

Operational Planning of Supply Chains in a Production and Distribution Center with Just-in-time Delivery

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Abstract:

Purpose: A supply chain consists of raw material suppliers, manufacturers and retailers where inventory of raw materials and finished goods are involved, respectively. Therefore, it is important to find optimal solutions, which are beneficial for both supplier, manufacturer and retailer.

Design/methodology/approach: This research focuses on a semi-continuous manufacturing facility by assuming that the production of succeeding cycle starts immediately after the production of preceding cycle. In reality, the inventory of a supply chain system may not be completely empty. A number of products may be left over after the deliveries are made. These leftover inventories are added to the next shipment after the production of required amount to make up a complete batch for shipment. Therefore, it is extremely important to search for an optimal strategies for these types production facilities where leftover finished goods inventory remains after the final shipment in a production cycle. Considering these scenarios, an inventory model is developed for an imperfect matching condition where some finished goods remains after the shipments.

Findings: Based on the previous observation, this research also considers a single facility that follows JIT delivery and produces multiple products to satisfy customers' demand. For this problem a rotational cycle model is developed to optimize the facility operations. Both problems are categorized as mixed integer non-linear programming problems which are to be solved to find optimum number of orders, shipments and rotational cycle policy for multiple products. Also, this solution will lead to estimate the optimum production quantity and minimum total system cost.

Research limitations: This research considers the supply chain based on manufacturers point of view and it does not consider the transportation cost associated with supply chain. Next study will be focused on issues with joint decision making, information sharing, and transportation decision.

Practical implications: This study will help the managers of refinery and paper industries in making their operation smooth by applying optimizing techniques and robust decision making.

Originality/value: Based on the literature, no research was found on continuous production system supply chain and its optimization with JIT delivery. This research will definitely provide a direction for such problem to the researchers.

Keywords: supply chain system, just-in-time delivery, initial inventory, minimum idle time and rotational cycle policy

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1. Introduction

Generally, Just-in-time (JIT) production systems have zero inventory systems and no buffer. In 1992, Golhar and Sarkers' observation stated that participation in JIT delivery system is economically disadvantageous for suppliers (Golhar & Sarker, 1992). In JIT system, the supplier has to coordinate his production with the buyer's demand so as to maintain zero inventory, but, in reality the supplier ends up with carrying large inventories to deliver limited shipments. In their work, an iterative solution is proposed to minimize the generalized total inventory cost model which is a piecewise convex function. Sarker and Parija (1994) developed a more general problem, which considered both supplier and buyer to determine optimal ordering policy for the raw material and manufacturing batch size to minimize the total cost. They also considered that, at the end of the delivery, a few finished goods are left over which is less than the shipment amount. They solved the problem in semi-closed form and found that the total cost function is piecewise convex. Figure 1 represents a typical supply chain operating under JIT delivery.

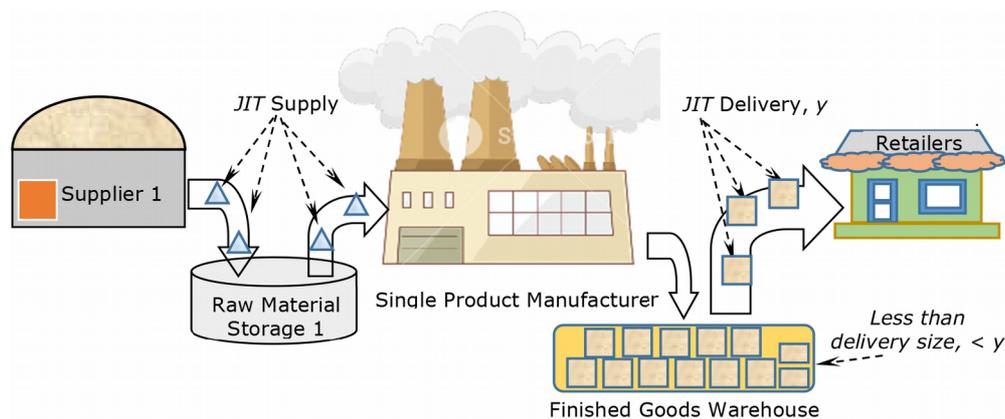


Figure 1. A typical JIT operated supply chain model with imperfect matching inventory

Hill (1996) modified the raw material ordering policy of Sarker and Parija (1994) by allowing a single order for multiple production cycles when the inventory cost for the raw material is much lower as compared to the ordering costs in each production cycle. Sarker and Khan (1999) proposed an ordering policy for raw materials to meet the requirements of a production facility that must deliver finished goods according to customers' demand at a fixed point of time. They considered that the products were supplied after processing the entire lot and quality certification of the products. They also evaluated relationships between production batch size, raw material quantity and delivery patterns. Biswas and Sarker (2005) studied a semi-continuous production system with JIT delivery and supply, where they proposed a total cost function of the system minimizing the downtime of the production facility and solved the problem with an integer approximation.

Sarker and Khan (1999) proposed an ordering policy for raw materials to meet the requirements of a production facility that must deliver finished goods according to customers' demand at a fixed point of time. They considered the finished products were supplied after processing the entire lot and quality certification and with just in time delivery. They evaluated relationships between production batch size, raw material quantity and delivery patterns. Few years later, Khan and Sarker (2002) developed another model for a manufacturing system which procures raw material from the suppliers in a lot and processes them as finished products. They estimated production batch sizes

for a JIT delivery system and incorporated a JIT raw material supply system. Diponegoro and Sarker (2002) developed a decision rule for the manufacturer to determine the delivery quantities, the production start time and batch sizes with trend demands during increasing, level and declining phases of the life cycle of a product. Chiu and Huang (2003) proposed an integrated just-in-time (JIT) inventory model of a multi-echelon with random delivery lead times for a serial supply chain with information exchange between members to make purchase, production, and delivery decisions jointly. The authors proposed search method for finding the optimal solution. Kim, Hong and Lee (2005) discussed the production and ordering policies of a single-manufacturer and single-retailer supply chain where retailer places orders based on the EOQ-like policy, and the manufacturer procures raw materials and distributes them to the multiple plants in parallel to satisfy retailer's demand. They proposed an efficient and effective heuristic algorithm to determine the near-optimal production allocation ratios. Diponegoro and Sarker (2006) developed an ordering policy for raw materials and determined an economic batch size for a product in a manufacturing system that supplies finished products to customers for a finite planning horizon. They considered the JIT delivery policy with lost sale problem due to shortage. Kim, Banerjee and Burton (2008) examined the relationship benefits of buyer-supplier over lot-for-lot with single setup single delivery systems. Also, they suggested two policies so that the supplier can satisfy customers' demand with single setup multiple delivery, multiple setup multiple delivery. Diponegoro and Sarker (2007) studied an operational policy for a lean supply chain system consisting of a manufacturer, multiple suppliers and multiple buyers. He dealt with three interrelated problems in supply chain. They are (a) single supplier and single buyer with fixed delivery size, (b) multiple suppliers and multiple buyers with individual delivery schedule and (c) time dependent delivery quantity with trend demand. He formulated these problems as mixed-integer, nonlinear programming problems with discrete, non-convex objective functions and constraints. Diponegoro and Sarker (2007) also developed a closed-form heuristic which provided near optimal solutions and tight lower bounds.

Hoque (2009) considered the single-vendor single-buyer integrated production inventory problem where the lot is transferred by some researchers in shipments of equal and/or unequal sizes. The author demonstrated that the minimal number (m) of unequal sized shipments selected might not be the minimal and developed a method for obtaining the minimal m and total number (n) of shipments. Mungan, Yu and Sarker (2010) developed an integrated inventory model for high-tech industries in JIT environment under continuous price decrease over finite planning horizon. They modelled an efficient algorithm to minimize the total cost of the system by determining optimal lot sizes for raw material procurement, and manufacturing batch under a finite planning horizon. Jha and Shanker (2009) presented a single-vendor single-buyer system-based production-inventory model for decaying items which deplete with constant decay rate. They formulated the model to find the optimal order quantity, lead time and the number of shipments from the vendor to the buyer during one production cycle which minimizes the total system cost. Sarker and Diponegoro (2009) addressed an optimal policy for production and procurement in a supply-chain system with multiple non-competing suppliers, a manufacturer and multiple non-identical buyers where the demand of finished product is given by buyers and the shipment size to each buyer is fixed. Chen and Sarker (2010) proposed a multi-vendor optimal model for deciding the batch size of vendor's production, and delivery frequencies of different vendors to the manufacturer including shared transportation costs.

