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Joint Optimization of Economic Production Quantity and Preventive Maintenance with Considering Multi-Products and Reserve Time

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Abstract:

Purpose: We deal with the problem of the joint determination of optimal economic production quantity (EPQ) and optimal preventive maintenance (PM) for a system that can produce multiple products alternately. The objective is to find the optimal number of production cycles and the PM policy simultaneously by minimizing the cost model.

Design/methodology/approach: Considering the products go through the system in a sequence and a complete run of all products forms a production cycle. In each cycle, beyond production time we also consider some reserve time for maintenance and setup, shortage and overproduction may occur. We study the integrated problem based on two PM policies, and explain the situation with the other PM policies. The delay – time concept is used to model PM decisions.

Findings: Using the integrated EPQ and PM model, we can calculate the optimal production planning and PM schedule simultaneously, especially we consider multiple products in each production cycle, which is more practical and economic than previous works.

Originality/value: In modern companies, the production planning and maintenance schedule share the same system, and traditional research about two activities is separated, that always generate conflicts, such as inadequate or excessive maintenance, and shortages, etc., so we develop the integrated EPQ and PM model to avoid these undesirable effects.

Keywords: economic production quantity, preventive maintenance, delay-time, inventory, product quality

1. Introduction

Traditionally, in modern companies, optimal production planning and optimal maintenance schedule are always studied separately, conflict is generated inevitably since the two activities share the same system. To avoid shortages, improper maintenance, or the other harmful effect caused by separated planning about production and maintenance, lots of researches of integrated production and maintenance are developed. Those researches can be categorized into two classes, the first one is about the integrated EPQ and maintenance model (Liu, Wang & Peng, 2015a), which is mainly based on the continuous modes of production, the second one is about the integrated capacitated lot sizing problem (CLSP) and maintenance model (Fitouhi & Nourelfath, 2014; Liu, Wang & Peng, 2015b), which is mainly based on the discrete modes of production. In this paper, we focus only on the first direction, and study the joint optimal EPQ and PM schedule in finite planning horizon for a multi-product system.

There are some related researches about integrated EPQ and PM problems. For instance, Lee and Rung (2000) studied lot-sizing policies in multi-stage serial production systems with the systems prone to failures. They concluded that the lot sizes in the unreliable systems could be smaller or larger than those in the classical EPQ model. Giri and Dohi (2004) proposed a net present value approach to determine the EPQ for an unreliable production system over an infinite planning horizon. Sami (2008) considered a system that deteriorates with an increasing failure rate, and proposed a model to determine the optimal number of the production runs and the PM schedule that minimize the long-term average cost. Chakraborty, Giriand and Chaudhuri (2008) presented a EPQ model for an unreliable production system in which the production facility may shift from an "in-control" state to an "out-of-control" state at any random time and may ultimately break down afterwards. Chakraborty, Giriand and Chaudhuri (2009) developed integrated production, inventory and maintenance models to study the joint effects of process deterioration, machine breakdown and inspections on the optimal lot-sizing decisions. Jafari and Makis (2015) developed the joint optimization of EPQ and PM policy for a production facility subject to deterioration and condition monitoring, and they proposed the proportional hazards model to consider condition monitoring information as well as the age of the production facility, the deterioration process is determined by the age and covariate values, the covariate process is modeled as a continuous-time Markov process, this work is extended by Jafari and Makis (2016), they modeled the covariate process as a Semi-Markov decision process. Bouslah, Gharbi and Pellerin (2016) studied the integrated design of production, continuous sampling inspection and preventive maintenance of a deteriorating production system.

The abovementioned works are more reasonable than the classical EPQ model as the system deterioration and maintenance are considered, however all these models are restricted to the one product case. Liu et al. (2015a) proposed an integrated EPQ and PM model for multi-products system, in this paper we will develop further research about the work they studied. We consider the reserve time which is decided by production planners for maintenance and system setup, the reserve time may be not exact since the production planner set it based on the history data and experiences, so the idle time or shortage may occur, this situation exists in reality but has not been studied in Liu et al. (2015a), we extend their work by considering the reserve time in this paper. In a finite planning horizon, several types of product should be produced according to their lot sizes, the demand for each product is fixed, and each product is produced once in a production cycle. PM is carried out at some set-up points for less interruption to the production. We also consider the unqualified products in the cost model. Our objective is to determine the optimal lot size for each product and the optimal PM policy, in reality, smaller lot sizes lead to smaller inventory costs, but more setup costs and more opportunities for PM, and vice versa, so it is necessary to model the integrated EPQ and PM schedule.

