

Supply Chain Single Vendor – Single Buyer Inventory Model with Price - Dependent Demand

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Abstract:

Purpose: The aims of this article are to develop an integrated production-inventory-marketing model for a two-stage supply chain, and to study the effect of coordination on the performance of the system. The demand rate of the end customer is assumed to be sensitive to the selling price. The inventory models are developed, and then optimal values of the selling price, order quantity and number of shipments for the independent and also joint supply chain are determined. In addition, the effects of coordination and the parameters of the model on the optimal solution and the performance of the supply chain are investigated.

Design/methodology: Mathematical modeling is used to obtain the profit functions of the vendor, the buyer and the whole supply chain. Then, the iterative solution algorithm is presented to solve the models and to determine the optimal solution in the coordinated/non-coordinated supply chain.

Findings: It is observed that under joint optimization, the demand rate and the supply chain's profit are higher than their values under independent optimization, especially for the more price sensitive demand. Therefore, coordination between the buyer and the vendor is advantageous for the supply chain. On the other hand, joint optimization will be less beneficial when there isn't a significant difference between the buyer's and the vendor's holding costs.

Originality/value: The main contribution of the article is to incorporate the pricing into ordering and shipping decisions of the supply chain with one vendor and one buyer, and also to investigate the effect of coordination on the optimal solution and performance. IsoIso.

Keywords: Integrated inventory, pricing, vendor, buyer

1. Introduction

In recent years, integrating traditional inventory management with other types of decisions made by the firm (e.g., pricing, quality level, guaranty period, etc) has attracted the attention of many researchers because these decisions must be compatible to each other in order to obtain maximal profit. In fact, setting prices and planning for how much inventory to hold are the two most strategic ones among the many decisions made by a manager. Keeping these facts in mind, practitioners and academics have focused on determining pricing strategy, which influences demands, and production-inventory decisions, which define the cost of satisfying those demands, simultaneously. The seminal work in this line of research is by Whitin (1955). He considered the economic order quantity (EOQ) model with pricing for a buyer that has a price dependent demand with a linear function. His work encouraged many researchers to investigate joint pricing and ordering problems. The focus of these models has been on demand functions (e.g., Rosenberg (1991), Lau and Lau (2003), on quantity discount (e.g., Burwell, Dave, Fitzpatrick and Roy (1997), Lin and Ho (2011), or on perishable inventories (e.g., Roy (2008), Khanra, Sana and Chaudhuri (2010)), among others. Chung and Wee (2008) developed joint pricing and ordering problems in another line of research in which multiple companies in a supply chain cooperate with each other. Actually, he inspired the idea of his work from Goyal (1976), which was the first study in the integrated vendor-buyer inventory models. The integrated inventory models, where the total cost of the supply chain is minimised, were developed to overcome the weakness of the traditional inventory management systems in which the members of the supply chain make their own optimal decisions independently, which may not be optimal for the whole system. Many researchers, such as Banerjee (1986), Hill (1997), Ouyang, Wu and Ho (2004), Rad, Khoshalhan and Tarokh (2011), Rad and Khoshalhan (2011) have then extended the work of Goyal (1976). Sajadieh and Jokar (2009) provided an integrated production-inventory-marketing model in which the optimal ordering, pricing and shipment policy are simultaneously determined to maximize the joint total profit of both the vendor and the buyer. Recently, Kim, Hong and Kim (2011) discussed joint pricing and ordering policies for price-dependent demand in a supply chain consisting of a single retailer and a single manufacturer. Some other researchers such as Ho, Ouyang and Su (2008), Chen and Kang (2010) and Chung and Liao (2011) also developed integrated inventory models that involve price-sensitive demands. The main focus of these works were on trade credit policies and they considered flexible production rates by assuming

that the production rate can be varied in the fixed ratios of the demand rate. We refer the readers to comprehensive reviews of joint operations-marketing models were done by Eliashberg and Steinberg (1993), Chan, Shen, Simchi-Levi and Swann (2004), Yano and Gilbert (2005) and Soon (2011) for more studies.