Hoque (2011) developed a generalized single-vendor multi-buyer supply chain model by synchronizing both the single-vendor single-buyer and the single-vendor multi-buyer integrated inventory supply chains by transferring the lot either only with equal-sized or only with unequal-sized sub-lot (batches) and presented logical development of their minimal cost solution techniques. Mansouri, Gallear and Askariadzad (2012) aimed to identify the gaps in decision-making support based on multiobjective optimization (MOO) for build-to-order supply chain management available in the literature. Sana (2012) investigated an economic order quantity/ economic production quantity model in three-layer (manufacturer, vendor and retailer) supply chain management with defective quality items while production rate, order quantity, number of shipments with equal sizes. Jonrinaldi and Zhang (2013) proposed a model and solution method for coordinating integrated production and inventory cycles in a whole manufacturing supply chain involving reverse logistics for multiple items with finite horizon period. The authors considered just in time delivery at the distributor's end to the retailers.

Modak, Panda and Sana (2015) studied a just-in-time based defective manufacturing environment including preventive maintenance. They incorporated just-in-time buffers for both the perfect and imperfect quality items to continue the normal operation during the shutdown period and optimized the just-in-time buffer to minimize the system running cost. Omar and Sarker (2015) considered a just-in-time (JIT) manufacturing system to synchronize JIT purchasing and selling in small lot size as a means of minimizing the total supply chain cost. The authors proposed an optimal policy where the shipment intervals as well as the lot sizes are varied. Fandel and Trockel (2016) focused on lot size planning of a two-firm supply chain expansion to include investment decisions and to coordinate the just-in-time-delivery with the lot size planning on the basis of the Harris- or the Wagner/Whitin-approach, which resulted the disposition benefits by Wagner/Whitin led to dominant solutions. Torkabadi and Mayorga (2017) considered on the implementation of Just-In-Time (JIT) in a multi-stage, and multi-product supply chain with Kanban, ConWIP, and a hybrid PCP. Considering the uncertainty, the authors evaluated performances of policies via a Fuzzy AHP method.

Wang and Ye (2018) studied the Just in time (JIT) and Economic order quantity (EOQ) models with carbon emissions in a two-echelon supply chain with single manufacturer and n retailers. In their model, they proposed that the manufacturer and retailers could adopt either a JIT mode or an EOQ mode in which every retailer could decide its own optimal lot size. Kim and Shin (2019) proposed that the third-party logistics service provider would determine the optimal order quantity, considering defective items under the VMI and JIT conditions. They designed a mathematical decision-making problem based on the EPQ /EOQ with defective items, which provides the optimal order quantity for TPL service providers under VMI and JIT. Mardani, Kannan, Hooker, Ozkul, Alrasheedi and Tirkolae (2020) presented a comprehensive review of the application of the Structural Equation Modelling (SEM) in the assessment of sustainable and green supply chain management (SCM) where these were systematically identified, reviewed, and categorized the SEM in the field of green SCM. Nobil, Sedigh and Cárdenas-Barrón (2020) developed a multiproduct economic production quantity inventory model for a vendor-buyer system in which several products are manufactured on a single machine and vendor delivers the products to customer in small batches. The aim of this study was to determine the optimal cycle length and the number of delivered batches for each product so that the total inventory cost is minimized.

During the model development, all of the researchers (Chen & Sarker, 2010; Mungan et al., 2010; Hoque, 2009; Diponegoro & Sarker, 2002, 2006, 2007; Sarker & Diponegoro, 2006, 2009; Sarker & Parija, 1994; Parija & Sarker, 1999; Khan & Sarker, 2002; Wang & Ye, 2018; Kim & Shin, 2019; Mardani et al., 2020) considered that the system remains idle until the shipments are made. Figure 2 shows the production with just-in-time (JIT) delivery.

In reality, the inventory of a supply chain system never becomes empty. A number of products are always left-over after the deliveries are made. These left-over amounts are added to the next shipment after the production of required amount to make-up a complete batch. An illustration may be observed in car manufacturing company such as Toyota, Honda, etc. where the new model cars go in production before the old model cars inventory runs out. Large industries (refineries, paper mills, etc.) also start their production before the finished product inventories fall to zero. Therefore, it is important to search for an optimal supply chain system for these kinds production facilities with left-over finished goods inventory. This research also focuses on the issue of minimizing the idle time of the production system. Based on the inventory decisions, the supply chain system of the production facility with JIT operation is discussed in the current research. A production facility (such as refinery, aluminium conversion industry) produces multiple products from a single type raw material (crude oil, large aluminium sheets, among others) and shipped to customers according to their demand. In the supply chain system of these categories of production facilities, the raw materials are ordered from the suppliers, and process the raw material into multiple finished products and deliver to the buyers or retailers. A single facility supply chain system is represented in Figure 1. For the last few decades, *just-in-time (JIT)* philosophy has played an important role in supply chain systems such as the manufacturing sectors. The successful implementations of JIT phenomena are frequent shipment of high-quality parts to the buyers and ordering raw materials in small batches whenever required to process finished products.

2. Problem Identification

When the production run stops, the on-hand inventory stops building up. Finished goods may be adequate for shipments after the production run is over as represented in Figure 1. In reality, the last shipment size may be less than the required amount. This situation leads to an *imperfect matching*. In this part of the research, the function is developed to find economic order quantity (*EOQ*) for the raw materials and an economic manufacturing quantity (*EMQ*) for the production facility with imperfect matching and minimal idle time. Here, raw material ordering cost, raw material inventory cost, manufacturing setup cost and finished goods inventory carrying cost are considered. In this section, an expression for the generalized cost function is developed that may be used to determine an optimal batch quantity for the production run.

2.1. Notation

To develop the model for determining the interactions between raw materials and finished goods demand, following definitions and notation are used:

C_O	= ordering cost of raw material, \$/order;
C_S	= manufacturing setup cost, \$/batch;
h_M	= holding cost of finished goods, \$/unit/year;
h_S	= holding cost of raw material, \$/unit/year;
D_M	= demand for finished goods, units/year;
D_S	= demand for raw materials, units/year;
f	= conversion factor of the raw materials; $f = D_M / D_S = Q'_M / Q'_S$;
L	= interval between deliveries, in years; $L = y/D_M$;
m	= number of <i>full</i> shipments of finished goods per cycle time;
n	= number of orders for raw materials; $m \geq n \geq 1$;
P_M	= production rate, units/year;
Q'_M	= quantity of finished goods manufactured per setup, units/batch;
Q'_S	= quantity of raw materials required for each batch; $Q'_S = Q'_M / f$;
I_{IM}	= total finished goods inventory for imperfect matching, units;
I_{IS}	= total raw materials inventory, units;
I_0	= quantity remained after the m number of shipments, or initial inventory, units;
T'	= total cycle time, years;
T_1	= production time (uptime), years; $T_1 = Q'_M / P_M = (m \cdot y + I_0) / P_M$;
T_2	= pure consumption time, years (downtime);
T_S	= setup time, years; $T_S < L$;
TC_{IM}	= total cost of finished goods, \$/year;
TC_{IS}	= total cost of raw materials, \$/year;
$TC_I(Q'_M, n)$	= total cost function for imperfect matching, \$/year;
y	= amount of finished goods shipped per time L , units/shipment.

2.2. Assumptions

To develop the mathematical model few important assumptions are:

1. Production rate is constant and finite and is greater than the demand rate.
2. Supply chain of the system considers JIT delivery and supply of finished goods and raw materials, respectively.
3. There is only one manufacturer and one raw material supplier.
4. A fixed quantity is left-over after required shipments and carried over to the succeeding cycle.
5. Production run of following cycle starts after the uptime of previous cycle and setup time.

2.3. Formulation of Cost Function Based on Average Inventory

In this case, the production rate, P , is assumed to be greater than the demand rate, D_M , so that there should be an inventory build-up during the production. Figure 2 shows the inventory build-up, where Figure 2(a) represents the inventory of the raw material supply and Figure 2(b) denotes the on-hand inventory of finished goods. Also, Figure 2(b) has an initial inventory, I_0 units, remaining at the end of m full shipments. That amount remained in cycle 1 is carried over to the start of the production uptime of cycle 2, and amount remaining at cycle 2 is carried over till the end of cycle 3, and so forth. Here, production starts after T_S time units and produces exactly $Q'_F (= my + I_0)$ amount to deliver $y/2$ after L time units. Hence, during time $L - T_S$ time the quantity produced is $y/2 - I_0$ at the rate of P , so that $I_0 + (y/2 - I_0)P_M \geq y/2$. Figure 1 is used to calculate the average on-hand inventory of the finished goods.

