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A Simple and Efficient Method to Allocate Costs and Benefits in Energy Communities

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Abstract:

Purpose: Define a simple and efficient method to allocate costs and benefits in energy communities, and characterize some of its key properties.

Design/methodology/approach: The approach is theoretical. We define an algorithm to allocate costs and benefits in energy communities, and derive some of its formal properties using mathematical reasoning. We also compare the proposed algorithm with several alternatives.

Findings: The proposed algorithm is simple and it ensures that the resulting distribution of costs and benefits is (i) beneficial for every member of the community, (ii) efficient, (iii) fair (in a formally defined sense), (iv) smooth (small changes in the consumption or in the generation of energy cannot lead to big changes in the allocation of costs and benefits), and (v) environmentally friendly in the sense that the individual allocated cost is a strictly increasing function of individual consumption.

Research limitations/implications: The properties of the proposed algorithm are satisfied for a specific type of energy community that is defined in the paper.

Practical implications: The algorithm is easy to implement in any energy community.

Social implications: The algorithm is highly relevant for any community of prosumers who are willing to exchange energy internally. It guarantees a number of desirable properties that are formally defined in the paper.

Originality/value: We prove that a simple algorithm to allocate costs and benefits in energy communities guarantees the fulfilment of several desirable properties.

Keywords: energy community, prosumer, allocation, costs, fairness, efficiency

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1. Introduction

There is an international consensus that recognizes the negative impact that humans have on Earth's climate, mainly due to our intensive use of fossil energy sources. The Intergovernmental Panel on Climate Change (IPCC) stipulates that actions must be taken to reduce the carbon footprint of human activity on the planet (Pörtner, Roberts, Tignor, Poloczanska, Mintenbeck, Alegría et al., 2022). Following this view, many developed countries have enabled ambitious political instruments aimed at developing an energy system based on renewable sources (European Commission, 2019; US Congress, 2022). The goal is challenging: to achieve climate neutrality by 2050 through an intensive roadmap in a context of energy crisis (International Energy Agency, 2022).

The transition from a carbon-based to a carbon-neutral economy requires the involvement of both large consumers (industry) and small consumers (households and retailers). Large consumers will be required to undertake decarbonization plans using their own financial resources. However, the involvement of small consumers is more delicate, since many of them cannot afford the necessary investments, and many others are not willing to run the risks associated with being an early adopter in a highly regulated and dynamic market.

To address this challenge, energy policies focus on the creation of new market agents from a consumer-centered perspective (European Parliament, 2019), aimed at empowering citizens as the main actors in the energy value chain (i.e. generation, commercialization and consumption). This implies a major change of paradigm, governed by new legislation that allows citizens to actively participate in the energy system through distributed renewable energy generation systems, thus gaining significant importance in the energy market. This involvement has a triple positive impact: firstly, the generation of clean energy close to the point of consumption; secondly, the promotion of local business development in these areas; and thirdly, the development of innovative energy management models at the local level, such as peer-to-peer energy exchange or local energy markets that may lead to further social and economic growth.

Naturally, this paradigm shift must be accompanied by a structural change in the energy market to manage this new dynamic of energy and money streams between citizens, in a way that favors their participation through both individual and collective initiatives (European Federation of Citizen Energy Cooperatives, 2021). In this context, energy communities (EC) play a major role.

An energy community is basically a set of participants –called prosumers– who can produce and consume energy in different volumes and time intervals. Prosumers can trade energy with an external market (to buy any energy deficit or sell any energy excess) and, crucially, they can also exchange energy between them. The legal possibility of exchanging energy between members of the community is the main novelty that energy communities introduce, through recent European Directive 2018/2001 Art.2 (European Parliament, 2018), which includes the definition of "peer-to-peer trading" in the European energy market. This definition, together with Directive 2019/944 (European Parliament, 2019) –which defines the term 'citizen energy community' as an enabling framework for consumer empowerment– are the two main ingredients that permit the creation and deployment of cost-effective local energy communities.

In principle, prosumers are free to set the rules for this internal exchange of energy as they wish. A specific set of rules is called here an *allocation system*, since these rules effectively allocate the costs and benefits of energy production and consumption for the members of the community. In this paper we analyze a specific family of (price-based) allocation systems that can be used to manage the internal exchange of energy within the members of a community. We prove that this family of allocation systems has a number of desirable properties, i.e. these allocation systems are:

1. *Beneficial*, i.e., the allocation system ensures that every individual benefits from participating in the community.
2. *Efficient*, i.e., there is no alternative such that at least one member is better off and no member is worse off.
3. *Fair*, i.e., if agent i overconsumes more than agent j, then agent i will be allocated a greater cost than agent j.

4. *Smooth*, i.e. a small change in any member's production or consumption of energy can only lead to a small change in every member's allocated costs and benefits.
5. *Environmentally friendly*, i.e., every member's allocated cost is a strictly increasing function of that member's energy consumption.

Besides the desirable properties outlined above (which are formally defined below), the proposed price-based allocation systems are simple to understand and to implement.

The rest of the paper is structured as follows. In Section 2, we analyze the conceptual framework of our field of study, identifying the main trends and research gaps that underpin our contribution in this paper. Section 3 describes the typology of energy community that we address here. In Section 4 we formulate a family of allocation algorithms that satisfies a set of desirable properties. Subsequently, Section 5 presents some alternative allocation systems and compares them with the main ones of the previous section. In Section 6 we present a case study to illustrate our main results. Finally, in Section 7 we summarize the most interesting conclusions of this study. All the formal proofs and the details on the case study are relegated to Appendices.

