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An Integrated Inventory and Order Pick-up Model Considering the Vendor's Capacities and the Vehicle Trips' Duration for the MVSB System

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Abstract:

Purpose: This research develops a model integrating inventory with order pick-up that considers both the vendor's capacities and the pick-up time for the multi-vendor single-buyer (MVSB) system.

Design/methodology/approach: A model of integrating inventory with pick-up of order considering both the vendor's capacities and the pick-up time for the MVSB's system is formulated as a mixed integer non-linear programming, and a heuristic-metaheuristic algorithm is proposed to solve the problem. This model applies a decomposition approach to minimize total relevant costs, which consist of the buyer's ordering, vendor's set-up, vendor's and buyer's holding, and pick-up.

Findings: This research proposes an inventory model integrated with the pick-up of orders, considering both the capacity of the vendor and the pick-up time for the MVSB system. This research also proposes a hybrid heuristic-metaheuristic algorithm that can obtain a solution in realistic computational time.

Originality/value: This paper proposes a mathematical model for the inbound inventory routing problem, considering both the capacities of vendors and the pick-up time for the MVSB system.

Keywords: JIT procurement, inventory, vendor's capacity, lot-size, vehicle, milk-run

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1. Introduction

Manufacturing industries prepare various types of parts that are needed either in-house or through outsourcing. Global competition often pushes the manufacturing industry to buy (known as outsource) parts that were previously made (Wee, Peng & Wee, 2009), with the possibility of switching to another vendor that is more competitive. The ratio of outsourcing in many industries had reached at least 60 percent, as reported by Muller (2009). Due to their decision to concentrate on their core competencies, companies have tended to outsource various types of parts (Glock & Kim, 2014).

Outsourcing requires the collaboration of vendors, buyers, and other parties (e.g., an expedition company) in a supply chain. As a result, the company left its self-serving and short-term purchasing strategy and switched to JIT procurement. Chen and Sarker (2010) and Moura and Botter (2016) recommended acquiring small lots more frequently in JIT procurement to minimize inventory costs. However, in reality, transportation and inventory costs were traded off. Small pick-up lot sizes effectively reduce inventory costs while simultaneously increasing transportation costs. Conversely, large pick-up lot sizes can reduce transportation costs but also increase inventory costs. To provide long-term benefits in JIT procurement, Goyal and Deskmukh (1992) and Beck, Glock and Kim (2017) recommend that the decision about inventory and transportation policy should consider the interests of all stakeholders.

Determining the lot-sizes while minimizing all relevant costs incurred by buyers and vendors is called joint economic lot-sizing (JELS) (Banerjee, 1986; Kim and Goyal, 2009; and Glock, 2012). There are four types of JELS based on vendor-buyer structure relationships; one of them, which is the focus of this study is multi vendors single buyer (MVSB) (Stacey, Natarajarathinam & Sox, 2007; Kim and Goyal, 2009; and Nemoto, Hayashi & Hashimoto, 2010). Although companies usually involve multi-vendors to provide parts, Glock (2011), who reviewed 155 papers discussing the integration of vendor and buyer, reported that the JELS model for MVSB's systems is still rarely found.

Stacey et al. (2007), Moin, Salhi and Aziz (2011), Glock and Kim (2014), and Beck, Glock and Kim (2017) proposed inventory models that integrated with pick-up consolidation for the MVSB's system. Through the consolidation strategy, the buyer uses large-capacity vehicles to pick-up orders from all of the vendor's location in small lot-sizes and then collectively brings them to the buyer's location. Toyota has effectively implemented the milk-run, one of the most popular forms of consolidation for pick-up (Nemoto et al., 2010).

Models that integrate inventory with order pick-up by implementing milk-run, which is called the inbound-inventory-routing-problem (IIRP), were developed by Natarajarathinam, Stacey and Sox (2012), Stacey (2007), Chen and Sarker (2014). The model for IIRP that considers vendor capacity for the MVSB system was developed by Marpaung, Aribowo, Suprayogi and Halim (2021). However, this model did not consider the pick-up time, which consisted of traveling, loading, and unloading time. The pick-up time needs to be considered to prevent delays in pick-up of parts, which cause disruptions to production activities for the buyers. Thus, an inventory model that is integrated with order pick-up, which considers both the capacities of vendors and the vehicle trips' duration for the MVSB system, is needed.

Marpaung et al. (2021) use an exact approach using Lingo 18.0 to find the optimum solution. It was reported that the global optimum solution was found in relatively short computing time for a very small-scale problem; but when attempted for medium-and large-scale problems, the computing time increases exponentially. Because the problem is NP-hard, it is no longer possible to use an exact approach for medium- and large-scale problems, as often found in the manufacturing industry. So, it is necessary to develop heuristics or metaheuristic algorithms to solve the problem.

This research continues the work of Marpaung et al. (2021) by developing a model of integrating inventory with order pick-up that considers both the vendor's capacity and the vehicle trips' duration simultaneously for the system of MVSB so that total costs of ordering, set-up, holding, and pick-up keeping are minimized. This research also developed a hybrid heuristic-metaheuristic algorithm to solve the problem.

2. Problem Statement

This research adopts the MVSB system, which involves multi-vendors providing parts for single-buyer. The buyer acts as a manufacturer that assembles the parts bought from the vendors. Each vendor also acts as a manufacturer that has limited production capacity, so the supply of one type of part must involve several vendors.