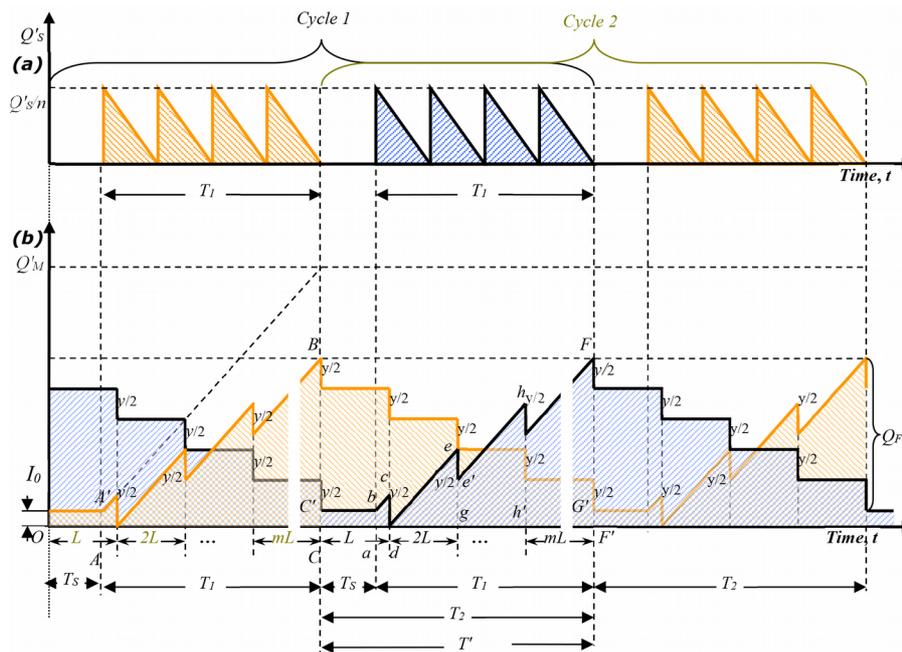


Figure 2. (a) Raw material inventory; (b) Finished goods inventory

\hat{I}_{IM} , \hat{I}_{IP} , and \hat{I}_{ID} are the total inventory, uptime inventory and downtime inventory for the imperfect matching, respectively. Therefore, the total inventory can be calculated as

$$\hat{I}_{IM} = \hat{I}_{IP} - \hat{I}_{ID} \tag{1}$$

From Figure 1, it can be found that

$$\hat{I}_{IP} = \text{area } C'bac + \text{area } abcd + \Delta deg + \text{area } ge'hFF' \tag{2}$$

Therefore, the total produced inventory can be found as

$$\hat{I}_{IP} = \frac{my}{2}T_1 + \frac{my}{2}T' + 2I_0T' \tag{3}$$

Again, the total inventory shipped can be calculated from Figure 2(a) as

$$\begin{aligned} \hat{I}_{ID} &= L(y/2) + 2L(y/2) + \dots + (m-1)L(y/2) + L(y/2) + 2L(y/2) + \\ &\dots + (m-1)L(y/2) = 2 \frac{m(m-1)}{2} \frac{yL}{2} = \frac{my^2}{2D_M}(m-1) \end{aligned} \tag{4}$$

where $L = y/D_M$.

Hence, the total inventory for time period, T' , can be calculated by combining Equations (3) and (4) as

$$\hat{I}_{IM} = \frac{Q_M'^2}{2D_M} + \frac{Q_M'}{2D_M} (4I_0 + y - D_M T_S) + \frac{I_0}{2D_M} (D_M T_S - I_0 - y), \tag{5}$$

where $T' = Q'_M / D_M$ and $Q'_M = my + I_0$.

Again, the total cycle time for imperfect matching case can be calculated as

$$T' = Q'_M / D_M. \tag{6}$$

Hence, the average inventory for imperfect matching case is

$$\begin{aligned} \bar{I}_{IM} &= \frac{\hat{I}_{IP} + \hat{I}_{ID}}{T'} = \frac{D_M}{Q'_M} \left[\frac{Q_M'^2}{2D_M} + \frac{Q'_M}{2D_M} (4I_0 + y - D_M T_S) - \frac{I_0}{2D_M} (I_0 + y - D_M T_S) \right] \\ &= \frac{Q'_M}{2} - \frac{I_0}{2Q'_M} (I_0 + y - D_M T_S) + \frac{1}{2} (4I_0 + y - D_M T_S). \end{aligned} \tag{7}$$

Therefore, the total cost function for the finished products can be written as

$$\begin{aligned} TC_{IM} &= \frac{D_M}{Q'_M} K_S + \bar{I}_{IM} H_M \\ &= \frac{D_F}{Q'_F} K_S + H_F \left[\frac{Q'_F}{2} - \frac{I_0}{2Q'_F} (I_0 + x - D_F T_S) + \frac{1}{2} (4I_0 + x - D_F T_S) \right]. \end{aligned} \tag{8}$$

During the production time or uptime, T_1 , the raw materials are ordered to produce finished products, which require Q'_S units of raw materials to produce Q'_M units of finished goods and they are instantaneously replenished by the outside supplier in n batches. Also, f units of raw materials required to produce one unit of finished product, i.e., $fQ_S = Q_M$. As a result, *raw materials* inventory of entire cycles can be expressed as

$$\bar{I}_{IS} = \left(\frac{Q'_S T_1}{2n} \right) = \left(\frac{Q'_S T_1}{2nf} \right) = \frac{Q_M'^2}{2nfP_M}, \tag{9}$$

where $Q'_S = Q'_M / f$, and $T_1 = Q'_M / P_M$.

Therefore, the total cost for the raw material can be expressed as

$$TC_{IS} = \frac{D_S}{Q'_S / n} C_0 + \bar{I}_{IS} h_S = \frac{nD_S}{Q'_M} C_0 + \frac{Q_M'^2 h_S}{2nfP_M} \tag{10}$$

Where $\frac{Q'_M}{Q'_S} = \frac{D_M}{D_S} = f$.

Therefore, the total cost for this imperfect matching case can be found as

$$\begin{aligned} TC_I(Q'_M, n) &= TC_{IS} + TC_{IM} = \frac{nD_M}{Q'_M} C_0 + \frac{Q_M'^2 h_S}{2nfP} + \frac{D_M}{Q'_M} C_S \\ &\quad + h_M \left[\frac{Q'_M}{2} - \frac{I_0}{2Q'_M} (I_0 + y - D_M T_S) + \frac{1}{2} (4I_0 + y - D_M T_S) \right], \end{aligned} \tag{11}$$

which yields

$$\begin{aligned} TC_I(Q'_M, n) &= \frac{Q_M'^2 h_S}{2nfP_M} + \frac{nD_M C_0}{Q'_M} + \frac{1}{2} Q'_M h_M \\ &\quad + \frac{1}{Q'_M} \left\{ D_M C_S - \frac{I_0 h_M}{2} (I_0 + y - D_M T_S) \right\} + \frac{h_M}{2} (4I_0 + y - D_M T_S). \end{aligned} \tag{12}$$

Replacing the constant term with appropriate notation and simplifying Equation (12), it can be re-written as

$$TC_I(Q'_M, n) = B_1(Q_M'^2 / n) + B_2(n / Q'_M) + B_3 Q'_M + B_4 / Q'_M + B_5, \tag{13}$$

where $B_1 = \frac{h_s}{2fP_M}$, $B_2 = D_M C_0$, $B_3 = \frac{1}{2}h_M$, $B_4 = D_M C_S - \frac{I_0 h_M}{2}(I_0 + y - D_M T_S)$, and, $B_5 = \frac{h_M}{2}(4I_0 + y - D_M T_S)$.