In this paper, we use the delay–time concept to model the PM policy for the system, this concept has been widely applied in maintenance modeling and optimization (Wang, 2012), many case studies have shown the validity of the delay-time-based models (Fu, Wang & Shi, 2012; Wu & Wang, 2011). The delay–time concept considers the failure process as a two stage process: the first stage from new to an initial point of the defect, usually referred to as the normal stage, in this stage, the defect can be identified and removed by PM; and then the second stage from the initial point to failure is the delay–time stage with the defect in unattended. The delay–time models can be divided into two categories: a single–component system model (Baker & Wang, 1993; Fu et al., 2012) and a complex system model (Wang & Banjevic, 2012; Wang, Banjevic & Pecht, 2010). In this paper, we use the complex system delay–time model since typical production systems are equipped with many components.

Given the above explanations, the main contribution of this paper can be stated as follows. 1) Developing an integrated EPQ and PM model for multi-products system, not just for single-product system that is studied by lots of previous works. 2) Considering the reserve time in the integrated model, the scenarios of idle time or shortages are studied. 3) Using the delay-time concept in the integrate model for describing maintenance activities.

The rest of the paper is organized as follows. Model notations and assumptions are given in Section 2. The integrated cost model is proposed in Section 3. Numerical examples are presented in Section 4. Section 5 concludes the paper.

2. Notations and Assumptions

2.1. Notations

Below is the definition of main notations:

L: Length of the planning horizon.

D_i: Total demand of ith product during the planning horizon, $i = 1, 2, 3, ..., k$.

d_i: Consumption rate of ith product, where $d_i = D_i/L$.

p_i: Production rate of ith product.

$\hat{L}$: Production time of all product demands within the planning horizon.

Q_i: Production lot size of ith product.

n: Number of production cycles during the planning horizon.

τ_i: Actual production time of ith product in one production cycle.

T_i: Nominal production time of ith product in one production cycle, it contains τ_i and the downtime caused by failures during τ_i.

$F(\bullet)$: The cumulative density function (cdf) of the delay-time.

λ: The rate of the occurrence of defects.

d_s: The unit time per set-up.

d_f: The unit time to repair per failure.

d_p: The unit time per inspection at a PM.

h_i: The unit inventory holding cost of the ith product.

C_s: The unit set-up cost.

C_d: The unit cost of repairing a defective component that is identified at a PM.

C_f: The unit cost for repairing a failure.

C_p: The unit cost of an inspection at a PM.

C_{dp}: The unit cost of disposing a unqualified product.

C_0: The unit cost of shortage.

2.2. Assumptions

(1) The defects of the system arrive independently according to homogenous Poisson process (HPP).

(2) The delay-time of all defects is independent and identically distributed.

(3) The PM is carried out at some set-up points for less interruption to the production.

(4) The PM is perfect and renews the system.

(5) A minimal repair is always performed at a failure.

(6) Each product is produced once in a production cycle.

Assumptions (1) and (2) have been used in previous delay-time models. Assumption (3) is a fact observed in industry where for typical batch production the set-up window is also often the time to do some PM. Assumption (4) is for modeling simplification. Assumption (5) is widely used in maintenance modeling, where due to the time constraint and the need to resume the production as soon as possible. About assumption (6), in reality, some products may be produced zero or more than one lot size in a cycle, this particular scenario will be researched in a separated paper, we do not consider it in this paper.

3. The Models

In this section, we proposed the integrated EPQ and maintenance model for two cases, one case describes the scenario of carrying out PM at the end of a production cycle, and another case describes that of carrying out PM at each set-up point. It is certainly true that more cases exist in reality, and all of the cases can be modeled based on the same way as we developed in this section.

3.1. The PM is Carried Out at the End of a Production Cycle

For batch manufacturing and multi-product situation, the production planners must consider the production capacity within the planning horizon when they arrange the demands of all products, and they always set some reserve time for maintenance and setup based on history data and experience, but the reserve time may be not exact. The production time of all product demands within the planning horizon is

$$\hat{L} = \sum_{i=1}^{k} \frac{D_i}{p_i},$$

where $\hat{L} < L$, the reserve time is $L - \hat{L}$, so the reserve time in every production cycle is $\hat{T} = (L - \hat{L})/n$, and L/n is the length of one cycle. In a cycle, the total set-up time is kd_s, the inspection time at PM is d_p, we denote $\hat{T}_- = \hat{T} - kd_s - d_p$, and allocate $\hat{T}_-$ to every product according to the proportion of production time, so we have the nominal production time of ith product in one production cycle, that is

$$T_i = \frac{D_i}{np_i} + \frac{D_i/p_i}{\sum_{i=1}^{k} D_i/p_i}\hat{T}_-,$$

which contains the actual production time τ_i and the downtime caused by failures during τ_i. Set $\tau_0 = 0$, the relationship of τ_i and T_i can be expressed as

$$d_f \int_{\sum_0^{i-1}\tau_i}^{\sum_0^{i}\tau_i} \lambda F(x)dx + \tau_i = T_i, \tag{1}$$

where

$$,d_f \int_{\sum_0^{i-1}\tau_i}^{\sum_0^{i}\tau_i} \lambda F(x)dx$$

is the downtime caused by failures during τ_i (Wang, 2012), and τ_i can be calculated by Equation (1).