To the best of knowledge, none of the above-mentioned integrated production-inventory-marketing models focused on investigating the effects of coordination on the performance of the supply chain, especially when the demand rate has an iso-elastic function of the selling price. Therefore, the aims of this article are to study an integrated inventory model that considers operations and pricing decisions, and to investigate the effect of coordination on the system. End customer demand is assumed to be an iso-elastic function of the selling price to account for the impact of price changes on customer demand. Furthermore, the production rate is finite and proportional to the demand rate (see for example Ho, 2011; Chang et al., 2009). To optimize the joint total profit, the selling price, order quantity and number of shipments will be determined in this study. The study is organized as follows: In Section 2, assumptions and notations are provided. Section 3 develops the model for iso-elastic demand function. Section 4 presents numerical example and sensitivity analysis. Conclusions are summarized in Section 5.

2. Assumptions and notation

The mathematical models in this article are developed based on the following assumptions and notations:

- Single manufacturer-single buyer supply chain, which is the simplest and basic form of the supply chains and could be a start to present and to extend more complicated and real inventory models, is considered. This type of supply chain has been also considered by other researchers such as Banerjee (1986), Sajadieh and Jokar (2009), Rad et al.(2011), Xiao and Xu (2013) and were reviewed in Glock (2012).
- Shortage is not allowed.
- For each unit of product, the vendor spends \$ c in production and receives \$ w from the buyer. After that, the buyer sells it by \$ p to its customers. The relationship between them is $p > w > c$.
- The demand rate is a decreasing function of the selling price, $D(p) = \alpha p^{-\beta}$, where $\alpha > 0$ is a scaling factor and $\beta > 1$ is the index of price elasticity. This type of demand function, which has been used by researchers such as Hamasi, Ghfari, Hamdi and Biranvand (2006), Hays and DeLurgio (2009) and Lin and Ho (2011), for example, is commonly

referred to as iso-elastic demand function. For notational simplicity, $D(p)$ and D will be used interchangeably in this article.

- The inventory is continuously reviewed.
- The buyer orders Q quantity from the vendor. The vendor manufactures a production lot $Q_v = nQ$ at one setup, and dispatches it to the buyer in n shipments with size Q , where n is a positive integer.
- The ratio of the market demand rate, D , to the production rate, R , is shown by ρ i.e., $\rho = D/R$, where $\rho \leq 1$ and is fixed (see for example Ho et al. (2008), Chen and Kang (2010) and Chung and Liao (2011)).
- The buyer's inventory holding cost per item per unit time is h_b ; the vendor's inventory holding cost per item per unit time is h_v , and $h_b > h_v$.
- The time horizon is infinite.

The other parameters are:

S vendor's set up cost

A buyer's fixed ordering and transportation cost

v unit variable cost for handling or receiving an item

p buyer's unit selling Price (decision variable)

Q buyer's order quantity (decision variable)

n number of shipments (decision variable)

3. Mathematical Model

In this section, at first, the buyer's and the vendor's inventory models are derived whereas each of them optimizes its own profit independently. Finally, an integrated inventory model and its optimization algorithm are developed.

3.1 Buyer’s total profit

We assume that the buyer’s demand is an iso-elastic function, $D(p) = \alpha p^{-\beta}$. The buyer intends to maximize its average profit function, TP_B ,

$$\begin{aligned} \max TP_B(p, Q) &= (p-w-v)\alpha p^{-\beta} - \frac{\alpha p^{-\beta}}{Q} A - \frac{h_b Q}{2} \\ \text{S.T.} \\ p &> w + v \\ Q &> 0 \end{aligned} \tag{1}$$

For fixed p , TP_B is a concave function in Q . Hence, the optimal order quantity will be obtained

by solving $\frac{\partial TP_B}{\partial Q} = 0$. Therefore, the optimal order quantity of the buyer can be determined as follows:

$$Q^* = \sqrt{\frac{2\alpha p^{-\beta} A}{h_b}} \tag{2}$$

Substituting the obtained optimal order quantity into Eq.(1) and simplifying, we get

$$TP_B(p) = (p-w-v)\alpha p^{-\beta} - \sqrt{2\alpha p^{-\beta} Ah_b} \tag{3}$$

Taking the first-order and second-order partial derivatives of Eq.(3) with respect to p , we have