The total cost function for this part of research is a non-linear integer programming problem with two integer variables Q'_F and n . Let, the problem is defined as IM (imperfect matching). Again, the production quantity Q'_F and the number of raw material shipment cannot be less than or equal to 1. Hence, the Problem IM can be expressed with two constraints as

Problem IM: Find m , and n so as to

Minimize:

$$TC_I(Q'_F, n) = B_1(Q'_M)^2 / n + B_2(n / Q'_M) + B_3 Q'_M + B_4 / Q'_M + B_5, \tag{14}$$

Subject to:

$$Q'_M \geq 1, \tag{14a}$$

$$n \geq 1, \tag{14b}$$

$$Q'_M \text{ and } n \text{ are integer.} \tag{14c}$$

In the following section, the solution procedures for Problem IM are described in details.

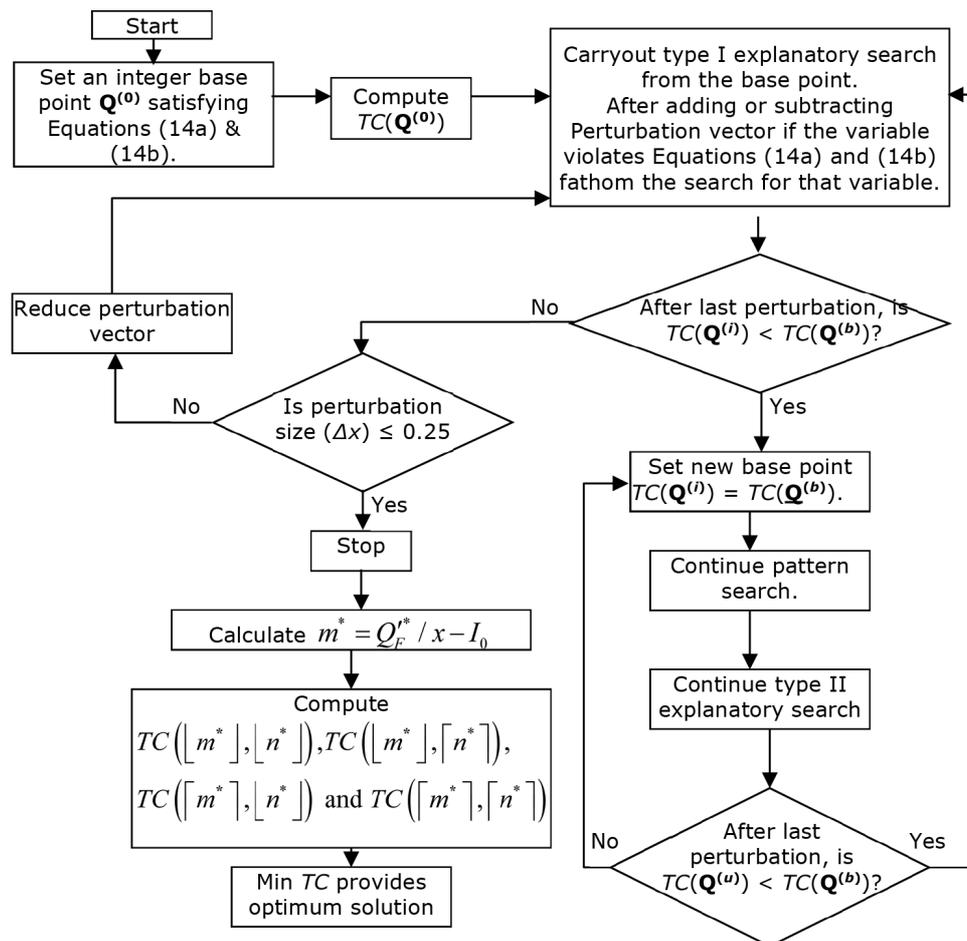


Figure 3. Flow chart of the Modified Hook and Jeeves algorithm

2.4. Solution Technique

The total cost function developed for imperfect matching problem is also a nonlinear integer programming (NLIP) problem and a non-convex function. To find the starting basic solution it is observed that one of the variables have roots of 4th degree polynomial Equation and also the variables are inter depended. Considering these situations, it is found that the Hook and Jeeves (1961) search algorithm is appropriate to solve the problem. The pattern of Hook and Jeeves search technique is a sequential method consisting of two kinds of moves in every step: (a) exploratory and (b) pattern move. The exploratory move is to explore the local behavior of the objective function and the pattern move is taking advantage of the pattern direction. This algorithm is used to solve unconstrained nonlinear programming problem, but the imperfect matching problem is a nonlinear integer programming problem (NLIP) with linear constraints. Therefore, the Hook and Jeeves search algorithm is modified to solve the NLIP with linear constraint as follows:

2.4.1. Modified Hook and Jeeves Algorithm: Determine the number of orders and shipments

Step 0: Given $P, D_F, x, I_0, H_R, K_0, K_S, T_S$ and f .

Step 1: Find the starting *base point* $\mathbf{Q}^{(0)} = (Q'_F{}^{(0)}, n^{(0)})$ using constraint given by Equations (14a) & (14b) and define an integer perturbation vector $\mathbf{P} = \Delta_i = (\Delta Q'_F, \Delta n)$ where $i = Q'_F$ & n (as the variables are integer) in each of the coordinate directions.

Step 2: Compute $TC_{IM} = TC(\mathbf{Q}^{(0)})$.

Step 3: Choosing each variable in turn an objective function evaluation is made at $\mathbf{Q}_i^{(1)} = \{(Q'_F{}^{(0)} + \Delta Q'_F{}^{(0)}, n^{(0)})$ or $(Q'_F{}^{(0)}, n^{(0)} + \Delta n)\}$ (to satisfy the constraint given in Equation (14a)). If $TC(\mathbf{Q}_i^{(1)}) < TC(\mathbf{Q}^{(0)})$, set $TC(\mathbf{Q}_i^{(1)}) < TC(\mathbf{Q}^{(0)})$.

Step 4: Using $\mathbf{Q}_i^{(1)}$ as current temporary base point to obtain new temporary base point as follows:

$$\mathbf{Q}_i^{(v)} = \begin{cases} \mathbf{Q}_i^{(v)} + \Delta_i, & \text{if } TC^+ = TC(\mathbf{Q}_i^{(v)} + \Delta_i) < TC(\mathbf{Q}_i^{(1)}) \\ \mathbf{Q}_i^{(v)} - \Delta_i, & \text{if } TC^- = TC(\mathbf{Q}_i^{(v)} - \Delta_i) < TC(\mathbf{Q}_i^{(1)}) < TC^+ = TC(\mathbf{Q}_i^{(v)} + \Delta_i). \\ \mathbf{Q}_i^{(v)} & \text{if } TC(\mathbf{Q}_i^{(1)}) < \min(TC^+, TC^-) \end{cases} \quad (15)$$

If $(\mathbf{Q}_i^{(v)} - \Delta_i) <$ Equation (14a), terminate that search direction for constraint violation. This process of searching for new temporary base point is continued for $v = 1, 2, \dots, u$ (m, n) is perturbed to find $\mathbf{Q}_i^{(u)}$, call this point as \mathbf{Q}^1 label this point as *Base 1*.

Step 5: If the point $\mathbf{Q}_i^{(u)} = \mathbf{Q}_i^{(0)}$. The pattern move phase is implemented by reducing the integer perturbation vector $\mathbf{P} = \Delta_i = (\Delta Q'_F, \Delta n)$ by $\mathbf{P} = \Delta_i / 2 = (\Delta Q'_F / 2, \Delta n / 2)$ where $i = Q'_F$ and n and go to *Step 3*.

Step 6: If $\mathbf{Q}_i^{(u)}$ is different from $\mathbf{Q}_i^{(0)}$, find a new base point as $\mathbf{Q}^1 = \mathbf{Q}_i^{(0)} + 2(\mathbf{Q}_i^{(u)} - \mathbf{Q}_i^{(0)}) = 2(\mathbf{Q}_i^{(1)} - \mathbf{Q}_i^{(0)})$. Indicate this point as *Base 2*, and go to *Step 3*.

Step 7: If at the end of *Step 4* $TC(\mathbf{Q}_i^{(u)}) < TC(\mathbf{Q}_i^{(0)})$, take the new base point as $\mathbf{Q}^2 = \mathbf{Q}_i^{(u)}$ and go to *Step 6*. If $TC(\mathbf{Q}_i^{(u)}) > TC(\mathbf{Q}_i^{(0)})$, set $\mathbf{Q}^3 \equiv \mathbf{Q}^2$ and go to *Step 5*.

Step 8: If solution does not improve, stop; that is the optimal solution; otherwise go to *Step 5* and continue until $\Delta_i < \varepsilon = 0.025$.