Each vendor performs a set-up before starting production. Parts produced by each vendor in one production cycle are grouped into several same-sized lots and picked up every common pick-up cycle-time using homogeneous vehicles using a milk-run transportation system. This research produces an inventory model that is integrated with the pick-up of orders for the MVSB system to keep all relevant costs to a minimum. The decision variables used for this model are the common cycle-time of order pick-up, frequency and lot-size of pick-up, vendor's lot-size production, and routes of each vehicle.

Assumptions are used in the development of the model: 1) Only one type of component is produced by each vendor. 2) Production rate of each vendor and the buyer's demand rates are constant and without defective parts. 3) The production rate of each vendor is smaller than the demand rate of the buyer. 4) Shortages and backlogs are not allowed. 5) Every vendor uses JIT delivery so that there is a common cycle time for pick-up. 6) Parts are picked up from the vendor's location using a milk-run system. 7) There is only one type of vehicle that is available in unlimited quantities.

3. Mathematical Model

3.1. Indexes, Parameters, and Decisions Variables

The following notations are used in this paper:

Indexes:

i, j = Index of vendor, $i, j = 0, \ldots, n$
o = Index of part, $o = 1, \ldots, O$
v = Index of vehicle, $v = 1, \ldots, V$

Parameters:

D_o = Rate of demand for Part o ($o = 1, \ldots, O$), (unit/year)
W_o = Weight of Part o (kg/unit)
P_{io} = Rate production each vendor for each part ($i = 0, \ldots, n; o = 1, \ldots, O$), (unit/year)
S_{io} = Cost for set-up at each vendor for each part ($i = 0, \ldots, n; o = 1, \ldots, O$), ($/*set-up*)
HM_o = Cost for holding Part o at the buyer's location ($/unit/year)
HV_{io} = Cost for holding of each part at each vendor ($i = 0, \ldots, n; o = 1, \ldots, O$), ($/unit/year)
A_i = Ordering costs for each vendor ($/order)
d_{ij} = Distance (in km), ($i, j = 0, \ldots, n$)
C = Capacity of each vehicle, (kg/vehicle)
F_o = Cost of the fixed transportation ($/vehicle)
F_y = Cost of the variable's transportation ($/kg/km)
t_{ij} = Traveling time of the vehicle from Vendor-i to Vendor-j, ($i, j = 0, \ldots, n$), (km)
N = A big number (use:1,000,000)
LT_i = Loading time at Vendor i ($i = 1, \ldots, n$), (hour)
ULT_i = Unloading time at Vendor i ($i = 1, \ldots, n$), (hour)

Decision Variables:

T = Common cycle time for order pick-up (in year)
m_{io} = Frequency of pick-up of each part for each vendor, ($i = 0, \ldots, n; o = 1, \ldots, O$), (times/cycle of production)
q_{io} = Lot-size for pick-up of each part for each vendor (unit/pick-up)
Q_{io} = Production lot-size of Vendor i for Part o, $Q_{io} = q_{io} m_{io}$, (unit/*batch*)
$x_{ijv} = \begin{cases} 1, \text{ if for Route } v, \text{ Vendor } j \text{ is visited immediately after Vendor } i, \\ 0, \text{ for otherwise} \end{cases}$
CL_{iv} = Cumulative load of Vehicle v when leave from Vendor i (kg)
u_{jv} = A variable to eliminate subtours

3.2. The Objective Function

For every common pick-up cycle-time T, the assembler, who acts as a buyer, orders parts from Vendor i with order cost A_i. The ordering cost from all vendors in one year is stated in Equation (1).

$$OC = \frac{\sum_{i=1}^{n} A_i}{T} \tag{1}$$

Before starting production to meet the order for Part *o*, Vendor *i* has to do set-up with cost S_{io}. The total set-up cost incurred in one year is stated in Equation (2).

$$SC = \sum_{i=0}^{n} \sum_{o=1}^{O} \frac{S_{io}}{T.m_{io}} \tag{2}$$

After doing the set-up, Vendor *i* starts production for Part *o* until one production cycle ends. Before being delivered to the buyer's location, the part is held in the Vendor *i* location with a holding cost HV_{io}. The total holding cost when Part *o* is stored at Vendor *i* for each cycle of production is a multiplication of HV_{io} and the average of the inventory amount of Vendor *i* in one production cycle, in literature known as the time-weighted-inventory (TWI).

In the previous study, Chen and Sarker (2014) assumed that the rate production of the vendor was higher than the rate production of the buyer ($P > D$), so that one vendor could fulfill the buyer's demand. Different from that study, this research assumes ($P < D$) therefore, procurement for one type of part must involve multiple vendors. That is why the system is called multi-vendor single-buyer (MVSB). For estimating TWI, we adopt a formula from Joglekar (1988) that assumes $P < D$, as stated in Equation (3).

$$TWI_{io} = \frac{1}{2}(q_{io}m_{io})^2\left(\frac{1}{P_{io}} - \frac{m_{io}-1}{m_{io}D_o}\right) \tag{3}$$

By using Equation (3), the total holding cost of all vendors in one year is stated in Equation (4).

$$HC = \frac{1}{2}\sum_{i=1}^{n}\sum_{o=1}^{O} HV_{io}\,(q_{io}m_{io})^2\left(\frac{1}{P_{io}} - \frac{m_{io}-1}{m_{io}D_o}\right) \tag{4}$$