2. Literature Review

This literature review section is divided into three parts; firstly, we provide an overview of publications related to prosumers and energy communities. Secondly, we present a brief review of the main research trends related to our field of study. And thirdly, we explain the research gap on which we focus here.

2.1. General Overview of the Research Field

Work related to prosumers and energy communities represents a novel and constantly growing field of research. Figure 1 shows that this field has rocketed since the approval of the European regulations on energy transition in 2018 (European Parliament, 2018, 2019). Nowadays, in Web of Science we can find more than 5000 publications that deal with the concepts of prosumer or energy community. Specifically, 194 publications mention both concepts in the same document.

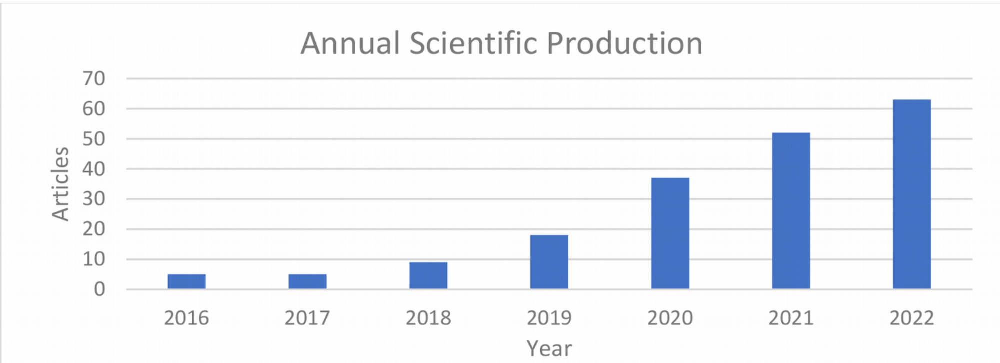

Figure 1. Annual scientific production for publications that mention the concepts "prosumer/s" and "energy community/ies" in their title, abstract or keywords

Despite the fact that the deployment of renewable energy does not discriminate any type of Renewable Energy Source (RES), the vast majority of publications focus on the implementation of solar photovoltaic (PV) energy systems supported by Battery Energy Storage Systems (BESS). Thus, PV and BESS seem to be the main tools available for consumers to gain empowerment in the energy market. This trend is supported by the evolution of the implementation of this type of technology compared to others in recent years (Eurostat, 2022).

Regarding the outcomes of publications consulted, there is a general consensus in two main aspects. On the one hand, coordinated collective initiatives are more economically efficient than individual behaviors. On the other hand, large RES generation assets operated cooperatively have a better economic performance than small individual

RES facilities of equivalent power, even when they are operated in a coordinated manner for the benefit of the community (Norbu, Couraud, Robu, Andoni & Flynn, 2021).

2.2. Main Research Trends

Going deeper into this field, different management strategies of solar RES have been investigated, ranging from a) system sizing in terms of PV power and BESS capacity (Mulder, Six, Claessens, Broes, Omar & Mierlo, 2013), b) system optimization (Heinisch, Odenberger, Göransson & Johnsson, 2019) and optimal asset management through AI-based forecasting tools (Hernandez-Matheus, Löschenbrand, Berg, Fuchs, Aragüés-Peñalba, Bullich-Massagué, 2022), given the intrinsic unpredictability of RES, c) the development of new services and business models adapted to the new energy paradigm, e.g., demand response, aggregators and flexibility (Honarmand, Hosseinnezhad, Hayes, Shafie-Khah & Siano, 2021) and d) interaction of the different roles of the energy system (i.e. consumer, prosumer, aggregator, distributor, trader) in the different market layers through new business models (Reis, Gonçalves, Lopes & Henggeler-Antunes, 2021). Our field of interest lies within this last line of research.

Specifically, we focus on the optimal economic management of a local energy market (LEM) consisting of an ecosystem of consumers and prosumers. There are two main trends in this part of the literature. First, those papers related to the management of LEM; and second, publications focused on the development of peer-to-peer (P2P) energy exchange strategies (Maldet, Revheim, Schwabeneder, Lettner, del Granado, Saif et al., 2022). In both cases, the problem of clearing the market is always present (Javadi, Gough, Nezhad, Santos, Shafie-khah & Catalão, 2022). Furthermore, these two trends have two main points in common: on the one hand, there is the objective of minimizing the energy purchasing cost from the grid through the internal use of energy through P2P transactions, which is understood as an increase in the efficiency of the local electricity system (Jasiński, Kozakiewicz & Sołtysik, 2021). On the other hand, there is the need for the existence of a figure that centralizes and coordinates energy transactions within the EC (Moret & Pinson, 2019), which is assumed to have a certain arbitrage capacity and authority to set the conditions of the transaction.

Most of the publications reviewed follow a modelling strategy that develops very general models with multitude of technical features and user preferences (Goia, Cioara & Anghel, 2022; Paudel, Chaudhari, Long & Gooi, 2019). This has the great advantage of achieving precise models of a distributed system taking into account its numerous degrees of freedom; however, this approach also brings the disadvantage of adding complexity to the models (together with a high computational cost), which prevents drawing conclusions about the properties of these models. In fact, many papers achieve the goal of predicting with high certainty the positive impact of the management strategies in specific collaborative systems; however, some authors, such as Henni, Staudt and Weinhardt (2021), point out that the individual impact of the benefits over each one of the participants has not been sufficiently analyzed in the literature.

