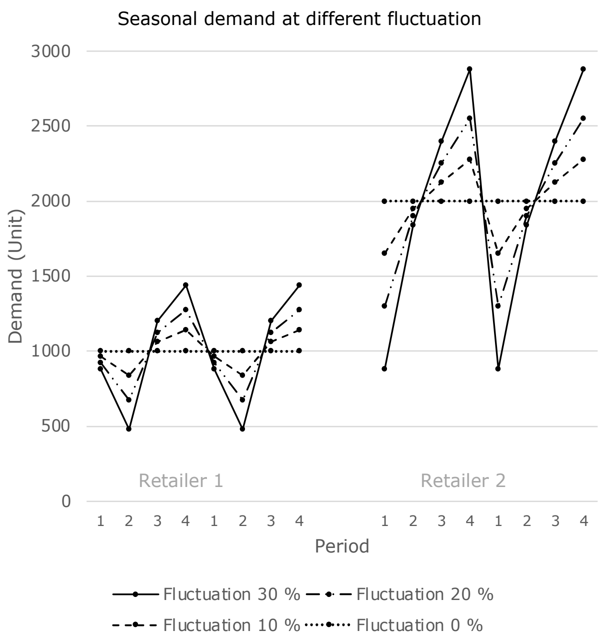

Figure 10. Seasonal demand at different fluctuation

Figure 11 illustrates a trend of increasing special mode orders as demand fluctuation decreases, observed consistently across both setting 1 and setting 2. This phenomenon arises from a corresponding decrease in safety stock as fluctuation diminishes, leading to a heightened risk of stockouts. However, the dynamic policy consistently demonstrates a lower number of special orders compared to the static policy, with the gap between the two policies narrowing as fluctuation decreases. This gap measures approximately 2% for setting 1 and approximately 5% for setting 2. Notably, when demand fluctuation is entirely absent (0%), the dynamic and static policies exhibit no discernible difference, as the dynamic policy effectively functions as a static policy with a single set of ordering policies.

Therefore, under conditions of low demand fluctuation, a static policy is generally favored due to its ease of implementation. However, in scenarios characterized by high demand fluctuation, a dynamic policy can potentially yield lower ordering costs, presenting a trade-off between its complexity and potential cost savings.

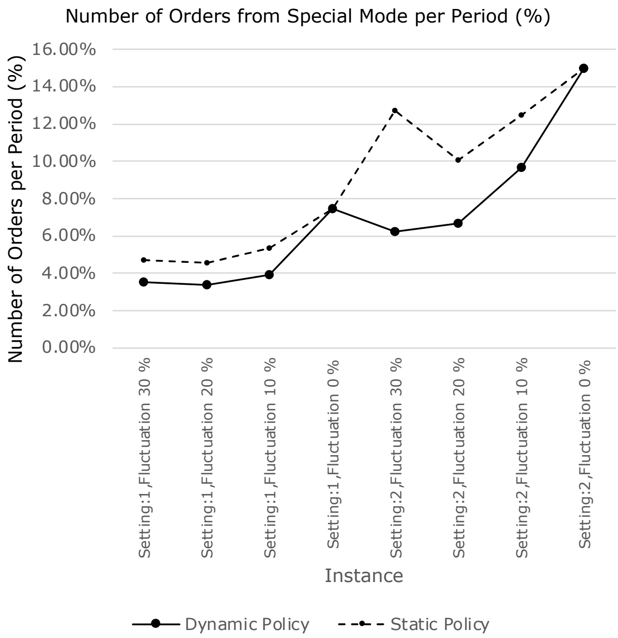

Figure 11. Number of orders from special mode at different fluctuation

6. Conclusion

Emergency and transshipment modes are categorized as special modes designed to prevent stockouts. Our experiment demonstrates that implementing special modes can effectively reduce lost sales, albeit at the cost of slightly increasing the holding cost for each instance. However, the benefit extends beyond just sales: special modes also offer the ability to maintain service levels even when safety stock is reduced. Therefore, allowing special modes to handle stockouts and minimizing safety stock emerges as a viable strategy for lowering holding costs.

The number of special orders tends to be lower under conditions of higher demand deviation due to the increased safety stock levels. Conversely, under demand settings where the regular mode places orders frequently, retailers are more prone to stockouts, consequently triggering more special orders. This increased risk arises because frequent regular orders often deplete on-hand inventory down to the reorder point, making stockouts more likely.

Two primary policies are considered for special replenishing modes: 1) Static policy applies a single policy across all periods in the demand cycle, offering ease of implementation and 2) dynamic policy employs different policies for each period within the cycle, potentially yielding better results but requiring more complex implementation.

Due to the system's prioritization of emergency mode over transshipment mode, the number of transshipment orders is approximately 18% of emergency orders under dynamic policies and 5% under static policies. This ratio tends to increase in scenarios with higher demand deviation. While both emergency and transshipment orders decrease under high demand deviation, the decrease in emergency orders occurs at a more rapid pace.

Deciding between a static and dynamic inventory policy is not a singular, predetermined course of action. This decision involves a trade-off between the simplicity of a static policy and the potential cost savings of a dynamic one, dependent on three key factors: cycle length and demand deviation in each period and demand fluctuation within each cycle.

In shorter cycles, the benefits of diversification offered by dynamic policies diminish. Static policies become preferable due to their simplicity and reduced risk of implementing an unsuitable policy for a limited timeframe. Conversely, higher demand variability requests increased safety stock, which, surprisingly, can translate to fewer special orders, especially for static policies. Interestingly, in scenarios with both high demand variability and short cycles, the gap between the special order frequencies of static and dynamic policies shrinks. This suggests that, in such situations, static policies can emerge as practical alternatives, offering ease of implementation without a significant cost penalty.

Demand seasonality also plays a crucial role in determining the optimal policy. Generally, dynamic policies generate fewer orders than static policies. This difference widens further when seasonal demand exhibits high fluctuations. Conversely, when seasonality is absent, both policies yield the same number of orders. Therefore, for scenarios with low demand fluctuation, a static policy is often preferred due to its ease of implementation. However, in the face of high demand fluctuation, a dynamic policy can offer lower ordering costs, albeit at the cost of increased complexity.

While this paper focuses on a 2-echelon system under seasonal demand, removing several limitations offers fruitful avenues for future research. 1) unit-size replenishment: this paper considers only replenishing policies with a unit size of items. However, practical systems often implement batch-size constraints, which could even differ between the warehouse and retailers. Analyzing the system's performance under various batch-size scenarios would provide valuable insights. 2) stochastic lead times: further research could explore how the system performs when each location faces stochastic lead times, introducing an element of real-world uncertainty. Investigating the impact and potential mitigation strategies for such variability would be highly valuable. 3) capacitated space: real-world systems often have capacity constraints at both the warehouse and retail locations. Developing ordering policies and optimization strategies for such systems would be a significant contribution to practical inventory management.

Declaration of Conflicting Interests

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