$$\frac{\partial TP_B(p)}{\partial p} = \alpha p^{-\beta} - \alpha \beta p^{-\beta-1}(p-w-v) + \frac{\alpha \beta p^{-\beta-1} Ah_b}{\sqrt{2\alpha p^{-\beta} Ah_b}} \tag{4}$$

$$\frac{\partial^2 TP_B(p)}{\partial p^2} = -2\alpha p^{-\beta-1} + \alpha \beta(\beta+1)p^{-\beta-2}(p-w-v) - \frac{\beta(\beta+2)\sqrt{\alpha p^{-\beta} Ah_b}}{2\sqrt{2}p^2} \tag{5}$$

TP_B is a concave function in p because $\frac{\partial^2 TP_B(p)}{\partial p^2} < 0$ (see appendix). Thus, p can be obtained by setting the Eq.(4) equal to zero and solving it for p .

3.2 Vendor’s total profit

After determining the optimal order quantity and the selling price by the buyer, the vendor receives the orders. At first, the vendor produces the first Q units and delivers it to the buyer as soon as possible. Then, it makes delivery on a known interval Q/D until the inventory level falls to zero (Fig.1). Hence, the vendor’s average inventory can be calculated as Eq.(6).

$$AI_v = \frac{D}{nQ} \left[\left[nQ \left(\frac{Q}{R} + (n-1) \frac{Q}{D} \right) - \frac{n^2 Q^2}{2R} \right] - \left[\frac{Q^2}{D} (1+2+\dots+(n-1)) \right] \right] = \frac{Q}{2} \left[(2-n) \frac{D}{R} + n-1 \right] \tag{6}$$

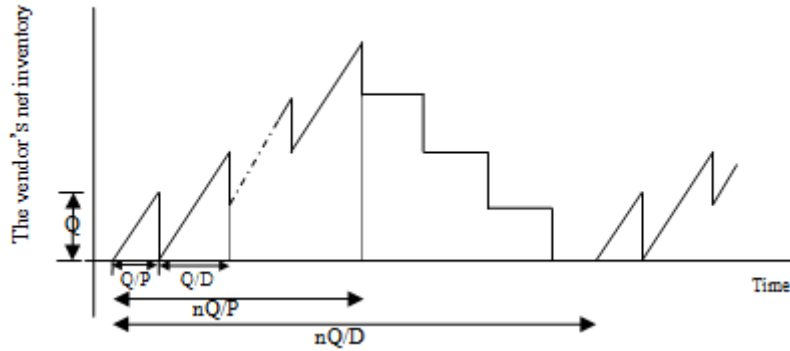


Figure 1. The vendor's inventory system

By considering fixed and given ρ , the vendor's average profit function can be presented by the following equation.

$$\begin{aligned} \max TP_V(n) &= (w - C)\alpha p^{-\beta} - \frac{S\alpha p^{-\beta}}{nQ} - \frac{h_v Q}{2}[(2-n)\rho + n - 1] \\ \text{S.T.} \quad & w > c \\ & n \in N \end{aligned} \tag{7}$$

It can be shown that $TP_V(n)$ is a concave function in n while n is assumed as a real number. Thus, the following equation for n can be obtained by setting the first derivate of Eq. (7) equal to zero and solving it for

$$n = \frac{\sqrt{2S\alpha p^{-\beta}}}{h_v Q^2(1-\rho)} \tag{8}$$

If n in Eq.(8) is not an integer number, we choose n^* which is the optimum integer value of n . n^* is determined such that yields $TP_V(n^*) = \text{Max} \{TP_V(n^-), TP_V(n^+)\}$ in Eq.(7) with regarding this fact that $TP_V(n)$ is a concave function, where n^+ and n^- represent the nearest integers \square larger and smaller than the optimal n .