2.3. Our Contribution

In this paper we focus on a particular type of energy community, formally defined in Section 3. The assumptions we impose on the type of community allow us to derive strong theoretical results and, at the same time, they are sufficiently general and plausible to make our work highly relevant in present day. In particular, we propose a method to allocate costs and benefits among the members of the community, which can be used in a wide range of communities, and we prove a number of desirable properties that this allocation method satisfies. These desirable properties are motivated by the shortcomings identified in the research field. In this sense, our results shed light on one of the main problems highlighted by Norbu et al. (2021), who point out that it is necessary to develop redistribution mechanisms that are fair, computationally affordable and easy to implement. Below we discuss some of the key properties that an allocation system should satisfy.

Early on in this field, it has been shown that cooperation between prosumers and consumers to share energy, especially in residential environments, is more efficient than a set of individualistic behaviors at the system level (De Almansa, Campos, Doménech & Villar, 2019). A key point to foster this cooperation is to define an appropriate system to allocate costs and benefits within the community (which may include a price-setting method, incentives, additional charges, etc.), since this system will determine the individual outcome for each participant

(Herenčić, Kirac, Keko, Kuzle & Rajšl, 2022; Grzanic, Morales, Pineda & Capuder, 2021). In this regard, it is important that every member of the community is better off within the community than outside. This is one of the main requirements that any allocation system should fulfil.

Another important property is "efficiency". Several authors mention the energy surplus of prosumers as one of the keys to the profitability of the EC (Faria, Barreto & Vale, 2019). According to several authors (Jasiński et al., 2021), efficient behaviors from the point of view of an EC try to maximize the consumption of RES energy generated within the local grid through internal energy storage and transactions. This implies minimizing energy purchases from the external grid, which has several advantages, including minimizing energy transport losses, and reducing CO_2 emissions.

In addition to this, considering the social dimension of an energy community, it is desirable to provide participants with simple and understandable tools that are perceived to be fair (Norbu et al., 2021). To this end, a "fairness" property is defined in this paper, which allows to give a direct and logical answer to questions such as why certain members are paying more than others. This reinforces the citizens' confidence and understanding of the system and therefore the likelihood of a successful implementation. The term "fairness" is a concept covered by several publications, such as Henni et al. (2021), which indicate that fair pricing systems and the distribution of associated benefits is a field that requires further research.

Taking into account that the outcome of transactions within a LEM are strongly conditioned by the pricing system and that households can be very sensitive to high price volatility (Ceglia, Marrasso, Pallotta, Roselli & Sasso, 2022), the fourth property that arises is "cost smoothness". This desirable property dictates that allocated costs are continuous functions. As will be shown later, the goal is to avoid that small changes in individual behavior or energy generation can result in large negative impacts on anyone, as well as achieving that similar consumption patterns lead to similar costs.

Finally, we consider an "environmental friendliness" property, which dictates that higher energy consumption must imply greater cost, since two of the main motivation factors for participation in a EC are cost savings and CO_2 reduction (Henni et al., 2021).

As for the plausibility of the assumptions we make, in the following table we simply outline some situations to which our work can be applied and discuss their relevance.

Field	Description
Typology of participants	People living in a residential building in an average high-density population city. This is a common case, as 46% of European population lives in flats (Eurostat, 2021), representing a significant part of the society where, in order to implement solar photovoltaic energy, a shared community facility must be chosen.
Grid configuration	We propose a community-shared facility, that fits into the "Energy cooperative" archetype according to Reis et al (2021), where the participants of the community own a part of the facility through a percentage share.
Community manager	An arbitrage and management figure is needed, playing the role of LEM coordinator with the same competences as the actual energy traders (European Parliament, 2019). This agent receives the power flow data from the distribution system operator (DSO) and computes the cash flow for every participant according to the allocation system selected.
Generation costs for RES	Most papers –especially for PV systems– assume that RES production costs should somehow reflect their initial investment costs through a "degradation cost" (Norbu et al., 2021), combining a fixed cost that does not depend on the energy production with the marginal cost of actual energy production. In this study, we separate the "investment decision" cost problem from the "facility management" problem. Thus, we assume that the facility has already been built, so the installation cost is a sunken cost, leaving only the –usually low– marginal cost of energy production. This will be further explained in Section 3.
Internal market rules	As in every market, cash flows are calculated according to different prices that are applied to different energy flows depending on their quantity and origin (i.e., local RES, external grid, BESS). The price setting mechanism may use exogenous parameters and endogenous parameters. This is detailed in Section 3.

Table 1. Situations where the proposed allocation system is relevant

3. A Simple Energy Community

In this section we describe the type of energy community we consider in this paper. The most important assumption is that members of the community can trade energy among themselves through a centralized Peer-to-Peer Energy-Trading (P2P-ET) market (Domènech-Monfort, De Jesús, Wanapinit & Hartmann, 2022; Zhou, Wu, Long & Ming, 2020). The market is said to be centralized because there is a supervising entity (sometimes called community manager) which coordinates energy trading and allocates costs and benefits for the members of the community. The role of the community manager could perfectly be played by a digital platform.