Then we discuss the inventory of products in a cycle, assume that failures are centered at the middle of production cycle of each product (see Figure 1), this is an approximation for reducing the comutation complexity sicne the failure point is random, the inventory situation of ith product may have three scenarios, one is the precisely production capacity (see Figure 1(a)), one is the shortage scenario (see Figure 1(b)), and the express production scenario (see Figure 1(c)).

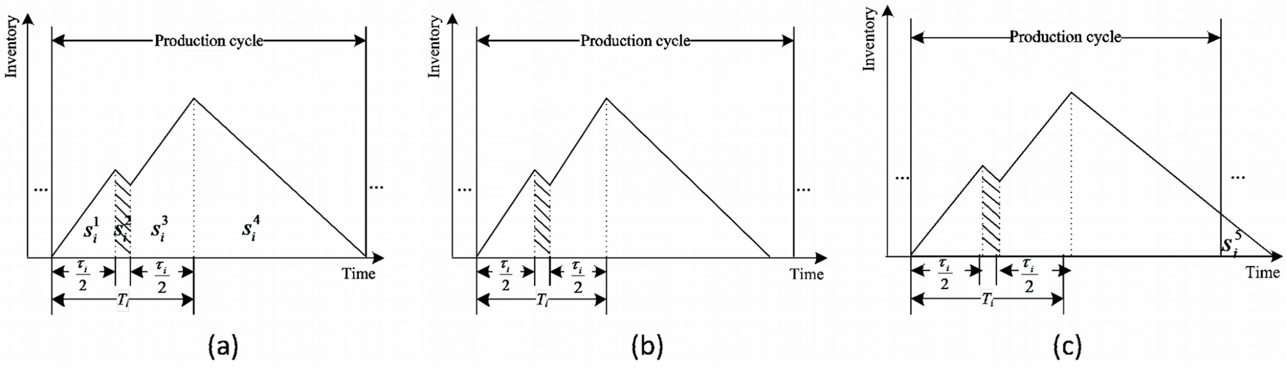

Figure 1. The inventory scenarios of product i.

We use $s_i^1, s_i^2, s_i^3, s_i^4$ and s_i^5 to denote the different areas in Figure 1, we have

$$s_i^1 = \frac{1}{2}\frac{\tau_i}{2}\left[(p_i - d_i)\frac{\tau_i}{2}\right],\ s_i^2 = \frac{\left[(p_i - d_i)\frac{\tau_i}{2}\right] + \left[(p_i - d_i)\frac{\tau_i}{2} - d_i(T_i - \tau_i)\right]}{2}(T_i - \tau_i),$$

$$s_i^3 = \frac{2\left[(p_i - d_i)\frac{\tau_i}{2} - d_i(T_i - \tau_i)\right] + \left[(p_i - d_i)\frac{\tau_i}{2}\right]}{2}\frac{\tau_i}{2},\ s_i^4 = \frac{\left[(p_i - d_i)\frac{\tau_i}{2} - d_i(T_i - \tau_i) + (p_i - d_i)\frac{\tau_i}{2}\right]^2}{2d_i}.$$

In a cycle, the production quantity of product i is $p_i\tau_i$, but the real demand is $d_i(L/n)$, if $p_i\tau_i = d_i(L/n)$, there is no shortage or express production, if $p_i\tau_i > d_i(L/n)$, express production is occur, the express quantity is $I_i^+ = \max\{0, p_i\tau_i - d_i(L/n)\}$, and if $p_i\tau_i < d_i(L/n)$, the shortage is occur, the shortage quantity is $I_i^- = \max\{0, d_i(L/n) - p_i\tau_i\}$. Then we have $s_i^5 = (I_i^+)^2/2d_i$, the expected inventory cost of product i in a cycle can be expressed as

$$E_C(h_i) = \left(s_i^1 + s_i^2 + s_i^3 + s_i^4 - s_i^5\right)h_i + I_i^+ h_i^+. \tag{2}$$