Parts produced by each vendor are then brought to the buyer's location using a pick-up vehicle. Every unit of Part *o* that is stored at the buyer's location incurs storage costs HM_o. Since the average amount of Part *o* are stored at the buyer's location is $\frac{1}{2}D_o$ units, the total parts' holding cost at the buyer's location in one year is stated in Equation (5).

$$THM = \frac{T}{2}\sum_{o=1}^{O} HM_o\,D_o \tag{5}$$

Parts from vendors are picked up by vehicles using a milk-run transportation system. This study uses Chen and Sarker's (2014) to determine pick-up costs, consisting of fixed and variable pick-up costs. The fixed pick-up cost F_o is the cost of renting a vehicle, and the variable pick-up cost F_y (also known as the freight rate) is the vehicle's operational costs. The pick-up fixed and variable costs in one year are stated in Equation (6) and Equation (7), respectively.

$$FCP = \frac{1}{T}\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{v=1}^{V} F_o x_{ijv}\right) \tag{6}$$

$$VCP = \frac{1}{T}\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{v=1}^{V} F_y x_{ijv} d_{ij} CL_{ikl}\right) \tag{7}$$

The objective function minimizes all relevant costs, as formulated in Equation (8).

$$Min\ TC^{Sys} = \frac{1}{T}\left(\sum_{i=1}^{n}\sum_{o=1}^{O}\frac{S_{io}}{m_{io}} + \sum_{i=1}^{n}A_i\right) + \frac{T}{2}\sum_{o=1}^{O}HM_o\ D_o + \frac{1}{2}\sum_{i=1}^{n}\sum_{o=1}^{O}HV_{io}\,(q_{io}m_{io})^2\left(\frac{1}{P_{io}} - \frac{m_{io}-1}{m_{io}D_o}\right) + \frac{1}{T}\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{v=1}^{V}F_o x_{ijv}\right) + \frac{1}{T}\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{v=1}^{V}F_y x_{ijv} d_{ij} CL_{ijv}\right) \quad (8)$$

3.3. Constraints

The constraints functions refer to Marpaung et al.'s (2021) work with one new constraint: Constraints (24). The additional Constraint (24) ensures that vehicles will not arrive late at each vendor and buyer location, which can disrupt the buyer's assembly activities. The formulations of all constraints are stated as follows:

Constraint 1: The number of parts in one pick-up cycle is at least the number of buyer's demands in one order cycle, stated in Constraint (9).

$$\sum_{i=0}^{n} q_{io} \geq D_o T\ for\ \forall\ o \quad (9)$$

Constraint 2: The number of parts picked up from one vendor is limited by its production capacity, as stated in Constraint (10).

$$q_{io} \leq P_{io} T\ for\ \forall\ \text{i, o} \quad (10)$$

Constraint 3: Each vendor is served or visited only by one vehicle, as stated in Constraint (11) and (12).

$$\sum_{v=1}^{V}\sum_{i=1}^{n}\sum_{j=0}^{n} x_{ijv} = 1\ \ for\ \ \forall\ i, j \neq i \quad (11)$$

$$\sum_{v=1}^{V}\sum_{i=1}^{n}\sum_{i=1}^{n} x_{ijv} = 1\ \ for\ \forall\ j, i \neq j \quad (12)$$

Constraint 4: The buyer's location is the starting and finishing point of each vehicle, as stated in Constraints (13) and (14).

$$\sum_{j=1}^{n} x_{ijv} \leq 1\ for\ i = 0, \forall\ v \quad (13)$$

$$\sum_{i=1}^{n} x_{ijv} \leq 1\ for\ j = 0, \forall\ v \quad (14)$$

Constraint 5: Continuity of route, meaning that after visiting a vendor's location, the vehicle immediately leaves that location after loading parts, as stated in Constraint (15).

$$\sum_{i=1}^{n} x_{ihv} - \sum_{j=1}^{n} x_{hjv} = \ 0\ for\ h = 1, \ldots, n;\ i, j \neq h, \forall\ v \quad (15)$$

Constraint 6: The elimination of sub-tours is stated in Constraints (16)-(19).

$$U_{jv} \geq\ U_{jv} + \sum_{o=1}^{O} q_{jo.W_o} - N\left(1 - x_{ijv}\right) for\ \forall\ i;\ j = 1, \ldots, n, \forall\ v \quad (16)$$

$$U_{iv} \geq \sum_{o=1}^{O} q_{io.W_o}\ \ for\ i = 1, \ldots, n; \forall\ v \quad (17)$$

$$U_{iv} \leq C \; for \; \forall \; i, v \tag{18}$$

$$U_{iv} = 0 \; for \; i = 0 \; and \; \forall \; v \tag{19}$$

Constraint 7: Cumulative vehicle's load

Constraint 7.1: The load of the vehicle when leaving the buyer's location is empty, as stated in Constraint (20).

$$CL_{iv} = 0 \; for \; i = 1; \; and \; \forall \; v \tag{20}$$

Constraint 7.2: The cumulative load of each vehicle at a vendor's location is the sum of the parts to be picked up from that vendor and the load carried from the previously visited vendor's location, as stated in Constraint (21).

$$CL_{jv} = (\textstyle\sum_{i=1}^{n} \sum_{o=1}^{O} q_{io} \,. W_o + CL_{iv}). x_{ijv} \; for \; j = 1, \ldots, n, i \neq j \; and \; \forall \; o \tag{21}$$

Constraint 7.3: Each vehicle's total load, which is at least equal to the load of the visiting vendor, restricts its capability, as stated in Constraints (22) and (23).

$$CL_{iv} \leq C \; for \; i = 1, \ldots, n, and \; \forall \; i, v \tag{22}$$

$$CL_{iv} \geq q_{io}. W_o \; for \; i = 1, \ldots, n, and \; \forall v \tag{23}$$

Constraint 8: The vehicle trips' duration is no more to the common cycle-time for order pick-up, as stated in Constraint (24).