The type of energy trading considered here (centralized P2P-ET) can be found in the literature under various other names, such as *community-based market* (Muhsen, Allahham, Al-Halhouli, Al-Mahmodi, Alkhraibat & Hamdan, 2022). Domènech-Monfort et al. (2022) provide a comprehensive review of the different names that are used in the literature for different types of energy trading models and the actors within them.

Besides P2P-ET, we also assume that there is one single energy-generating facility, so the cost of production of one unit of energy at any given time slot is the same for every member of the community. A common example of this type of community is a group of neighbors who decide to install a photovoltaic facility in their building.

3.1. Elements of a Simple Energy Community

In this section we describe the elements that comprise the simple energy community we consider here (see Figure 2). The energy community is formed by a set of *members* that require a certain amount of energy at each moment in time. The community comprises an *internal facility* that generates energy at a certain cost; this energy may or may not be enough to cover the demand of all the members of the community at certain times. The community has also access to an *external grid* with which the community can trade energy at prices that are set externally and may vary in time. Finally, the community may also comprise a *battery* where members can store energy.

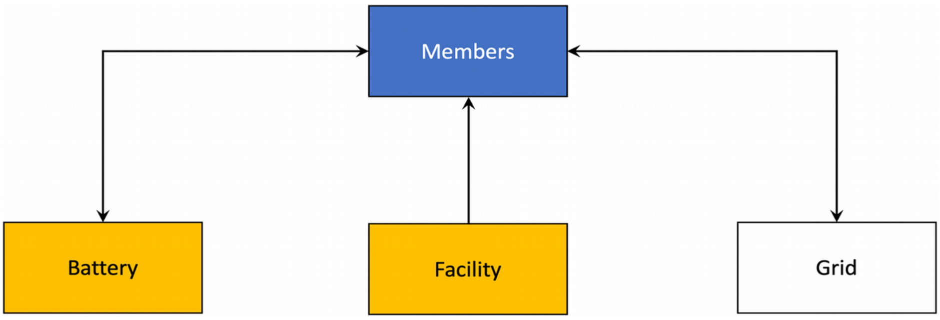

Figure 2. Different elements of an energy community. Arrows denote possible transfers of energy. Orange boxes indicate limited sources of energy, while the white box (Grid) indicates a source of energy that is assumed to be unlimited

Thus, in general, members of the community can obtain the energy they require a) from the internal facility, b) from the internal battery, or c) from the external grid (which is assumed to be an unlimited source of energy). It is also possible that, at certain times, the energy produced by the facility exceeds the demand of the community members. In that case, the excess energy can be stored in the internal battery or sold to the external grid at a certain price (and this sale generates a profit). In this paper, we study different ways in which this type of energy community can be managed, i.e. different ways to allocate costs and benefits among its members. The allocation is conducted *ex-post*, i.e., once the prices of trading with the grid are known.

3.2. The Internal Facility and its Associated Costs

3.2.1. Net Producers and net Consumers

Let A denote the community. The energy generated by the internal facility at time slot t (often, an hour) is denoted $E^g_{A,t}$. A time slot (or compensation period) is a period of time over which the energy consumed by the community can be directly compensated with energy generated in the community. The maximum duration of a time slot in communities that are connected to a grid is often determined by law.

The energy $E^g_{A,t}$ generated by the internal facility is distributed among the members of the community, according to some exogenous rule that we consider given (nonetheless, we comment a natural way of doing this below). The energy obtained from the facility by member $a \in A$ is denoted $e^g_{a,t}$. Naturally, $\sum_{i \in A} e^g_{i,t} = E^g_{A,t}$. It will be useful to define $\alpha_{a,t}$ as the fraction of the total energy generated by the facility that member $a \in A$ gets at time slot t, i.e. $\alpha_{a,t} = e^g_{a,t} / E^g_{A,t}$. The energy $e^g_{a,t}$ obtained from the facility by member a can be consumed, transferred to another member of the community, sold to the grid, stored in a battery, or any combination of these.

The energy consumed by member a at time t is denoted by $e^c_{a,t}$. This energy must be obtained directly from the internal facility ($e^g_{a,t}$), from another member of the community, from the internal battery, from the external grid, or any combination of these. The total energy consumed by the members of the community A at time t is $E^c_{A,t} = \sum_{i \in A} e^c_{i,t}$.

At each time slot t, it is useful to divide the members of the community into two groups: *net producers* (those for who $e^g_{a,t} \geq e^c_{a,t}$), and *net consumers* (those for who $e^g_{a,t} < e^c_{a,t}$). See Figure 3.