Step 9: Compute $m^* = Q'_F / x - I_0$. If $\lfloor m^* \rfloor < m^*$ and $\lfloor n^* \rfloor < n^*$ or $\lceil m^* \rceil > m^*$ and $\lceil n^* \rceil > n^*$, and compute $TC(\lfloor m^* \rfloor, \lfloor n^* \rfloor)$, $TC(\lfloor m^* \rfloor, \lceil n^* \rceil)$, $TC(\lceil m^* \rceil, \lfloor n^* \rfloor)$ and $TC(\lceil m^* \rceil, \lceil n^* \rceil)$ and take the smallest TC for optimal solution of integer variables.

The following section represents the numerical computation for six data sets to assess the efficiency of the algorithm described in this section. Otherwise, a random search procedure may be applied to find the local optimal solutions.

2.5. Computational Results

The modified Hook and Jeeves search algorithm is used to solve the imperfect matching problem. Using the same sets of data used in Section 3.5 and the Modified Hook and Jeeves algorithm, the computational results are given below in Table 1:

Parameters	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6
P (units/year)	3,600	3,600	6,000	7,000	8,000	11,000
D_M (units/year)	2,400	2,400	3,000	5,200	5,200	7,200
C_0 (\$/order)	150	100	150	200	200	300
C_S (\$/setup)	50	100	60	70	200	250
h_S (\$/unit/year)	1	10	3.5	4	4	10.5
h_M (\$/unit/year)	2	10	5	15	25	45
f	2	3	3	2.5	3	4
y (units/shipment)	100	100	150	200	300	350
I_0	25	30	50	80	90	100
L (year/shipment)	0.04167	0.04167	0.0500	0.03846	0.05769	0.04861
T_s (years)	0.001	0.002	0.002	0.003	0.005	0.006

Table 1. Data set for computation of Problem IM

Parameters	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6
Q^*	660.94	295.73	483.24	405.00	364.68	367.15
m^*	6.61	2.96	3.22	2.02	1.22	1.05
$\lfloor m^* \rfloor$	6	2	3	2	1	1
$\lceil m^* \rceil$	7	3	4	3	2	2
n^*	1	1	1	1	1	1
$TC_I^*(Q_M^*, n^*)$	\$1,610.48	\$4,154.74	\$3,344.32	\$9,914.21	\$17,075.31	\$32,472.76
$TC\{\lfloor m^* \rfloor, n^*\}$	\$1,612.82	\$4,255.80	\$3,345.81	\$10,003.83	\$19,010.69	\$32,818.16
$TC\{\lceil m^* \rceil, n^*\}$	\$1,616.94	\$4,174.05	\$3,458.00	\$10,767.26	\$17,096.01	\$37,785.24
$TC_I^*(m^*, n^*)$	\$1,612.82	\$4,174.05	\$3,345.81	\$10,003.83	\$17,096.01	\$32,818.16

Table 2. Results obtained by using modified Hook and Jeeves Algorithm

Using both GAMS and LINGO, the same results (given in Table 2) are found. Hence the solutions for this data set are optimal.

2.6. Special Case

As discussed in Section 3, when plant is idle during the downtime period only (Figure 4), the inventory model transforms to the imperfect matching model with idle time.

Therefore, when $T_s \rightarrow T_s + T_D = T_s + T_S + T_P = 2T_s + my / P = 2T_s + (Q'_M - I_0) / P_M$, the total cost function expressed in Equation (12) transforms to

$$\begin{aligned}
 TC_{IS}(Q'_M, n) = & \frac{Q'^2_M h_S}{2nfP_M} + \frac{nD_M C_0}{Q'_M} + \frac{1}{Q'_M} \left\{ D_M C_S - \frac{I_0 h_M}{2} (I_0 + y - 2D_M T_s) \right\} \\
 & + \frac{Q'_M h_M}{2} \left(1 - \frac{D_M}{P_M} \right) + \frac{h_M}{2} \left\{ 4I_0 + y + D_M \left(\frac{I_0}{P_M} - 2T_s \right) \right\}.
 \end{aligned} \tag{16}$$

which is the modified Sarkers and Parija (1994) model.

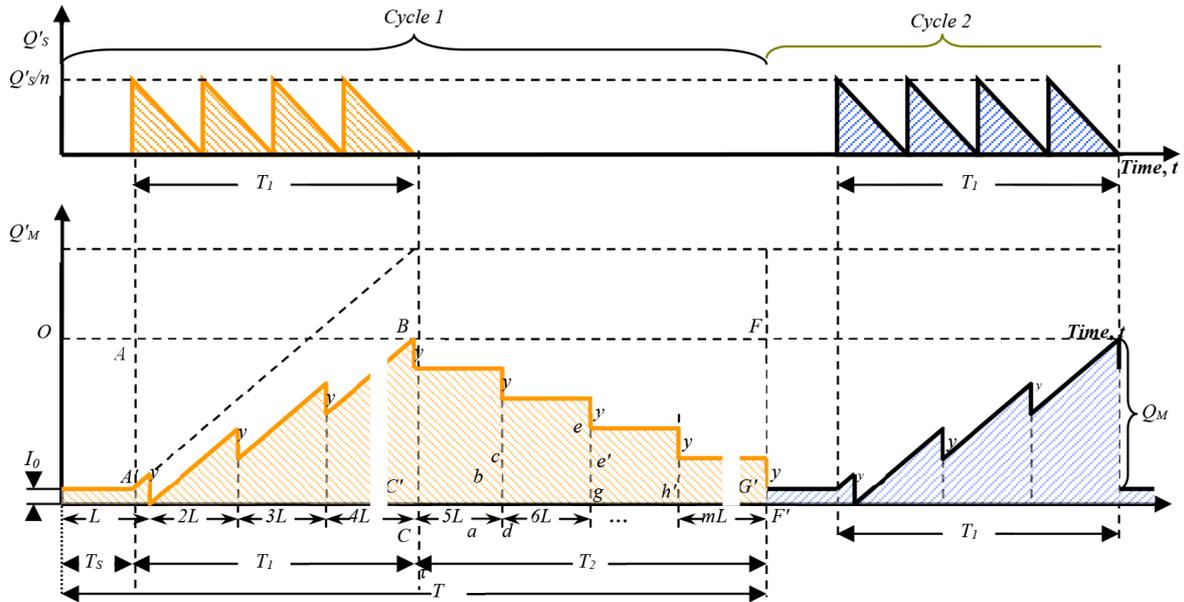


Figure 4. (a) Finished products inventory with idle time; (b) Raw material inventory.

3. Single Facility Lot Sizing Model

In a single facility lot sizing model, multiple products are produced in a time span where completion of the production of any product can meet the customers demand during that time span. After the time span the product goes to production again to meet the next time span. This time span is referred as rotational cycle. This section considers a single production facility that produces K products with a constant demand of D_{Mk} units per year for product k (where $k = 1, 2, \dots, K$), and k product is produced at a constant rate of P_k units per year to satisfy the demand D_{Mk} . All products are delivered at a fixed amount of y_k units after every L_k time units. According to the assumption, production of all k items must meet customers' demand and $\sum_{k=1}^K (D_{Mk} / P_k) \leq 1$.

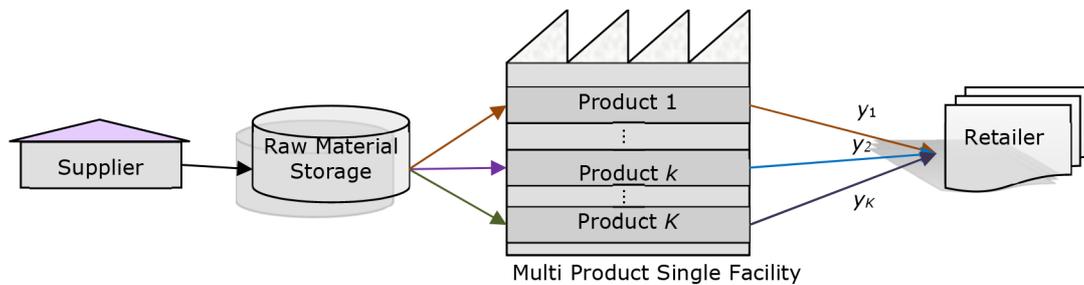


Figure 5. Single-supplier, multi-product and Single-buyer supply chain system

Also, due to rotational cycle policy, all products with the same production cycle time, T_C , and a lot of each product is produced during this time period. Due to the rotational, the products are produced in a fixed order, which is repeated from cycle to cycle. Without permitting any shortages, it is a problem to determine the time of production and optimum number of units to produce for each item which was defined as rotational cycle by Johnson and Montgomery (1974). In their research they considered a single facility lot sizing model based on classical inventory model. In real life, the single production facility, a number of products are always left-over after the possible deliveries are made. These left-over amounts are added to the next shipment after the production of required amount to make-up a complete batch. An illustration may be observed in retail stores such as Albertson's, Target, Wal-Mart, etc. This research incorporates the inventory model with JIT delivery and imperfect matching inventory situation.