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_{ijv} \left(t_{ij} + LT_i + ULT_i \right) \leq T * 24 * 365 \; for \; \forall \; v \tag{24}$$

Constraint 9: The decision variable restrictions are stated in Constraint (25).

$$x_{ijv} \in (0{,}1) \; for \; \forall \; i, j, v, \; \mathrm{T} > 0, \; \mathrm{q}_{io} \geq 0, \; \mathrm{m}_{io} > 0 \; and \; integer \; for \; \forall \; i, o \tag{25}$$

The mathematical formulation described before is categorized as mixed-integer-nonlinear-programming (MINLP), consists of the inventory problem and the vehicle-routing problem (VRP), both of which are NP-hard problems. This model is classified as NP-hard, meaning it is not easy to obtain the solution with an analytic and/or exact approach. Therefore, in the following section, a heuristic-metaheuristic algorithm will be developed for solving this problem.

4. Heuristic-Metaheuristic Algorithms

4.1. Propositions

In this research, the proposed algorithm requires three propositions as the basis for developing it. Proposition 1 concerns the allocation of orders; Proposition 2 considers the behavior of the relevant total costs; and Proposition 3 concerns the set of common pick-up times (*T* point) that has the potential to become the optimum solution. The three propositions are described as follows:

Proposition (1). Although the composition technique developed by Park, Kim and Hong (2006) to find the optimum order allocation is focused only on the inventory model, it can still be used for model inventory that integrates with VRP. The mathematical model of Park et al. (2006) is expressed in Equations (26) and Constraints (27)-(29). Proof of Proposition 1 is stated in Appendix 1.

$$Min\ Z = -\sum_{o=1}^{o}\sum_{i=1}^{n} \frac{S_{io}\left(\frac{P_{io}}{2D_o}\left(1-\frac{D_o}{S_{io}}\right)\frac{\left(\frac{P_{io}}{2D_o}-2\right)}{\frac{P_{io}}{S_{io}}}-\lambda_{io}\right)^2}{\frac{P_{io}}{D_o}} \tag{26}$$

Subject to:

$$\sum_{i=1}^{n}\lambda_{io} = 1\ for\ \forall\ o \tag{27}$$

$$\lambda_{io} \le \frac{P_{io}}{D_o}\ for\ i = 1,\ldots n, \forall\ o \tag{28}$$

$$0 \le \lambda_{io} \le 1 \tag{29}$$

Proposition (2). The total relevant costs are at a minimum when the vehicle's utility is close to 100%.

Proof: The common cycle-time of pick-up (*T*) is a decision variable that is continuous in the interval 0 to 1. The increase in computation time to solve the integrated inventory-VRP model is caused by the process of finding the optimum value of *T* in the interval range (0.1). Then an analysis is carried out to determine the relationship between *T* and the cost of elements of the objective function, as shown in Figure 1.

Figure 1 denotes that inventory costs are monotonically decreasing from *T* value 0 to a certain *T* value and then change the pattern to be monotonically increasing. The variable pick-up costs are monotone and non-increasing. Meanwhile, fixed pick-up costs still have a local optimum point that occurs when the pick-up vehicle's utility reaches 100 percent or full capacity. It can be seen that the total relevant costs have a local optimum point following the pattern of fixed pick-up costs. Therefore, when the vehicle's utility is close to 100% or full capacity, the optimum total relevant cost is obtained.

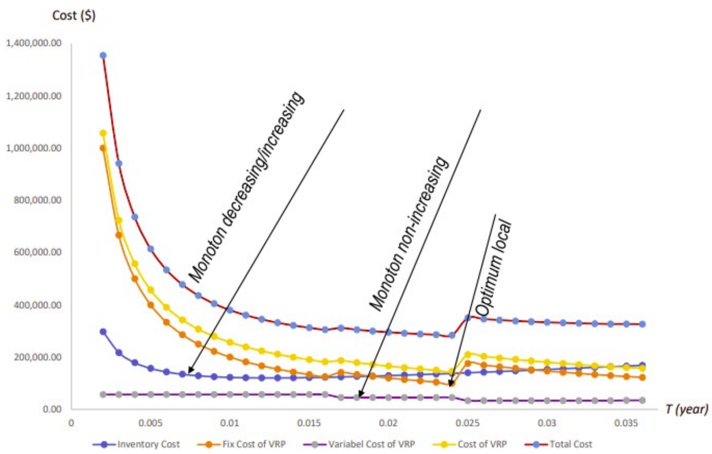

Figure 1. The relationship between T and relevant costs

Proposition (3). The points *T* so that the utility of vehicle is close to 100 percent is the ratio between the entire available vehicle's capacity and the total weight of the part for pick-up. The set of points *T's* is stated in Equation (30).

$$\tau = \left\{T(v) = \frac{C.v}{\sum_{o=1}^{o} D_o.W_o}\ for\ v = 1, \ldots, n,\ o = 1, \ldots, O\right\} \tag{30}$$

Proof: If the annual demand for Part o for the buyer is D_o, and each part unit weights W_o, then the total weight of Part o in one year is $D_o \cdot W_o$, so the total weight of all parts imported from vendors is $\sum_{o=1}^{O} D_o . W_o$. For the pick-up vehicle that arrives at every pick-up cycle-time (T), the total weight of the load carried by the vehicle is $\sum_{o=1}^{O} D_o . W_o$.