From Equation (2), we can get the inventory costs of all products in a cycle,

$$E_C(h) = \sum_{i=1}^{k} E_C(h_i).$$

The shortage cost in a cycle is

$$C_o \sum_{i=1}^{k} I_i^-.$$

The set-up cost is kC_s. The inspection cost at PM is C_p. The failure and defects repair cost can be expressed as

$$C_f \int_0^{\sum_{i=1}^{k}\tau_i} \lambda F(x)dx$$

and

$$C_d \int_0^{\sum_{i=1}^{k}\tau_i} \lambda(1 - F(x))dx$$

respectively, where

$$\int_0^{\sum_{i=1}^{k}\tau_i} \lambda F(x)dx$$

is the expected number of failures and

$$\int_0^{\sum_{i=1}^{k}\tau_i} \lambda(1-F(x))dx$$

is the expected number of defects identified at PM. At last, we assume that the percentage of unqualified products is proportional to the number of faiures, it lies in the fact that more failures mean more defective components within the system, which furthermore leads to the production of more unqualified products (Liu et al., 2015b), a coefficient β is used to consturct the reationship between the expected number of unqualified product *i*, $E(N_i^{dp})$, and the number of failures, that is

$$E(N_i^{dp}) = \beta\int_{\sum_0^{i-1}\tau_i}^{\sum_0^{i}\tau_i} \lambda F(x)dx,$$

so the expected cost incurred due to unqualified products in a cycle is

$$\sum_{i=1}^{k} C_{dp}E(N_i^{dp}).$$

Based on the above analysises of the cost in a cycle, the expected total cost during the planning horizon can be given by

$$E_C(n) = n\left(\begin{array}{l}\sum_{i=1}^{k} E_C(h_i) + C_o\sum_{i=1}^{k} I_i^- + kC_s + C_f\int_0^{\sum_{i=1}^{k}\tau_i} \lambda F(x)dx + \\ C_d\int_0^{\sum_{i=1}^{k}\tau_i} \lambda(1-F(x))dx + \sum_{i=1}^{k} C_{dp}E(N_i^{dp}) + C_p\end{array}\right) \quad (3)$$

3.2. The PM is Carried Out at Each Set-up Point

If the PM is carried out at each set-up point, some models are the same as that proposed in Section 3.1, so we just present the different models in this section, such as $\hat{T}_- = \hat{T} - kd_s - kd_p$, the relationship of τ_i and T_i can be expressed as

$$d_f\int_0^{\tau_i} \lambda F(x)dx + \tau_i = T_i. \quad (4)$$

In a cycle, the failure and defects repair cost can be expressed as

$$C_f\sum_{i=1}^{k}\int_0^{\tau_i} \lambda F(x)dx$$

and

$$C_d \sum_{i=1}^{k} \int_0^{\tau_i} \lambda(1 - F(x))dx$$

respectively, the inspection cost at PM is kC_p, the expected number of unqualified product is

$$E(N_i^{dp}) = \beta \int_0^{\tau_i} \lambda F(x)dx,$$

so the expected cost of unqualified products is

$$\sum_{i=1}^{k} C_{dp} E(N_i^{dp}).$$

The expression of the expected inventory holding cost, the shortage cost and the set-up cost are the same as that in Section 3.1, we do not repeat that again in this section. So the expected total cost during the planning horizon can be given by

$$E_C(n) = n \begin{pmatrix} \sum_{i=1}^{k} E_C(h_i) + C_o \sum_{i=1}^{k} I_i^- + kC_s + C_f \sum_{i=1}^{k} \int_0^{\tau_i} \lambda F(x)dx + \\ C_d \sum_{i=1}^{k} \int_0^{\tau_i} \lambda(1 - F(x))dx + \sum_{i=1}^{k} C_{dp} E(N_i^{dp}) + kC_p \end{pmatrix} \tag{5}$$

Equation (3) and Equation (5) are the integrated cost models proposed by two PM policies (either at the end of production cycle, or at each set-up point), the decision variable is n, we can calculate the optimal n by minimizing the integrated cost model, and furthermore, the optimal EPQ and PM policies can be calculated and presented. It is clear that some other PM policies can be considered, and the integrated model based on other PM policies can be proposed in the similar way.

4. Numerical Example

In this section, we present a numerical example to illustrate the models in Section 3. The probability density function of the delay-time is assumed to follow an exponential distribution with parameter α, and this distribution has been widely used in previous case studies based on delay time, and was chosen based on the best fit to the actual data (Wang, 2012). The parameters are presented in Table 1 and Table 2, where the number of products is three, the time unit is one day, the production quantity unit is “ton”.