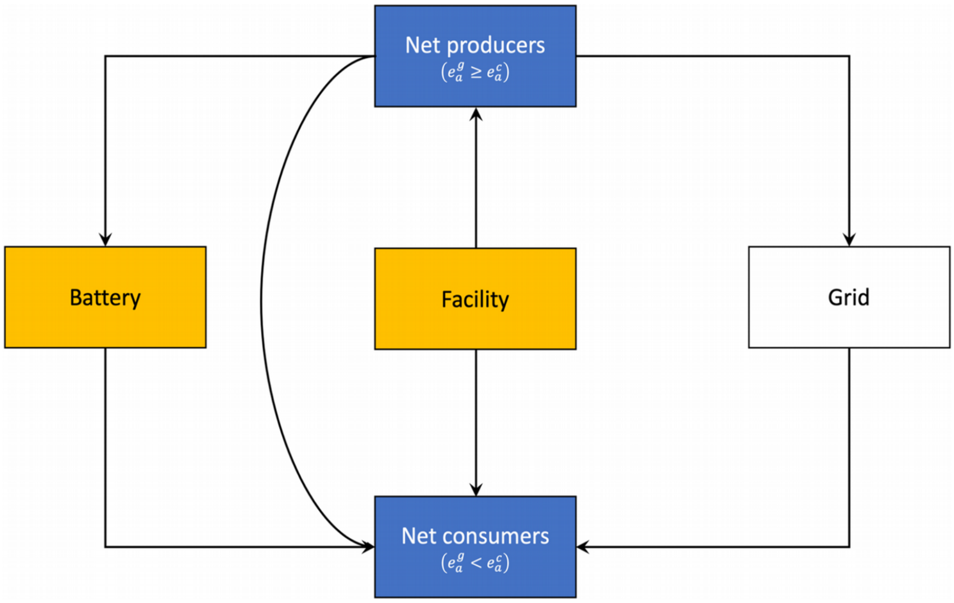

Figure 3. Different elements of an energy community, and different types of members of the community. Arrows denote possible transfers of energy. Blue boxes indicate different types of members (net producers and net consumers in the time slot). Orange boxes indicate limited sources of energy. The white box (Grid) indicates a source of energy that is assumed to be unlimited

3.2.2. A Common Way of Distributing the Energy Produced by the Facility

In many cases, the facility is owned by the members of the community. In this case, it is natural to set $\alpha_{a,t}$ according to ownership shares. To be sure, if α_a is the share of member a in the ownership of the facility, then $\alpha_{a,t} = \alpha_a$ and $e^g_{a,t} = \alpha_a \cdot E^g_{A,t}$ at all times t.

In other cases, the facility belongs to an external firm and members of the community can buy unlimited energy from the external firm. This situation would be a simpler case than the one we consider here.

3.2.3. About Installation and Maintenance Costs

Installation and maintenance costs of the facility may be relevant only if the facility is owned by the members of the community. Thus, in this section we assume this is the case.

We assume that the facility that generates the energy has already been installed and has already been financed by the members of the community (or there is a binding commitment to finance it), so the cost of installation is a sunk cost that should not affect any subsequent decision. Installation costs are relevant to decide whether to install the facility or not, but once the facility has been installed, they are irrelevant for any subsequent decision.

Nonetheless, the way the facility was financed is likely to determine its ownership. Here we assume that every member a owns a fixed fraction α_a of the energy-producing facility, potentially different for different members. This fraction α_a could well be the proportion of installation costs paid by member a, but this is an unnecessary presumption, and we can dispense of it. In any case, this fraction α_a could determine member a's share of the energy produced by the facility, as explained above.

Ownership fraction α_a could also determine member a's share of the facility's maintenance costs. Maintenance costs are fixed costs (since they do not depend on how much energy is produced), but –unlike installation costs– are recurrent. Besides, members are free to stop incurring in them by renouncing to keep on operating the facility. Maintenance costs are relevant to decide whether to continue operating the facility, or not. As long as the maintenance costs over a (sufficiently long) period of time T are less than the savings provided by the facility over that period T, then it makes sense to operate the facility (which involves paying the maintenance cost). Note that the savings obtained by each member are somewhat dependent on the allocation system, so the allocation system may influence whether operating the facility makes sense or not for some members of the community. Here we assume that running the facility makes economic sense (otherwise there would be no need for an allocation system), but this is something that should be checked in any practical situation where an allocation system is implemented.

4. A Family of Allocation Systems Based on Prices

An *allocation system* is an algorithm that determines the different flows of energy between the entities of the community (members, facility, battery and grid) at each time slot, and allocates costs and benefits among its members. In this paper we study allocation systems that operate *ex-post*, i.e., once the prices of trading energy with the grid are known. In this section we propose and analyze a family of allocation systems that satisfy several desirable properties.

4.1. Definition

The family of *price-based allocation systems* determines energy flows and distributes costs and benefits by setting different prices for the energy traded between the elements of the community in each time slot (see Figure 4). Different ways of setting the transfer price at which members of the community trade energy among them give rise to different instances within this family of *price-based allocation systems*.

It is assumed that, at any time slot, net consumers will buy energy at the lowest possible price and net producers will sell any excess at the highest possible price.

We explain each of the prices below. For the sake of notational clarity, we do not explicitly include the dependency of prices on slot *t* in the notation whenever it is not necessary. All prices are assumed to be non-negative.

- $p_{buy}^{Facility}$: marginal cost of production of one unit of energy by the facility. This may well be null, or nearly null up to $E^g{}_A$. Since this is marginal cost, it does not include installation costs or maintenance costs. If the facility is owned by an external firm, then $p_{buy}^{Facility}$ is equal to the price charged per unit of energy by the external firm.
- p_{sell}^{Grid} and p_{buy}^{Grid} are the prices at which individuals can sell energy to the grid (p_{sell}^{Grid}) or buy energy from the grid (p_{buy}^{Grid}). These are set exogenously, and the difference between them is what makes energy internal trading beneficial for the members of the community. It is assumed that $p_{buy}^{Facility} < p_{sell}^{Grid} < p_{buy}^{Grid}$. This implies that it is optimal for the community as a whole to satisfy as much demand as possible using the energy produced, before selling or buying any energy from the grid.
- $p_{tr}, p_{sell}^{Battery}$ and $p_{buy}^{Battery}$ are endogenous, i.e. determined by the allocation system. p_{tr} is the price at which energy units are traded within the community. $p_{sell}^{Battery}$ and $p_{buy}^{Battery}$ are the prices at which prosumers sell and buy energy from the battery, respectively.