There are v unit vehicles, and each of them has a capacity C, so that the total available vehicle capacity in one common pick-up cycle-time is $C \cdot V$. In every common pick-up cycle-time T, the load amount is $T . \sum_{o=1}^{O} D_o . W_o$, so $T . \sum_{o=1}^{O} D_o . W_o \leq C . V$, or $T \leq \frac{C.v}{\sum_{o=1}^{O} D_o . W_o}$. The minimum total vehicle capacity required in every pick-up cycle-time is $T . \sum_{o=1}^{O} D_o . W_o$. This means that the total vehicle capacity can exceed the minimum vehicle requirement, but this is a loss because the vehicle is not fully utilized.

The set of points T that potentially contain the local optimum point and/or the global optimum point is the set of points of the ratio between the total vehicle capacity to $\sum_{o=1}^{O} D_o . W_o$. Thus, the set of T points that has the potential to become a local optimum point and/or a global optimum point is $\tau = \left\{ T(v) = \frac{C.V}{\sum_{o=1}^{O} D_o . W_o} \; for \; v = 1, \ldots, n, o = 1, \ldots, O \right\}$.

4.2. Algorithm Proposed

The algorithm that is proposed uses a decomposition technique, which splits the problem into both inventory and VRP subproblems. Each subproblem is solved separately but is united by two shared decision variables (known as global variable): the common pick-up cycle-time (T) and order allocation (λ_{io}). The pick-up frequency (m_{io}) and the vehicle route (x_{ijv}) are decision variables that only exists for inventory and VRP subproblem, respectively (known as local variables).

The objective function of the inventory subproblem is to minimize set-up, ordering, and holding costs for vendors, and holding costs for buyers, as stated in Equation (31). Its constraints are only variable restrictions for T and m_{io} stated in Constraint (32).

$$Min \; TC^{Inv.} = \frac{1}{T}\left(\sum_{i=1}^{n}\sum_{o=1}^{O}\frac{S_{io}}{m_{io}} + \sum_{i=1}^{n} A_i\right) + \frac{T}{2}\sum_{o=1}^{O} HM_o \; D_o + \frac{1}{2}\sum_{i=1}^{n}\sum_{o=1}^{O} HV_{io} \, (\lambda_{io} . D_o . T . m_{io})^2 \left(\frac{1}{P_{io}} - \frac{(m_{io} - 1)}{m_{io} D_o}\right) \tag{31}$$

Subject to:

$$T > \; 0, mio > \; 0 \; and \; integer \tag{32}$$

The objective function of the VRP subproblem is to minimize the fixed pick-up and the variable pick-up costs, as stated in Equation (33). The constraints for the VRP subproblem are Constraints (11)-(23) plus Constraint (34).

$$Min \; TC^{VRP} = \frac{1}{T}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{v=1}^{V} F_o x_{ijv}\right) + \frac{1}{T}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{v=1}^{V} F_y x_{ijv} d_{ij} CL_{iv}\right) \tag{33}$$

Subject to:

$$T > \; 0, \; x_{ijv} \in (0,1) \; for \; \forall \; i, j, v \tag{34}$$

Based on the three propositions above, the heuristic-metaheuristic algorithm is developed in the following stages:

1. Determine the order allocation (λ_{io}) using Equations (26), (27), (28) and (29).
2. Determine T_{max} using Equation (35).

$$T_{max} = \underset{\forall i,o}{\text{Min}} \left(\frac{C}{\lambda_{io} . D_o . W_o}\right) \tag{35}$$

3. Determine the set of point T using Equation (30).
4. For $r = T(v = 1)$ determine:
   a) Value of m_{io} using Equation (31) and Constraint (32)
   b) Value of x_{ijr} using Equation (33), Constraints (11)-(23), and Constraint (34).
   c) Find the minimum value $MinTC^{Sys}(r)$ using Equation (36)