3.3 The joint total profit

If the buyer chooses its selling price and ordering quantity (p, Q), and the vendor determines its number of shipment n , then the total system profit under independent optimization, $TP_I(p, Q, n)$, is equal to the sum of the buyer's and the vendor's profits, i.e., $TP_I(p, Q, n) = TP_B(p, Q) + TP_V(n)$. Consider the situation where the vendor and the buyer decide to coordinate and share information with each other to determine the best policy for the integrated supply chain system. Therefore, the average joint total profit function is given by

$$\max JTP(p, Q, n) = (p - c - v) \alpha p^{-\beta} - \frac{(S/n + A) \alpha p^{-\beta}}{Q} - \frac{h_b Q}{2} - \frac{h_v Q}{2} [(2 - n) \rho + n - 1]$$

S.T.

$$p > c + v$$

$$Q > 0$$

$$n \in N$$
(9)

To investigate the effect of the number of shipments on the joint total profit, the second-order derivative of $JTP(p, Q, n)$ with respect to n is calculated as follow:

$$\frac{\partial^2 JTP(p, Q, n)}{\partial n^2} = \frac{-2S \alpha p^{-\beta}}{Q n^3} < 0$$
(10)

It is noticeable that although the number of shipments, n , is an integer, it is considered as a real number in order to solve Eq(9) and obtain Eq. (10). In addition, as the result shows that $JTP(p, Q, n)$ is a concave function in n for fixed p and Q , the search for the optimal number of shipments, n^* , is reduced to find a nearest-integer to the local optimal solution (see for example Kim & Ha, 2003; Chen & Kang, 2010). Next, by taking the first-order and second-order partial derivatives of $JTP(p, Q, n)$ with respect to Q for fixed n and p , we have:

$$\frac{\partial JTP(p, Q, n)}{\partial Q} = \frac{(S + nA) \alpha p^{-\beta}}{nQ^2} - \frac{h_b}{2} - \frac{h_v}{2} [(2 - n) \rho + n - 1]$$
(11)

$$\frac{\partial^2 JTP(p, Q, n)}{\partial Q^2} = -\frac{2(S + nA) \alpha p^{-\beta}}{nQ^3}$$
(12)

$JTP(p, Q, n)$ is a concave function in Q for fixed n and p because $\frac{\partial^2 JTP(p, Q, n)}{\partial Q^2} < 0$. Thus, there exists a unique value of Q (denoted by Q^*) which maximizes $JTP(p, Q, n)$. Q^* can be obtained by solving the equation $\frac{\partial JTP(p, Q, n)}{\partial Q} = 0$ in (11), and is given by Eq. (13).

$$Q^* = \sqrt{\frac{2 \alpha p^{-\beta} (S/n + A)}{\{h_v [(2 - n) \rho + n - 1] + h_b\}}}$$
(13)

Substituting (13) into (9), we can get the following joint total profit which is the function of the two variables p and n .

$$JTP(p, n) = (p - c - v) \alpha p^{-\beta} - \sqrt{2 \alpha p^{-\beta} (S/n + A) \{h_v [(2 - n) \rho + n - 1] + h_b\}}$$
(14)

For the fixed n , taking the first-order partial derivative of $JTP(p, n)$ in (14) with respect to p and setting it equal to zero (i.e., necessary condition for optimality), and solving for p , we obtain the following formula:

$$\frac{\partial JTP(p, n)}{\partial p} = \alpha p^{-\beta} - \alpha \beta p^{-\beta-1}(p-c-v) + \frac{\alpha \beta p^{-\beta-1}(S/n+A)\{h_v[(2-n)\rho+n-1]+h_b\}}{\sqrt{2\alpha p^{-\beta}(S/n+A)\{h_v[(2-n)\rho+n-1]+h_b\}}} = 0 \quad (15)$$

As the second order derivative of Eq. (14) is negative, the sufficient condition for optimality is met, and the joint total cost function $JTP(p, n)$ is concave at p .

$$\frac{\partial^2 EJTP(p, n)}{\partial p^2} = -2\alpha p^{-\beta-1} + \alpha \beta (\beta+1) p^{-\beta-2}(p-c-v) - \frac{\beta(\beta+2)\sqrt{\alpha p^{-\beta}(S/n+A)\{h_v[(2-n)\rho+n-1]+h_b\}}}{2\sqrt{2}p^2} < 0 \quad (16)$$

3.4 Solution Algorithm

We apply the following iterative algorithm which is used in some articles such as Ray, Gerchak and Jewkes (2005), Sajadieh and Jokar (2009) and Chen and Kang (2010) to find the optimal solution (p^*, Q^*, n^*) .