3.1. Notation and Assumptions

The notation used to develop this model are described where used. To develop the mathematical model the assumptions considered are

1. Production rates are constant and finite and greater than the demand rates, $P_1 > D_{M1}$,
2. Production of all K items must meet customers' demand,
3. Production facility considers as just-in-time (JIT) delivery and supply of finished products and raw materials, respectively,
4. Production run of a product starts immediately after the uptime or production run of previous product and setup time,
5. Multiple products are produced in each rotational cycle, and
6. A fixed quantity is left-over after required shipments and carried over to the succeeding cycle.

3.2. Raw Material Inventory and Cost Function

To develop the mathematical model the assumptions considered are (a) production rates are constant and finite and greater than the demand rates, $P_1 > D_{M1}$, (b) production of all K items must meet customers' demand, (c) production facility considers as just-in-time (JIT) delivery and supply of finished products and raw materials, respectively, (d) production run of a product starts immediately after the uptime or production run of previous product and setup time, (e) multiple products are produced in each rotational cycle, and (f) a fixed quantity is left-over after required shipments and carried over to the succeeding cycle.

An inventory diagram of a single facility lot sizing model is presented in Figure 6. The pattern of raw material inventory is shown in Figure 6(a) where Q_S is the raw materials required from the supplier during T_{P1} time period. These Q_S units are ordered in n_1 batches in instantaneous replenishments of Q_S/n_1 units. It is assumed that each unit of product 1 produced requires f_1 units of raw material, so that $Q'_{M1} = f_1 Q_S$. Again, in this research the raw materials are ordered and converted to finished goods during the production time or uptime, T_{P1} . Thus, the time weighted inventory, \bar{I}_R of raw material held in a cycle of product 1 is given by

$$\bar{I}_{S1} = Q'_S T_{P1} / (2n_1) = Q'_{M1} T_{P1} / (2n_1 f_1) = Q'^2_{M1} / (2n_1 f_1 T_{P1}), \tag{17}$$

Where $Q'_{Mk} / Q'_S = D_{Mk} / D_{Mk} = f_k$, $T_{pk} = Q'_{Tpk} / P_k$ and $k = 1, \dots, K$.

Therefore, the total cost for the raw material k can be expressed using Equation (17) as

$$TC_{Sk} = \frac{n_k D_{Sk} C_{0k}}{Q'_S} + \bar{I}_{Sk} h_S = \frac{n_k D_{Mk} C_{0k}}{Q'_{Mk}} + \frac{Q'^2_{Mk} h_S}{2n_k f_k P_k}. \tag{18}$$

3.3. Finished Goods Inventory and Cost Function

According to the JIT delivery schedule, fixed amount of y_k units of product k will be delivered after every L_k time units. The lot size for product k must be equal to the demand during the rotational cycle, T_C , without permitting shortages as

$$Q'_{Mk} = (n_k y_k + I_{0k}) = T_C D_{Mk}. \tag{19}$$

According to Figure 6(b), at point A_1 production of product 1 starts with P_1 units/year after T_{S1} time units and produces exactly Q'_{M1} ($= n_1 y_1 + I_{01}$) amount to deliver y_1 after L_1 time units. Hence, during time $L_1 - T_{S1}$ time the quantity produced is $x_1 - I_{01}$ at the rate of P_1 , so that $I_{01} + (y_1 - I_{01})P_1 \geq y_1$. The first shipment of y_1 units of product 1 can be delivered at point B_1 after L_1 time units combining with the left over inventory of I_{01} from the previous cycle. Again, production continues and the inventory builds up as $P_1 > D_{F1}$ and another shipment of x_1 amount is made at point C_1 after L_1 time units.

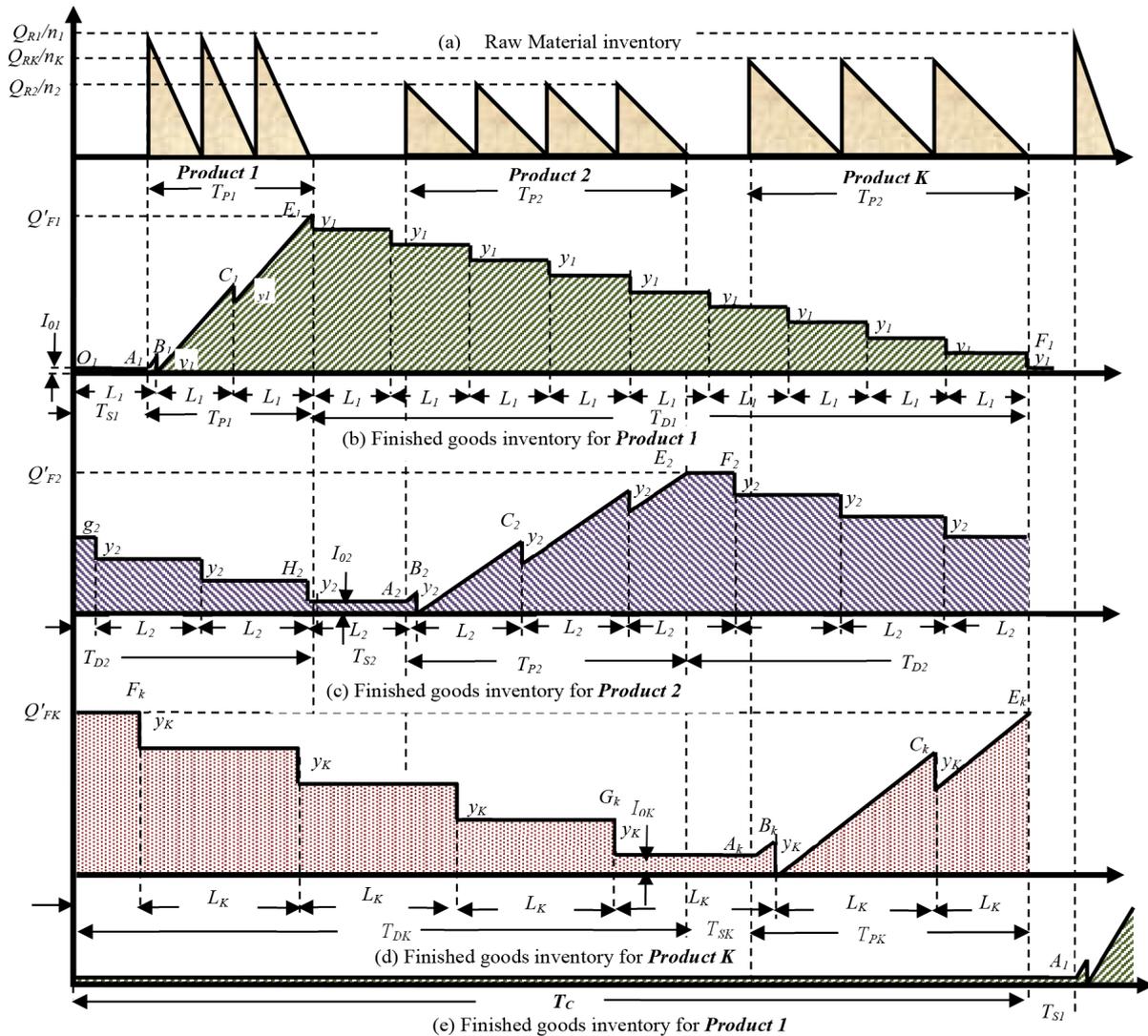


Figure 6. Rotational cycle inventory formation

Thus, after the uptime T_{P1} and point E_1 , production of product 1 stops and the inventory forms a saw-tooth pattern. After point E_1 , y_1 amount is shipped in every L_1 time units to the customer from built-up inventory during downtime T_{D1} . During downtime T_{D1} , the inventory forms as stair case pattern. At the end of T_{D1} time and all possible shipments I_{01} amount of inventory left out in the warehouse as $y_1 > I_{01}$, which is carried over to the next production cycle of product 1. At the end of T_{P1} and after T_{S2} , the production of product 2 starts and delivers y_2 units of product 2 after L_2 time units [Figure 6(c)]. The production of product 2 continues until T_{P2} time units followed by the downtime of T_{D2} . Thus, the process continues for product K from point A_k to E_k according to Figure 6(d). It should be noted that during T_{P2} and T_{PK} , the amount of product 2 and product K produced, respectively, must satisfy customers' demand of these products throughout rotational cycle, T_C . After T_{PK} , the production of product 1 starts again. Figure 6(b) is used to calculate the average on-hand inventory of the finished goods. I_{T1} , I_{P1} , and I_{D1} are the total inventory, uptime inventory and downtime inventory for product 1, respectively. Therefore, the total inventory can be calculated as