$$Min\ TC^{Sys}(r) = Min\ TC^{Inv}(r) + Min\ TC^{VRP}(r) \tag{36}$$

5. Repeat step 4 for $r = r + 1$
6. Determine the minimum value $MinTC^{Sys}$ using Equation (37)

$$Min\ \ TC^{Sys} = \underset{\forall r \in \tau}{Min}\{TC^{Sys}(r)\} \tag{37}$$

7. The flowchart of the algorithm that is proposed is shown in Figure 2.

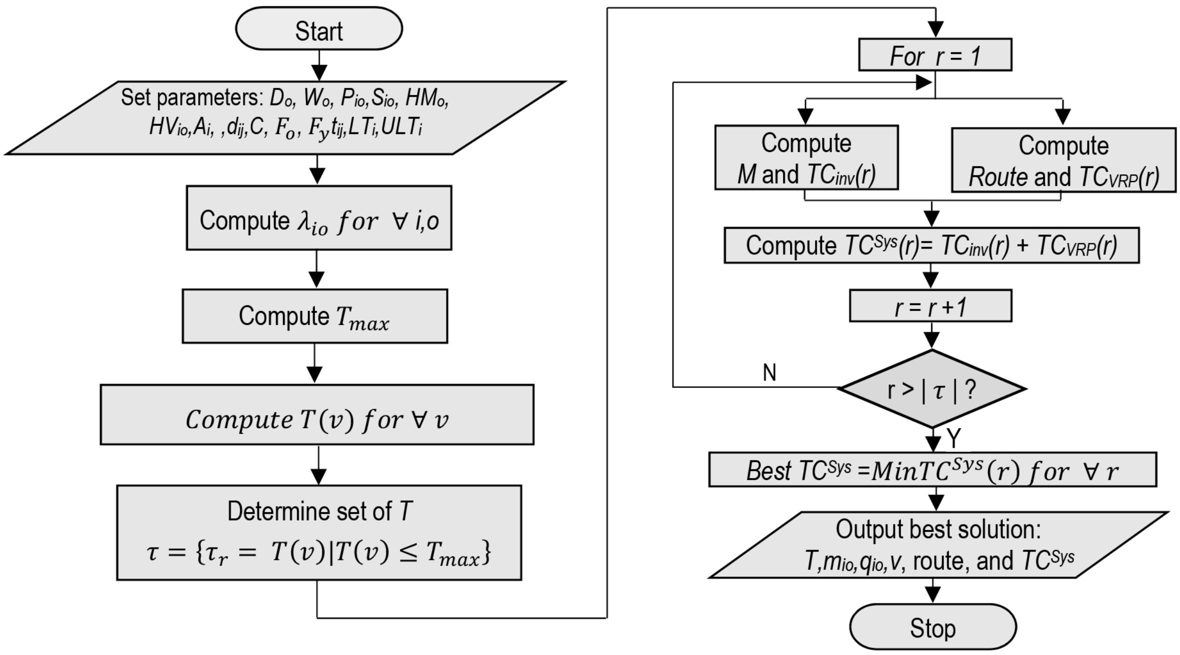

Figure 2. Flowchart of the heuristic-metaheuristic algorithm

The computation time for the subproblem of inventory when solved with the help of Lingo 18.0 is still rational for various scales of problems. On the contrary, as the problem scales increase, the time computation increases exponentially. Furthermore, the VRP subproblem's solution is found by ant colony optimization (ACO). In contrast to Chen and Sarker (2014), who used ACO for IIRP problems, the proposed algorithm only uses ACO for the VRP subproblem. The parameters of ACO used are: N is 10, α is 1, β is 5, ρ is 0.1, Φ1, and NI_{max} is 1,000. Figure 3 is the pseudo-code for the VRP subproblem.

4.3. Testing the Proposed Algorithm's Efficiency and Effectiveness

The problem situation discussed in this study occurs in the manufacturing industry (such as automotive, electronics, machine tools, and heavy equipment), which implements JIT production-procurement. In JIT procurement, a milk-run logistics system has been implemented to reduce transportation costs. Based on the existing problems, this research develops a model to integrate inventory and order pick-up policies, minimizing all relevant costs in the supply chain system.

Two numerical examples (that are categorized as small-scale problems) that can be solved by the exact method are used to assess the proposed algorithm's efficacy and efficiency. In the first scenario, there are three vendors offering one kind of component using one kind of vehicle, 1P-3V-1TV; and in the second scenario, one type of component is offered by four vendors using one type of vehicle, 1P-4V-1TV.

Parameter data consisted of five sets generated for both scenarios, which were each replicated 10 times. Using the computer's configuration (Intel Core/CPU1.60GHz/ RAM4GB), both numerical examples were solved by both the exact method and the proposed algorithm. The objective function average's (OF) and the computation time average's (CT) for the exact method and heuristic algorithm for each set of data parameters are compared. The experimental results are stated in Table 1.

```
Begin: input data and initialization of parameters
    ACO's set parameters
    Set parameters of model
NI = 1; (iterations initialization)
While NI ≤ NI_max; (number of maximum iterations)
  | For i = 1: N; (number of ant population)
  |   | Route construction considering capacity of vehicle and vehicle duration trip
  |   | Compute f; (objective function)
  |   | Find the best solution
  |   | Record the best solution
  | End for
  | Entering policy of adaptable control, ρ:
  | If NI > Nic (if the solution not improve after N_ic iteration)
  |   | If (f_best (NI) = f_best (NI − Nni); f_best (NI) is the best fitness for iteration NI)
  |   |   | ρ = 0.90*ρ
  |   | End if
  |   | If ρ ≤ ρmin
  |   |   | ρ = ρmin
  |   | End if
  | End if
  | Update pheromone
  | NI = NI + 1
  | Find the best solution for this iteration
End while
Output best solution
```