Step0. Let $n = 0$ and set $JTP(p^{(n)}, Q^{(n)}, n) = 0$.

Step1. Set $n = 1$.

Step 2. Determine $p^{(n)}$ by solving Eq. (15).

Step 3. Compute the value of $Q^{(n)}$ using Eq. (13).

Step 4. Calculate $JTP(p^{(n)}, Q^{(n)}, n)$ using Eq. (9).

Step 5. If $JTP(p^{(n)}, Q^{(n)}, n) \geq JTP(p^{(n-1)}, Q^{(n-1)}, n-1)$, then go to step 6. Otherwise, the optimal solution is $(p^*, Q^*, n^*) = (p^{(n-1)}, Q^{(n-1)}, n-1)$.

Step 6. Let $n = n + 1$, then go to step 2.

4. Numerical Example and Sensitivity Analysis

Referring to the existing literature such as Sajadieh and Jokar (2009) and Chen and Kang (2010), we discuss an example with the following data: $A=\$200/\text{order}$, $S=\$1200/\text{setup}$, $C=2.5\$/\text{unit}$, $v=\$1/\text{unit}$, $w=\$5/\text{unit}$, $\rho=0.8$, $h_b=\$0.5/\text{unit}/\text{year}$, $h_v=0.25/\text{unit}/\text{year}$, $\alpha=300,000$ and $\beta=1.245$. Therefore, $D(p)=300,000p^{-1.25}$.

The percentage improvement i.e., $PI = \frac{JTP - TP_I}{TP_I} \times 100$ is calculated to shed light on the benefits of joint optimization. It's noticeable that JTP and TP_I represent the total system profit under joint and independent optimization, respectively. The optimal values of ρ , Q , n and the total system profit under independent optimization are: 31, 1825.3, 8 and 112,976, respectively. The equivalent values for joint optimization are: 18.6, 2188.4, 9 and 116,600. Therefore, Coordination provides 3.21% improvement for the total system profit. The vendor's profit improvement resulting from the joint approach amounts to about 93.68%. The buyer, however, is at an inconvenience and his/her profit shows a 5.18% decrease. The total improvement should then be shared in some equitable manner and some kind of profit-sharing mechanism such as a side payment from the vendor to the buyer, or a price discount scheme needs to be employed in order to encourage cooperation and entice the buyer to change his/her lot size (see for example Ouyang, Wu & Ho, 2004; Goyal, 1976; Sajadieh and Jokar, 2009).

In order to emphasise the role of coordination in the total profit, a selection of randomly generated problem instances are solved and summarized in Table 1. In a second step, some of the model parameters are varied and their impact on the optimal solution and the total profit are studied.

α	β	ρ	A	S	h_b	h_v	v	w	c	TP_I	JTP	PI (%)
381330	1.32	0.8	2508	11247	1.78	1.32	15.3	17.8	13.2	56,567	56,686	0.2
764660	2.59	0.8	2576	2394	1.68	0.82	5.8	16.8	8.2	349	439	25.9
568210	1.62	0.85	1786	2202	6.90	1.64	10.6	69.0	16.4	15,136	20,249	33.8
833750	1.33	0.9	1640	8993	3.61	0.77	17.9	36.1	7.7	118,870	128,620	8.2
723330	1.46	0.9	1297	1375	1.04	0.54	5.6	10.4	5.4	89,635	92,705	3.4
847210	1.30	0.9	1840	3000	5.04	1.70	18.7	50.4	17.0	126,469	133,930	5.9
555060	2.08	0.85	2764	6123	1.97	0.53	4.7	19.7	5.3	4,327	6,575	52.0
723330	2.83	0.9	1297	1375	1.04	0.54	5.6	10.4	5.4	388	481	24.1
109520	1.16	0.95	2756	15136	1.32	0.53	12.7	13.2	5.3	41,425	41,689	0.6
369700	1.89	0.7	2367	2514	4.59	0.93	10.0	45.9	9.3	2,811	4,258	51.5