$$I_{T1} = I_{P1} - I_{D1}. \quad (20)$$

From Figure 6(b), it can be found that

$$I_P = n_1 y_1 T_{P1} / 2 + n_1 y_1 T_{D1} + I_{01} T_C. \quad (21)$$

Using Equation (21) upon simplification and the inventory can be found as

$$I_P = Q'_{M1} [3 / (2D_{M1}) - 1 / P_1] - I_{01} Q'_{M1} [1 / (2D_{M1}) - 1 / P_1] + (I_{01} - Q'_{M1}) T_{S1} \quad (22)$$

Again, the total inventory shipped can be calculated from Figure 6(b) as

$$I_D = L_1 y_1 + 2L_1 y_1 + \dots + (m-1)L_1 y_1 = \frac{m_1(m_1-1)L_1 y_1}{2} = \frac{m_1 y_1^2 (m_1-1)}{2D_{M1}} \quad (23)$$

Where $L_1 = y_1 / D_{M1}$.

Hence, the average inventory for product 1, \hat{I}_{T1} , and time period, T_C , can be calculated by combining and simplifying Equations (21), (22) and (23) as

$$\hat{I}_{T1} = \frac{Q'_{M1}}{2D_{M1}} \left[Q'_{M1} + 4I_{01} + y_1 - 2D_{M1} T_{S1} - \frac{D_{M1}(Q'_{M1} - I_{01})}{P_1} \right] - \frac{I_{01}}{2D_{M1}} \left[I_{01} + y_1 - 2D_{M1} T_{S1} - \frac{D_{M1}(Q'_{M1} - I_{01})}{P_1} \right] \quad (24)$$

Using Equation (24), the total cost function of product k can be expressed as

$$TC_k(Q'_{Mk}) = \frac{1}{Q'_{Mk}} \left\{ D_{Mk} C_{Sk} - \frac{I_{0k} h_{Mk}}{2} (I_{0k} + y_k - 2D_{Mk} T_{Sk}) \right\} + \frac{Q'_{Mk} h_{Mk}}{2} (1 - D_{Mk} / P_k) + \frac{h_{Mk}}{2} \{ 4I_{0k} + y_k + D_{Mk} (I_{0k} / P_k - 2T_{Sk}) \} \quad (25)$$

Now, replacing Q'_{Mk} from Equations (18) and (25) by using the relationship presented in equation (19), the new equations can be expressed as

$$TC_{Sk} = \frac{n_k C_{0k}}{T_C} + \frac{h_S T_C^2 D_{Mk}^2}{2n_k f_k P_k}, \text{ and} \quad (26)$$

$$TC_k(Q'_{Mk}) = \frac{1}{T_C D_{Mk}} \left\{ D_{Mk} C_{Sk} - \frac{I_{0k} h_{Mk}}{2} (I_{0k} + y_k - 2D_{Mk} T_{Sk}) \right\} + \frac{h_{Mk}}{2} \left[T_C D_{Mk} (1 - D_{Mk} / P_k) + \{ 4I_{0k} + y_k + D_{Mk} (I_{0k} / P_k - 2T_{Sk}) \} \right] \quad (27)$$

Using Equations (26) and (27) and simplifying, the total cost function of finished product k can be expressed as

$$TC_k(Q'_{Mk}, m_k) = \frac{T_C^2 D_{Mk}^2 h_{Sk}}{2m_k f_k P_k} + \frac{n_k C_{0k}}{T_C} + \frac{T_C D_{Mk} h_{Mk}}{2} (1 - D_{Mk} / P_k) + \frac{1}{T_C} \left\{ C_{Sk} - \frac{I_{0k} h_{Mk}}{2D_{Mk}} (I_{0k} + y_k - 2D_{Mk} T_{Sk}) \right\} + \frac{h_{Mk}}{2} \{ 4I_{0k} + y_k + D_{Mk} (I_{0k} / P_k - 2T_{Sk}) \} \quad (28)$$

3.4. Objective Functions and its Constraints

Combining all costs for all K products and using Equation (28), the objective function can be written as

$$C_T(T_C, m_1, \dots, m_K) = \sum_{k=1}^K \left[\frac{T_C^2 D_{Mk}^2 h_{Sk}}{2m_k f_k P_k} + \frac{n_k C_{0k}}{T_C} + \frac{T_C D_{Mk} h_{Mk}}{2} (1 - D_{Mk} / P_k) + \frac{1}{T_C} \left\{ C_{Sk} - \frac{I_{0k} h_{Mk}}{2D_{Mk}} (I_{0k} + y_k - 2D_{Mk} T_{Sk}) \right\} + \frac{h_{Mk}}{2} \{ 4I_{0k} + y_k + D_{Mk} (I_{0k} / P_k - 2T_{Sk}) \} \right] \quad (29)$$

Before minimizing the problem, it is necessary to study the constraints related to the rotational cycle policy, such as the setup times the number of raw material deliveries for each product. If the setup time for product k is T_{Sk} , then the total setup time per cycle and the total production time per cycle must be smaller or equal to the rotational cycle length. Therefore, the following constraint on T_C will be

$$T_C \geq \sum_{k=1}^K [T_{Sk} + Q'_{Mk} / P_k]. \tag{30}$$

Replacing Q'_{Mk} by using Equation (19), it can be re-written as

$$T_C \geq \sum_{k=1}^K \frac{T_{Sk}}{1 - \sum_{k=1}^K [D_{Mk} / P_k]} \equiv T_{\min} \geq 0 \tag{31}$$

Also, the number of raw material delivery, n_k for product k cannot be less than 1 and should be an integer variable. Hence, the constraint on n_k is

$$n_k \geq 1 \text{ and is an integer, for } k = 1, 2, \dots, K \tag{32}$$

Using these two constraints defined in Equations (31) and (32), the objective function can be formulated as

Minimize:

$$C_T(T_C, n_1, \dots, n_K) = \sum_{k=1}^K \left[\frac{T_C^2 D_{Mk}^2 h_{Sk}}{2m_k f_k P_k} + \frac{n_k C_{0k}}{T_C} + \frac{T_C D_{Mk} h_{Mk}}{2} (1 - D_{Mk} / P_k) \right. \\ \left. + \frac{1}{T_C} \left\{ C_{Sk} - \frac{I_{0k} h_{Mk}}{2D_{Mk}} (I_{0k} + y_k - 2D_{Mk} T_{Sk}) \right\} + \frac{h_{Mk}}{2} \{ 4I_{0k} + y_k + D_{Mk} (I_{0k} / P_k - 2T_{Sk}) \} \right] \tag{33}$$

Subject to:

$$T_C \geq T_{\min} \geq 0, \tag{33a}$$

$$n_k \geq 1 \text{ and is an integer, for } k = 1, 2, \dots, K. \tag{33b}$$

Therefore, the problem becomes a mixed integer non-linear programming problem and the solution procedure to this problem is discussed in the next section.