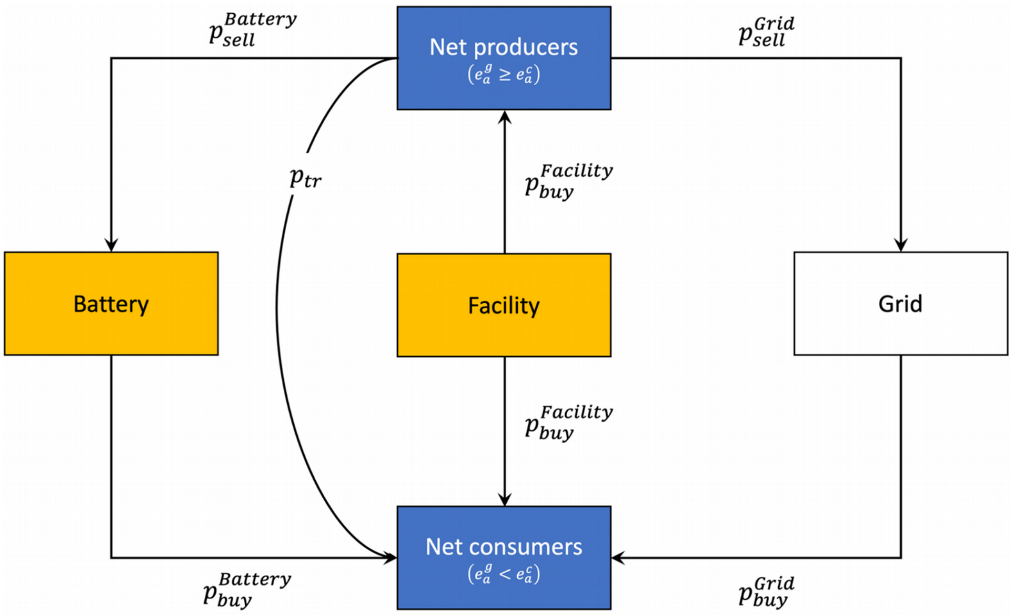

Figure 4. The figure refers to one compensation time slot t. Arrows denote transfers of energy, at the price indicated in the label. Blue boxes indicate different types of members (net producers and net consumers in the time slot). Orange boxes indicate limited sources of energy, while the white box (Grid) indicates a source of energy that is assumed to be unlimited. Price labels (e.g. "sell" or "buy") refer to actions taken by the members of the community

We assume that $p_{sell}^{Grid} < p_{tr} < p_{buy}^{Grid}$. Otherwise, some members could be better off if they did not join the community. Given a certain p_{tr}, if supply and demand for energy in the internal market do not match, the greater of the two is prorated. This ensures that the maximum number of energy units are traded (maximum efficiency) and every net producer (resp. net consumer) trades the same proportion of their excess (resp. demand).

4.2. Analysis of Price-Based Allocation Systems with no Battery

It is informative to start analyzing the family of price-based allocation systems assuming there is no battery (Figure 5).

In this case, all net producers try to sell their overproduction $[e^g_a - e^c_a]^+$ in the internal market (since $p_{sell}^{Grid} < p_{tr}$) and all net consumers try to buy their overconsumption $[e^c_a - e^g_a]^+$ in the internal market (since $p_{tr} < p_{buy}^{Grid}$). The operator $[\ ]^+$ is defined as $[x]^+ = \max(x,0)$.

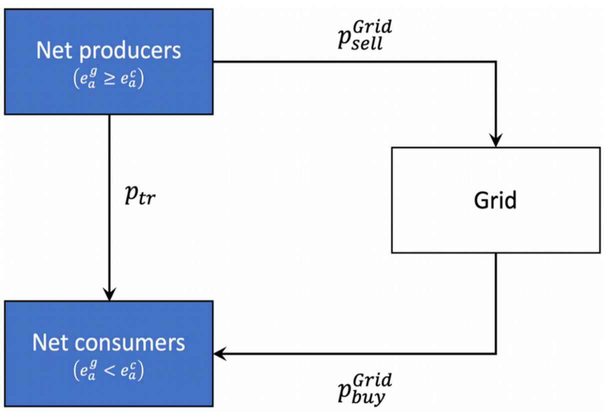

Figure 5. Illustration of a price-based allocation system assuming there is no battery

4.2.1. Energy Traded in the Internal Market

The energy actually traded in the internal market by member a is then:

$$e_a^{tr} = \left(\frac{\left[e_a^g - e_a^c\right]^+}{\sum_{i\in A}\left[e_i^g - e_i^c\right]^+} + \frac{\left[e_a^c - e_a^g\right]^+}{\sum_{i\in A}\left[e_i^c - e_i^g\right]^+} \right) E_A^{tr}$$

Where $E^{tr}{}_A$ is the total number of energy units traded in the internal market, which equals:

$$E_{A,t}^{tr} = \min\left(\sum\nolimits_{i\in A}\left[e_i^g - e_i^c\right]^+, \sum\nolimits_{i\in A}\left[e_i^c - e_i^g\right]^+\right)$$

Using NP to denote the set of net producers and NC to denote the set of net consumers, note that $\sum_{i\in NP} e_{i,t}^{tr} = \sum_{i\in NC} e_{i,t}^{tr} = E_{A,t}^{tr}$.