C_s	C_f	C_d	C_p	C_o	C_{dp}	d_s	d_p	d_f
25	500	100	10	5	2	0.2	0.4	0.6

Table 1. The parameters of cost and time

	D_i	p_i	h_i	h_i^+	λ	α	β	L
Product 1	6000	60	0.005	0.01	0.0416	0.0833	2	400
Product 2	10000	80	0.007	0.014				
Product 3	6000	40	0.006	0.012				

Table 2. The parameters of production and system

Using the parameters in Table 1 and Table 2, optimizing Equation (3) and Equation (5) respectively, we can get the optimal results. Figure 2 shows the result of Equation (3), which according to the situation that carrying PM at the end of production cycle, the optimal value of n is 20, $E_C(n^* = 20) = 7787$ is the minimum value, correspondingly, the optimal lot sizes of three products are $Q_1 = 6000/20 = 300$, $Q_2 = 500$ and $Q_3 = 300$. Similarly, Figure 3 shows the result of Equation (5), which according to the situation that carrying PM at each set-up point, the optimal result is $E_C(n^* = 12) = 6503$, and the optimal results are $Q_1 = 6000/12 = 500$, $Q_2 = 833$ and $Q_3 = 500$

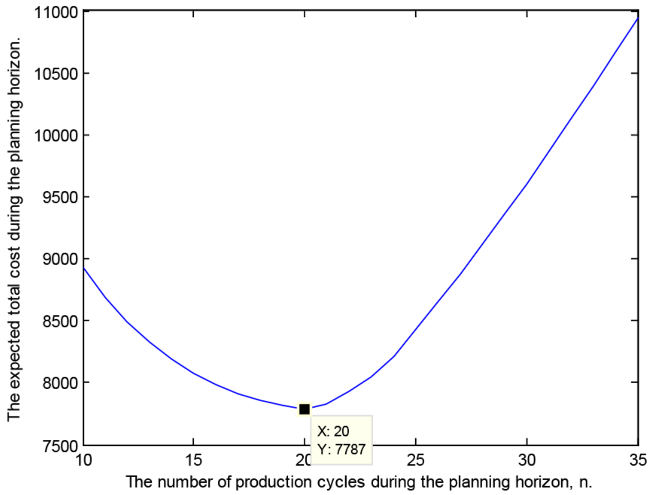

Figure 2. Excepted total costs of Equation (3)

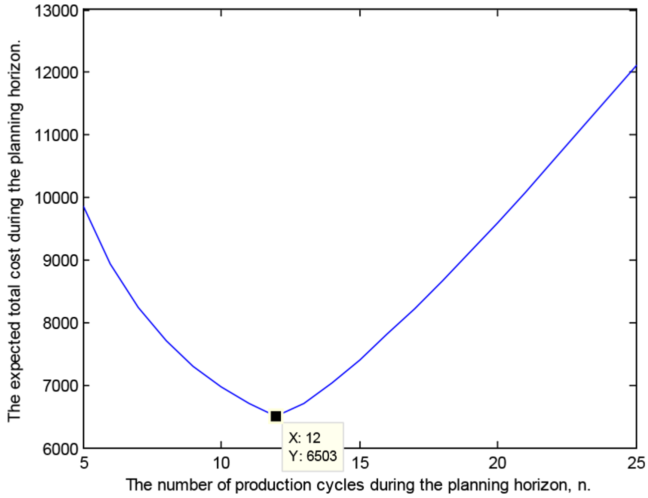

Figure 3. Excepted total costs of Equation (5)

Comparing the results of the two equations, it is clear that the expected total cost calculated by Equation (5) is better than that of Equation (3), so in this example carrying PM at each set-up point is the better choice.

5. Conclusion

In this paper, we studied the integrated problem of maintenance and EPQ for a multi-product system that subject to PM with two cases, one is carrying PM at the end of the production cycle, another one is carrying PM at each set-up point. The optimal result can be obtained by minimizing the total maintenance and production cost during the planning horizon, for each maintenance policy, we first calculate the optimal value of the number of production cycles during the planning horizon, then the optimal economic production quantity can be calculated. At last, we compare the results based on the two PM policies, choosing a policy as the optimal one based on the minimizing total cost. We also explain that the integrated model based on the other PM policies can be proposed in a similar way. The results shows that the optimal maintenance schedule and production planning can be calculated simultaneously, shortages, improper maintenance, and the other harmful effect caused by separated planning can be avoided. Possible extensions of this work can be listed: 1) considering imperfect maintenance in the model; 2) considering several types of failures and defects, which may have different impacts on the cost and downtime of the systems; 3) considering the fluctuating demand in the model.

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