Table 1. A selection of randomly generated examples

4.1 Sensitivity analysis for the index of price elasticity β

As can be seen in Table 2, the percentage improvement, PI , increases by β . It means that for more price sensitive demands, joint optimization shows more improvements, and it is more beneficial. This result is similar to which Sajadieh and Jokar (2009) deduced for the integrated inventory model with the linear price sensitive demand. Therefore, regardless of type of the demand functions, more price sensitive demands lead to more benefits through coordination. Such a situation could occur, for example, in a high technology market where new competitors rapidly enter the market and the technological difference between the products offered in the market becomes smaller. Therefore, customers focus more on the price of the product. Another outcome can be inferred from Table 1 is that when the supply chain's members optimize their

inventory systems independently, the optimal selling price is higher than its values under joint optimization. In addition, as the sensitivity of demand to price increases, the difference between the selling price in independent and joint systems increases too. Consequently, cooperation leads to the higher demand, and so the total profit increases especially for more price sensitive demands.

β	Independent optimization							Joint optimization					PI(%)
	p	Q	n	TP_B	TP_V	TP_I	Q_v	p	Q	n^*	JTP	Q_v	
1.05	129.5	1205.6	8	223,740	3,984	227,724	9644.8	78.6	1363.4	9	228,950	12270.6	0.54
1.1	67.4	1528.3	8	178,610	6,597	185,207	12226.4	40.6	1758.8	9	187,350	15829.2	1.16
1.25	31	1825.3	8	103,390	9,586	112,976	14602.4	18.6	2188.4	9	116,600	19695.6	3.21
1.5	18.3	1747.8	8	46,260	8,771	55,031	13982.4	10.9	2240.6	9	59,218	20165.4	7.61
1.75	14.3	1509.9	8	22,924	6,434	29,358	12079.2	8.5	2063.6	9	33,170	18572.4	12.99
2	12.3	1257.7	8	11,863	4,375	16,238	10061.6	7.4	1831.0	9	19,362	16479.0	19.24
2.25	11.2	1027.8	8	6,287	2,795	9,082	8222.4	6.7	1592.7	9	11,547	14334.3	27.14
2.5	10.4	829.3	8	3,370	1,766	5,135	6634.4	6.2	1367.3	9	6,947	12305.7	35.28

Table 2. Sensitivity analysis for the index of price elasticity β

4.2 Sensitivity analysis for the purchasing price w

Increasing the unit purchasing price which is paid by the buyer to the vendor, w , increases the percentage improvement, PI . It is since under joint optimization, the purchasing price doesn't affect the joint total profit. On the contrary, under independent optimization, the buyer has to increase the selling price to compensate the augmentation of the purchasing price. Therefore, the demand as well as the total profit decreases under independent optimization. In addition, Fig.2 shows that for the more price sensitive demand, the slope of the percentage improvement against the purchasing price is faster. Hence, for more price sensitive demand, increasing of the purchasing price brings about much more percentage improvement. These results and which Sajadieh and Jokar (2009) obtained for the linear demand function are the same. Therefore, the effects of the purchasing price on the supply chain are independent of demand function's type.