4.2.2. Total Surplus Generated in the Internal Market

In this section we compute the total surplus generated in the internal market. The total surplus measures –in monetary terms– how much better off the members of the community are, in the aggregate, thanks to the possibility of internal energy trading (see e.g. chapter 9 in Pindyck & Rubinfeld, 2017). It stems from the fact that buyers in the internal market pay a lower price than the price they would have to pay if internal trading was not allowed $(p_{tr,t} < p_{buy,t}^{Grid})$ and, similarly, sellers in the internal market are paid a higher price than the price they would obtain if trading was not allowed $(p_{tr,t} > p_{sell,t}^{Grid})$. Note that some authors (e.g. Grzanic et al., 2021; Long, Wu, Zhang, Thomas, Cheng & Jenkins, 2017) use the term "surplus" in a different way, for "production over consumption"; here we use the terms "overproduction" or "excess" for that concept.

To compute the surplus generated in the internal market, first we have to consider the cost incurred by members if there was no trade:

$$cost_a^{NoTr} = p_{buy}^{Facility} \cdot e_a^g + p_{buy}^{Grid} \cdot \left[e_a^c - e_a^g\right]^+ - p_{sell}^{Grid} \cdot \left[e_a^g - e_a^c\right]^+$$

Thus, the total cost for the community if trading was not allowed would be:

$$Cost_A^{NoTr} = \sum\nolimits_{i\in A} cost_i^{NoTr} = p_{buy}^{Facility} \cdot E_A^g + p_{buy}^{Grid} \cdot \sum\nolimits_{i\in A}\left[e_i^c - e_i^g\right]^+ - p_{sell}^{Grid} \cdot \sum\nolimits_{i\in A}\left[e_i^g - e_i^c\right]^+$$

If trading is allowed, the community uses all the energy produced by the facility first and then, if needed, buys any shortage from the grid (since $p_{sell}^{Grid} < p_{tr} < p_{buy}^{Grid}$). Thus, the total cost for the community allowing for trading is:

$$Cost_A^{Tr} = p_{buy}^{Facility} \cdot E_A^g + p_{buy}^{Grid} \cdot \left[E_A^c - E_A^g\right]^+ - p_{sell}^{Grid} \cdot \left[E_A^g - E_A^c\right]^+$$

It is easy to prove that $Cost_A^{Tr} \leq Cost_A^{NoTr}$.

Now we can compute the surplus created by internal trading, which is:

$$\begin{aligned} Surplus_A^{Tr} &= Cost_A^{NoTr} - Cost_A^{Tr} = \\ &= p_{buy}^{Grid} \cdot \left(\sum\nolimits_{i\in A}\left[e_i^c - e_i^g\right]^+ - \left[E_A^c - E_A^g\right]^+\right) - p_{sell}^{Grid} \cdot \left(\sum\nolimits_{i\in A}\left[e_i^g - e_i^c\right]^+ - \left[E_A^g - E_A^c\right]^+\right) = \\ &= p_{buy}^{Grid} \cdot \sum\nolimits_{i\in NC} e_{i,t}^{tr} - p_{sell}^{Grid} \cdot \sum\nolimits_{i\in NP} e_{i,t}^{tr} = \left(p_{buy}^{Grid} - p_{sell}^{Grid}\right) \cdot E_{A,t}^{tr} \end{aligned}$$

This makes intuitive sense because surplus is created in each internal transfer, and every transfer creates the same surplus $p_{buy}^{Grid} - p_{sell}^{Grid}$.

4.2.3. Individual Allocated Cost in the Compensation Slot

Cost for net producers:

$$cost^{Tr}_{a\in NP,t} = e^{g}_{a,t} \cdot p^{Facility}_{buy,t} - e^{tr}_{a,t} \cdot p_{tr,t} - \left(e^{g}_{a,t} - e^{c}_{a,t} - e^{tr}_{a,t}\right) \cdot p^{Grid}_{sell,t}$$

Cost for net consumers:

$$cost^{Tr}_{a\in NC,t} = e^{g}_{a,t} \cdot p^{Facility}_{buy,t} + e^{tr}_{a,t} \cdot p_{tr,t} + \left(e^{c}_{a,t} - e^{g}_{a,t} - e^{tr}_{a,t}\right) \cdot p^{Grid}_{buy,t}$$

4.2.4. Distribution of Total Surplus Among Members of the Community

The surplus obtained by any individual member *a* from the internal market is the difference between her allocated cost in the absence of internal market (i.e. $cost^{NoTr}_{a\in NP}$) minus her allocated cost when energy trading is allowed. For net producers, this surplus equal:

$$cost^{NoTr}_{a\in NP} - cost^{Tr}_{a\in NP} = e^{tr}_{a} \cdot \left(p_{tr} - p^{Grid}_{sell}\right)$$