Figure 3. Pseudo-code algorithm of ACO for the VRP subproblem

Dataset name	Exact method		Proposed algorithm		Gap (Δ)	
	OF ($)	CT (second)	OF ($)	CT (second)	OF (%)	CT (%)
1P-3V-1TV (1)	229,389.90	55.00	229,389.90	17.41	0.00	- 40.91
1P-3V-1TV (2)	263,822.30	41.10	263,822.30	18.27	0.00	-55.54
1P-3V-1TV (3)	249,779.30	54.40	249,454.30	13.65	0.00	-74.90
1P-3V-1TV (4)	305,691.00	36.42	305,691.00	14.53	0.00	-60.10
1P-3V-1TV (5)	245,563.20	35.88	245,556.20	14.39	0.00	-59.88
1P-4V-1TV (1)	288,771.50	20,328.10	289,471.50	52.61	0.24	-99.74
1P-4V-1TV (2)	286,592.10	20,144.20	286,642.80	59.59	0.02	-99.70
1P-4V-1TV (3)	200,186.80	19,690.28	200,186.80	71.31	0.00	-99.64
1P-4V-1TV (4)	259,700.00	19,974.30	259,700.10	65.51	0.00	-99.67
1P-4V-1TV (5)	244,991.00	19,772.60	244,991.00	61.09	0.00	-99.69
Average	257,448.71	10,013.59	257,490.59	38.84	0.03	-78.98

Table 1 The experiment results

As can be seen in the experimental results above, the computation time, if solved by the exact method, has increased exponentially since increasing the number of vendors. It means solving the problem using the exact method, especially when dealing with a large-scale problem, is unrealistic. Conversely, the solution that was obtained using the heuristic-metaheuristic algorithm is almost near the global optimum (based on error in average OF < 1 %) and has a shorter computation time. So, the heuristic-metaheuristic algorithm is effective and efficient for solving problems on various scales.

Furthermore, three types of scales of problems (small, medium, and large) were conducted. The parameter data consists of five sets generated for all scenarios. For all of data sets, a heuristic-metaheuristic algorithm solution was obtained for 10 replications. The average ($\bar{x}$), standard deviation (SD), and coefficient of variation (CV) of both the objective function (OF) and computation time (CT) for all of data sets were recorded. The experimental results are stated in Table 2.

Dataset name	OF ($)			CT (second)		
	$\bar{x}$	SD	CV	$\bar{x}$	SD	CV
1P-5V-1TV (1)	304,495.60	3,124.5	0.0103	72.5	1.12	0.0154
1P-5V-1TV (2)	404,192.40	4,124.5	0.0102	74.5	1.47	0.0197
1P-5V-1TV (3)	502,765.90	5,215.4	0.0104	76.5	1.45	0.0190
1P-5V-1TV (4)	601,394.30	9,114.3	0.0152	78.2	1.26	0.0161
1P-5V-1TV (5)	214,555.80	4,124.5	0.0192	79.4	1.54	0.0194
5P-25V-1TV (1)	1,157,882.75	15,950.24	0.0138	126.4	1.67	0.0132
5P-25V-1TV (2)	1,217,682.85	14,850.27	0.0122	134.5	1.95	0.0145
5P-25V-1TV (3)	1,817,872.75	25,950.24	0.0143	127.5	1.65	0.0129
5P-25V-1TV (4)	1,157,882.75	16,875.64	0.0146	134.5	2.14	0.0159
5P-25V-1TV (5)	1,157,882.75	18,970.85	0.0164	142.3	2.45	0.0172
10P-50V-1TV (1)	2,313,544.53	45,801.15	0.0198	302.6	3.54	0.0117
10P-50V-1TV (2)	2,613,541.17	46,711.16	0.0179	314.2	3.75	0.0119
10P-50V-1TV (3)	2,113,241.75	35,501.47	0.0168	324.3	4.12	0.0127
10P-50V-1TV (4)	2,411,551.59	38,801.45	0.0161	316.8	5.11	0.0161
10P-50V-1TV (5)	2,713,575.35	46,505.46	0.0171	317.5	5.45	0.0172
Average	-	-	0.0150	174.8	-	0.0155

Table 2 The proposed algorithm's computational results

The computation results demonstrate that, as indicated by the CV's average of the OF and CT, the proposed algorithm yields reliable or consistent results. Overall, the values of objective function and time of computation have coefficients of variation of 1.02% and 1.98%, respectively. Therefore, the heuristic-metaheuristic developed in this paper may be used to solve this model for different sizes of problems.

5. Concluding Remark

This research produces an inventory model that is integrated with the pick-up of orders using milk-run logistics for the MVSB's system, considering both the vendor's capacity and the pick-up vehicle trips' duration so that the total relevant costs are kept to a minimum. By formulating it as a MINLP, this research succeeded in accommodating both the limitation of the vendor's capacity and the vehicle trips' duration simultaneously. The model outputs are common pick-up cycle-time, order allocation, parts quantity to be picked up from each vendor location, pick-up frequency in one production cycle for each vendor, and pick-up route of vehicles to minimize total relevant cost.

The results obtained showed that the effort to achieve the global optimum solution by the exact method caused exponentially increasing computational time as the problem scale increased. Under these conditions, a new approach is needed that can find the solutions in a reasonable amount of computation time. This research develops a heuristic algorithm combined with a metaheuristic approach. Using a decomposition technique, the hybrid heuristic-metaheuristic method divides the problem as inventory problem and VRP. While a solution for the inventory subproblem can still be obtained by the exact method with Lingo, the solution for the VRP subproblem was solved by the metaheuristic approach (in this case using the ACO with MATLAB).