3.5. Solution Technique of Rotational Cycle Problem

The formulation of the single facility lot-sizing problem for imperfect matching system can be categorized as a mixed-integer non linear programming problem where m_k 's are integer and T_C is a real variable and the number of variables are $(K + 1)$. Due to formulation of the problem, it cannot be solved using derivatives and a closed form solution cannot be determined. Using the Divide and Conquer rule, the objective function is divided into two parts (a) rotational cycle for finished products, and (b) number of raw material orders. The rotational cycle for the finished products (T_C) is the same for the raw material delivery, because the raw materials are delivered from the supplier by instantaneous replenishments. Again, the raw material for a product k is ordered when the finished product k goes in production. The solution procedures are shown as follows:

(a) Rotational Cycle for Finished Products

To solve the rotational cycle policy for the part finished product supply, the cost function from Equation (27) can be written as

Minimize:

$$C_M(Q'_{Mk}) = \frac{1}{T_C D_{Mk}} \left\{ D_{Mk} C_{Sk} - \frac{I_{0k} h_{Mk}}{2} (I_{0k} + y_k - 2D_{Mk} T_{Sk}) \right\} + \frac{h_{Mk}}{2} \left[T_C D_{Mk} (1 - D_{Mk} / P_k) + \{ 4I_{0k} + y_k + D_{Mk} (I_{0k} / P_k - 2T_{Sk}) \} \right], \quad (34)$$

Subject to:

$$T_C \geq T_{\min} \geq 0. \quad (34a)$$

It can be shown that the Equation (34) is a convex function for T_C ; therefore, it can be solved by differentiation with respect to T_C and equate it to zero, which yields

$$T_C^* = \sqrt{\frac{2 \sum_{k=1}^K [C_{Sk} - I_{0k} h_{Mk} (I_{0k} + y_k - 2D_{Mk} T_{Sk}) / (2D_{Mk})]}{\sum_{k=1}^K D_{Mk} h_{Mk} (1 - D_{Mk} / P_k)}}, \quad (35)$$

where $k = 1, \dots, K$.

Equation (35) has to satisfy the constraint given in Equation (34a). Using the optimal rotational cycle T_C^* , the number of shipments for different finished product can be obtained from Equation (19). The optimal rotational cycle, T_C^* is used to solve the optimal number of orders for raw materials in following section.

(b) Number of Raw Material Orders

As the raw materials order policy is instantaneous, the production rate for the raw material is ∞ ; therefore, this also satisfies the condition for rotational cycle. Now, applying the value of T_C^* from Equation (35), the total cost/objective function for raw material k can be written as [from Equation (26)]

Minimize:

$$C_R(n_k) = \frac{n_k C_{0k}}{T_C^*} + \frac{T_C^{*2} D_{Fk}^2 h_{Sk}}{2n_k f_k P_k}, \quad (36)$$

Subject to:

$$n_k \geq 1 \text{ and integer, } \forall k = 1, \dots, K. \quad (36a)$$

This objective function [Equation (36)] is convex in n_k and the objective function is a discrete function, which cannot be solved using differentiation. Hence, the induction method is used to solve n_k . Using the induction method in Equation (36), the boundary condition for n_k^* is can be evaluated as

$$\left\lceil \frac{[\sqrt{1+4\Delta_k} - 1]}{2} \right\rceil \leq n_k^* \leq \left\lfloor \frac{[\sqrt{1+4\Delta_k} + 1]}{2} \right\rfloor, \quad (37)$$

Where $\Delta_k = \frac{h_{Mk} T_C^{*3} D_{Mk}^2}{2f_k P_k K_{0k}}$ and $k = 1, 2, \dots, K$. In addition, Equation (37) has to satisfy the constraint given in Equation (36a). Applying the boundary condition in Equation (37) the optimal objective function can be evaluated as well as the optimum number of orders m_k^* for raw material k , where $k = 1, \dots, K$. Hence, optimum total cost for all raw materials can be expressed as

$$C_R(n_1^*, \dots, n_K^*) = \sum_{k=1}^K \left[\frac{n_k^* C_{0k}}{T_C^*} + \frac{T_C^{*2} D_{Fk}^2 h_{Sk}}{2n_k^* f_k P_k} \right] \quad (38)$$

As discussed before, both n^*_k and T^*_C is may not be globally optimal. Therefore, another forward search is conducted using Equation (33), starting from the constraints for T^*_C and n^*_k [given in Equations (33a) and (33b)] and with step sizes 0.01 and 1, respectively, to evaluate the optimal T^{opt}_C and n^{opt}_k that will minimize the $C_T(T^{opt}_C, n^{opt}_1, \dots, n^{opt}_k)$.

3.6. Numerical Computation of Optimum Rotational Cycle

Six products, presented in Table 3, are being produced in a single facility manufacturing system with JIT delivery.

Parameters	Product 1	Product 2	Product 3	Product 4	Product 5	Product 6
P (units/year)	14,000	10,500	15,000	10,000	9,000	20,000
D_F (units/year)	2,000	1,500	3,000	1,800	1,200	2,200
K_o (\$/order)	150	100	150	200	200	300
K_S (\$/setup)	50	100	120	130	200	150
H_R (\$/unit/year)	1	10	3.5	4	4	10.5
H_F (\$/unit/year)	2	10	5	15	25	45
f	2	3	3	2.5	3	4
x (units)	100	100	150	200	300	350
I_o (units)	25	30	50	40	60	55
T_s (years)	0.001	0.002	0.002	0.003	0.005	0.006

Table 3. Data set for single facility lot-sizing model

Using these data from Table 3 and Equation (35), the T^*_C can be found as $T^*_C = 0.56$ years. Now using the value of T^*_C in Equation (37) the boundary conditions for n^*_k can be found as $n^*_1 = 1, n^*_2 = 1, n^*_3 = 1, n^*_4 = 1, n^*_5 = 1,$ and $n^*_6 = 1$ Using these values the total costs can be found as $C_T(T^*_C, n^*_1, \dots, n^*_6) = (0.56, 1, 1, 1, 1, 1) = \$33,928.26$ per year, and this is local optimum solution. Therefore, a forward search is conducted starting from $T^*_C = 0.21$ (with step size 0.01), and $m^*_k = 1$ (with step size 1) and the optimum solution is obtained in $C_T(T^{opt}_C, n^{opt}_1, n^{opt}_2, n^{opt}_3, n^{opt}_4, n^{opt}_5, n^{opt}_6) = (0.32, 1, 1, 1, 1, 1) = \$32,373.85$ per year. The detailed results of rotational cycle policy are presented with numerical values in Table 2. In this case, it is considered that all six products are produced in a single facility in a sequence and they will be delivered using just-in-time (JIT) policy. Also, the raw materials for each product will be ordered following multiple ordering policies. According to the constraint given in Equations (35a) it can be determined by using the data given in Table 3 that $T_C \geq 0.019/0.09 = 0.21$. Also, it is observed that $\sum_{k=1}^6 (D_{Mk} / P_k) = 0.91 \leq 1$ which satisfies the assumption for rotational cycle policy. The results for the single facility lot sizing models for imperfect matching case presented in Table 2.

Parameters	Product 1	Product 2	Product 3	Product 4	Product 5	Product 6
T^*_C , years	0.32					
m^*_k	1	2	2	1	1	1
n^*	6	4	6	2	1	1
Q^*_{Fk} , units/year	600	400	900	400	300	350
Q^*_{Rk} , units/year	300	133	300	160	100	88
$C^*_T(T^*_C, m^*_1, \dots, m^*_k)$	\$32,373.85					

Table 4. Optimum results for raw materials of imperfect matching case

Thus, this section concludes the research for rotational cycle policy of an imperfect matching situation when the system idle time is negligible.

4. Conclusion

In the past, researchers tried to develop the proper supply chain management with ideal conditions. This research more focused on real life situations where the production facility does not remain idle for longer period of time and does not become empty at the end of full shipments. In many industries, the production facilities stay idle only during the routine maintenance because of high costs to shut down and restarts the production facilities, such as refineries. This research will have a significant impact on the real life production facilities where the idle time of the facility is negligible. Also, this will help to develop a better supply chain management to any industry.

Also, the research extended incorporating an operation policy of a supply chain of a single facility lot-sizing model with just-in-time (JIT) deliveries with imperfect matching situations. Also, the current research considered a supply chain system that operates under a reduced idle time, where the production of a cycle of one product starts immediately after the end of production cycle of previous product. A set of problems are categorized as a serial system with a fixed quantity and a fixed delivery interval. The problem is solved for the optimum rotational cycle, optimum number of orders, optimum batch sizes, and optimum numbers of shipment evaluated to minimize the total system cost.

The operation policies prescribe the number of orders and the ordered quantities of raw materials from suppliers, production quantities, and number of shipments to the customers for an infinite planning horizon. Prospective research issues that can be pursued further concerning the supply chain system addressed in this research by incorporating time varying demand, variable production capacity and rate, transportation costs, and multi-stage systems.

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