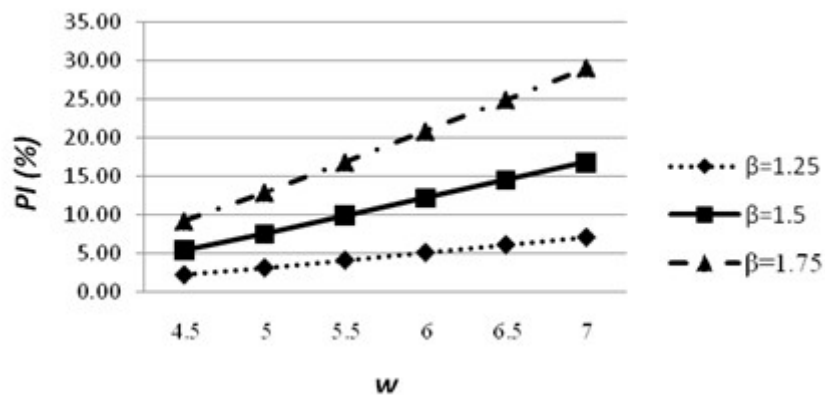


Figure 2. Effect of the buyer purchasing price on the percentage improvement

4.3 Sensitivity analysis for the unit inventory costs ratio h_v/h_b

Table 3 and Fig. 3 shows that the percentage improvement, PI , and the optimal number of shipments, n , decrease when $\alpha=h_v/h_b$ increases. In the other words, joint optimization will be less beneficial when there isn't a significant difference between the buyer's and the vendor's holding costs. Under independent optimization, the growth of h_v doesn't influence the buyer's selling price and the demand rate, but the vendor prefers to keep fewer stocks. So, it decreases the number of shipments, n , to reduce its inventory level. However, under joint optimization, the selling price increases besides decrease in the number of shipments. Therefore, the increase of JTP , where both the selling price and number of shipments change, is lower than the increase of TP_I , where the number of shipments only changes. Consequently, the percentage improvement reduces.

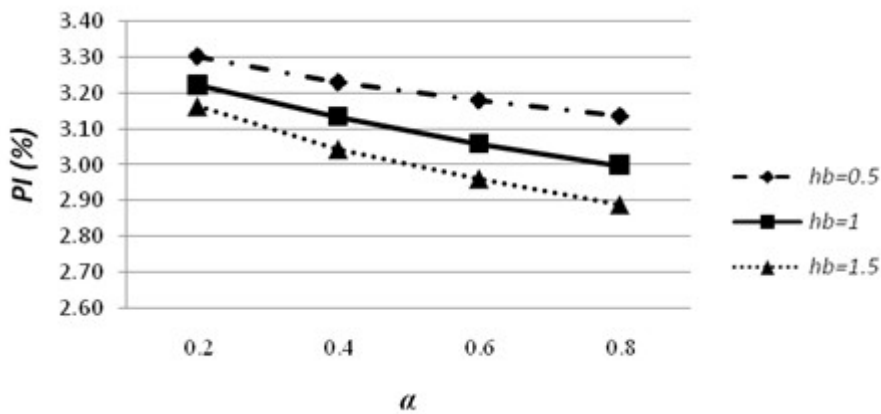


Figure 3. Effect of $\alpha=h_v/h_b$ on the percentage improvement

h_b	α	Independent optimization							Joint optimization					$PI(\%)$
		p	Q	n	TP_B	TP_V	TP_I	Q_v	p	Q	n	JTP	Q_v	
0.5	0.2	31	1825.3	12	103,390	9,928	113,318	21903.6	18.4	2385.7	13	117,060	31014.1	3.30
	0.4	31	1825.3	9	103,390	9,688	113,078	16427.7	18.5	2230.1	10	116,730	22301.0	3.23
	0.6	31	1825.3	7	103,390	9,491	112,881	12777.1	18.6	2180.7	8	116,470	17445.6	3.18
	0.8	31	1825.3	6	103,390	9,317	112,707	10951.8	18.7	2116.6	7	116,240	14816.2	3.14
1	0.2	31	1825.3	6	103,390	9,152	112,542	10951.8	18.7	1966	7	116,040	13762.0	3.11
	0.4	31.3	1284.6	12	103,020	9,600	112,620	15415.2	18.7	1672	13	116,250	21736.0	3.22
	0.6	31.3	1284.6	9	103,020	9,262	112,282	11561.4	18.8	1560.5	10	115,800	15605.0	3.13
	0.8	31.3	1284.6	7	103,020	8,986	112,006	8992.2	19	1524.1	8	115,430	12192.8	3.06
1.5	0.2	31.3	1284.6	6	103,020	8,740	111,760	7707.6	19.1	1477.8	7	115,110	10344.6	3.00
	0.4	31.3	1284.6	5	103,020	8,509	111,529	6423.0	19.2	1371.3	7	114,830	9599.1	2.96
	0.6	31.5	1045.1	12	102,730	9,363	112,093	12541.2	18.9	1355.8	13	115,640	17625.4	3.16
	0.8	31.5	1045.1	9	102,730	8,951	111,681	9405.9	19.1	1264	10	115,080	12640.0	3.04

Table 3. Sensitivity analysis for the unit inventory costs ratio $\alpha=h_v/h_b$

4.4 Sensitivity analysis for the set up cost toward the ordering cost S/A

As can be discerned from Fig.4, an increase in $\gamma = S/A$ increases the number of shipments and the vendor's production quantity. It is a reasonable result because under fixed value of A , higher γ means higher setup cost, S . In such a situation, it is expected from the vendor to increase its production quantity in each setup. Therefore, the number of shipments, n , and the vendor's production quantity, Q_v , increase. Furthermore, percentage improvement PI , decreases by γ (see Fig. 5). Hence, coordination of the supply chain is less attractive when the vendor's setup cost is considerably higher than the buyer's ordering cost.