In this study, experiments were carried out by developing three problem scale scenarios: small-scale (1P-5V-1TV), medium-scale (5P-25V-1TV), and large-scale (10P-50V-1TV). Based on the experimental findings, it was possible to obtain the best solution in an acceptable computing time for three different problem sizes. Thus, the proposed algorithm is reliable for solving IIRP on MVSB's system, which considers both the vendor capacity and the vehicle trips' duration.

Both vendor capacity limitations and vehicle trip duration are essential considerations in developing a model integrating inventory with pick-up of orders for the system of MVSB. In reality, in manufacturing industries, many vendors must be involved to meet buyer demand for one type of part because of limited vendor capacity. Actually, in reality, the buyer must also consider the duration of the vehicle trip to ensure the availability of parts when needed without increasing inventory.

However, this model still has limitations. Only one type of vehicle is considered in the model. In reality, manufacturing industry companies buy various types of parts from many vendors using heterogeneous vehicles. Consequently, the addition of vehicle types will increase the complexity of the model. It is suggested for future research to model the IIRP, which considers various types of vehicles and other aspects.

Declaration of Conflicting Interests

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Appendix 1. Proof of Proposition 1

The proof of Proposition 2 was carried out using a numerical approach. Two numerical examples were created by varying the number of vendors for the supply of one type of part using one type of vehicle (1P-3V-1TV, 1P-4V_1TV). Each numerical example was iterated until the optimum order of allocation (λ_{io}) was found. The results then were compared to the order allocation according to the model of Park et al. (2006).

For scenario 1P-3V-1TV, initialization was carried out by setting the initial lambda (λ_{io}) as the ratio of each vendor's production capacity to the total production capacity. The values λ_{io} obtained were then used as input to get T, m_{io}, and x_{ijv}. Furthermore, the values of T, m_{io}, and x_{ijv} were used as input to get λ_{io}. Thus, the iteration was continued until the values of T, m_{io}, x_{ijv} and λ_{io} were steady. A similar method was used for scenario 1P-4V-1TV. Using Lingo 18.0, the results for scenarios 1P-3V-1TV and 1P-4V-1TV are stated in the following two tables.

Iteration	Input	Output
1	λ_{11} = 0.26, λ_{21} = 0.35, λ_{31} = 0.39	T = 0.03, m_{11} = 4, m_{21} = 4, m_{31} = 6, route: V_o-V_1-V_2-V_3-V_o
2	T = 0.03, m_{11} = 4, m_{21} = 4, m_{31} = 6, route: V_o-V_1-V_2-V_3-V_o	λ_{11} = 0.175, λ_{21} = 0.375, λ_{31} = 0.45
3	λ_{11} = 0.175, λ_{21} = 0.375, λ_{31} = 0.45	T = 0.20, m_{11} = 6, m_{21} = 4, m_{31} = 4, route: V_o-V_1-V_2-V_3-V_o
4	T = 0.20, m_{11} = 6, m_{21} = 4, m_{31} = 4, route: V_o-V_1-V_2-V_3-V_o	λ_{11} = 0.15, λ_{21} = 0.40, λ_{31} = 0.45
5	λ_{11} = 0.15, λ_{21} = 0.40, λ_{31} = 0.45	T = 0.20, m_{11} = 6, m_{21} = 4, m_{31} = 4, route: V_o-V_1-V_2-V_3-V_o

Table A. The input and output of scenario 1P-3V-1TV

Iteration	Input	Output
1	λ_{11} = 0.23, λ_{21} = 0.19, λ_{31} = 0.35, λ_{41} = 0.31	T = 0.016, m_{11} = 7, m_{21} = 8, m_{31} = 7, m_{41} = 6, route: V_0-V_3-V_2-V_1-V_4-V_0
2	T = 0.016, m_{11} = 7, m_{21} = 8, m_{31} = 7, m_{41} = 6, route: V_0-V_3-V_2-V_1-V_4-V_0	λ_{11} = 0.240, λ_{21} = 0.198, λ_{31} = 0.242, λ_{41} = 0.320
3	λ_{11} = 0.240, λ_{21} = 0.198, λ_{31} = 0.242, λ_{41} = 0.320	T = 0.016, m_{11} = 7, m_{21} = 7, m_{31} = 8, m_{41} = 6, route: V_0-V_3-V_2-V_1-V_4-V_0
4	T = 0.016, m_{11} = 7, m_{21} = 7, m_{31} = 8, m_{41} = 6, route: V_0-V_3-V_2-V_1-V_4-V_0	λ_{11} = 0.24, λ_{21} = 0.20, λ_{31} = 0.24, λ_{41} = 0.32
5	λ_{11} = 0.24, λ_{21} = 0.20, λ_{31} = 0.24, λ_{41} = 0.32	T = 0.016, m_{11} = 7, m_{21} = 7, m_{31} = 8, m_{41} = 6, route: V_0-V_3-V_2-V_1-V_4-V_0

Table B. The input and output of scenario 1P-4V-1TV

The optimum order allocation obtained by Park et al. (2006) for scenario 1P-3V-1TV was λ_{11} = 0.15, λ_{21} = 0.40, and λ_{31} = 0.45. The optimum order allocation for scenario 1P-4V-1TV was λ_{11} = 0.24, λ_{21} = 0.20, λ_{31} = 0.24, and λ_{41} = 0.32. The results show that the optimum order allocation value obtained from the iterative process for both scenarios was the same as the optimum order allocation using the model of Park et al (2006). Based on these results, the determination of the optimum order allocation in Park et al. (2006) model can be used for this research.

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