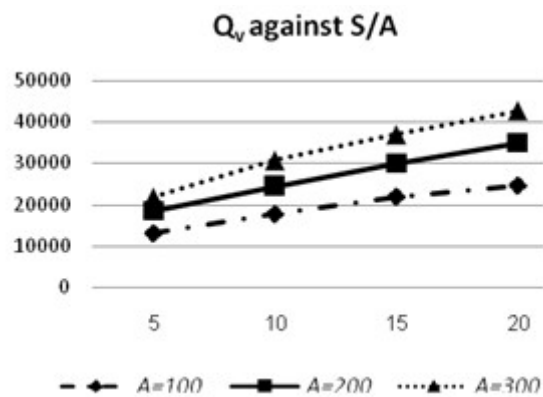


Figure 4. Effect of S/A on the vendor's production quantity

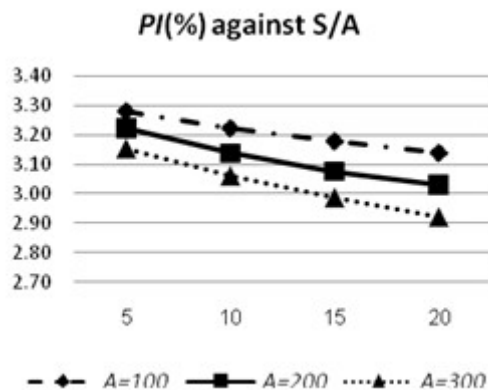


Figure 5. Effect of S/A on the percentage improvement

In this article, an integrated production-inventory-marketing model for a two-stage supply chain is presented. It is assumed the demand rate is an iso-elastic function of the selling price. Then, the total cost functions are developed, and the optimal values of the selling price, order quantity and number of shipments are obtained under independent and joint optimizations. A numerical example and the sensitivity analysis are done, and the main following findings are attained.

The optimal selling price under independent optimization is higher than its value under joint optimization, and so coordination increases the demand- and profit of the supply chain. Furthermore, supply chain's members can get more profits from coordination in a competitive market in which sensitivity of the demand to price is high. Another finding is that increasing the unit purchasing price, which is paid by the buyer to the vendor leads to increase in the percentage improvement. Finally, coordination of the supply chain is less attractive when the vendor's setup cost is considerably higher than the buyer's ordering cost.

Future research can be done for multi-vendors and multi-buyers supply chains. In addition, the model can be developed for imperfect products and also deteriorating items.

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Appendix

As $(\beta-1)$ is smaller than β , we have:

$$\alpha \beta (\beta-1) p^{-\beta-1} < \alpha \beta^2 p^{-\beta-1} \quad (\text{A.1})$$

Furthermore, we know that $p > w+v$. Therefore, it can be concluded:

$$-\alpha \beta (\beta+1)(c+v) p^{-\beta-2} < -\alpha \beta (\beta+1) p^{-\beta-1} \quad (\text{A.2})$$

After summation of Eqs. (A.1) and (A.2), we have:

$$-2\alpha p^{-\beta-1} + \alpha \beta (\beta+1) p^{-\beta-2} (p-w-v) < -\alpha \beta p^{-\beta-1} \quad (\text{A.3})$$

Eq. (A.3) shows that $-2\alpha p^{-\beta-1} + \alpha \beta (\beta+1) p^{-\beta-2} (p-w-v) < 0$. In addition, it is obvious that the

last part of Eq. (5), i.e., $-\frac{\beta(\beta+2)\sqrt{\alpha p^{-\beta} Ah_b}}{2\sqrt{2}p^2}$, is always negative. Consequently, it is proved that

Eq.(5) is always negative.