And the surplus obtained by net consumers equals:

$$cost^{NoTr}_{a\in NC} - cost^{Tr}_{a\in NC} = e^{tr}_{a} \cdot \left(p^{Grid}_{buy} - p_{tr}\right)$$

Thus, the set of net producers as a whole get a fraction $\frac{p_{tr}-p^{Grid}_{sell}}{p^{Grid}_{buy}-p^{Grid}_{sell}}$ of the total surplus, while net consumers get a fraction $\frac{p^{Grid}_{buy}-p_{tr}}{p^{Grid}_{buy}-p^{Grid}_{sell}}$ of the total surplus. Within each class (net producers and net consumers), surplus is distributed proportional to e^{tr}_{a} (which for net producers is proportional to overproduction $[e^{g}_{a} - e^{c}_{a}]^{+}$ and for net consumers it is proportional to overconsumption $[e^{c}_{a} - e^{g}_{a}]^{+}$).

We believe that this distribution of the surplus generated by the internal market among the members of the community (i.e. proportional to energy transferred) can be considered fair in the sense that the internal market is a legal possibility whose value should be distributed according to the usage of that legal possibility, i.e. according to how many energy units are traded in the internal market.

4.3. Properties Satisfied by Price-Based Allocation Systems with no Battery

Price-based allocation systems satisfy the following properties:

Property 1: “Participation is beneficial”. Every member is better off under the allocation system than in a situation where every member a gets $e^{g}_{a,t}$, and no energy is traded among the members. In other words, all participants are better off in the community than outside the community, i.e. in every time slot, no participant pays more than what the participant would pay outside the community, and it may well be the case that the participant pays less.

Property 2: “No Pareto improvement is possible”. The resulting outcome is Pareto optimal, i.e. there is no other allocation system under which at least one member pays less and no member pays more.

Property 3: “If $p^{Facility}_{buy}$ is sufficiently low, then there is a perfect rank correlation between individuals’ allocated cost and their overconsumption defined as $(e^{c}_{a,t} - e^{g}_{a,t})$ in every compensation time slot”. I.e., in every time slot, the more a member consumes above allocated production, the more the member pays.

Assuming p_{tr} is a continuous function, price-based allocation systems satisfy **Property 4**: “Individuals’ allocated cost is a continuous function”.

Assuming p_{tr} does not depend on energy consumption, price-based allocation systems satisfy **Property 5**: “Every individual’s allocated cost is a strictly increasing function of that individual’s consumption”. I.e., *ceteris paribus*, if a member increases her consumption, her individual allocated cost increases.

In the following two sections we consider two distinct ways of setting the transfer price. The first one is continuous, while the second one is not.

4.4. An Example of a Price-Based Allocation System with Continuous Transfer Price for Communities with no Battery

4.4.1. Definition

In this section we consider a subset of price-based allocation systems that set the transfer price $p_{tr,t}$ at time slot t according to the following formula:

$$p_{tr,t} = p_{sell,t}^{Grid} + \lambda \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right) \qquad \text{with } \lambda \in (0,1) \tag{1}$$

4.4.2. Properties

Since $p_{tr,t}$ (1) is a continuous function that does not depend on energy consumption, this allocation system satisfies Property 4 and Property 5 (besides properties 1-3). Also, note that the fraction $\frac{p_{tr}-p_{sell}^{Grid}}{p_{buy}^{Grid}-p_{sell}^{Grid}}$ of the total surplus that net producers get is λ, while net consumers get a fraction (1-λ) of the total surplus. If λ=1/2, surplus generated by trading is distributed proportional to $e^{tr}{}_{a,t}$ across all members. This makes intuitive sense, since surplus is created in each internal transfer, every transfer creates the same surplus $p_{buy}^{Grid} - p_{sell}^{Grid}$, and if λ=1/2, this surplus is shared equally between the seller and the buyer. Thus, if λ=1/2 surplus is distributed proportional to $e^{tr}{}_{a,t}$.

4.5. An Example of a Price-Based Allocation System with Discontinuous Transfer Price for Communities with no Battery

4.5.1. Definition

In this section we consider a specific price-based allocation system that sets the transfer price $p_{tr,t}$ at time slot t according to the following formula:

$$p_{tr,t} = \begin{Bmatrix} p_{sell,t}^{Grid} & \text{if } E_{A,t}^{g} \geq E_{A,t}^{c} \\ p_{buy,t}^{Grid} & \text{if } E_{A,t}^{g} < E_{A,t}^{c} \end{Bmatrix} \tag{2}$$

Equation (2) sets a transfer price equal to $p_{sell,t}^{Grid}$ in the time slots where the community as a whole generates more energy than its overall consumption, and sets the transfer price equal to $p_{buy,t}^{Grid}$ in the periods where the community as a whole consumes more energy than its overall production.

4.5.2. Properties

Note that $p_{transfer}$ (2) is a discontinuous function that depends on energy consumption. This implies that this price setting mechanism does not satisfy properties 4 and 5 above, and it also implies a number of drawbacks that we summarize below:

- An individual's increase in energy consumption (keeping everything else constant) may lead to a reduction of her individual's allocated cost.
- An individual's decrease in energy consumption (keeping everything else constant) may lead to an increase of her individual's allocated cost.
- An individual's allocated cost is not a continuous function of her own consumption (i.e. small changes in one individual's consumption may lead to big changes in that individual's allocated cost).
- An individual's allocated cost is not a continuous function of other individuals' consumption (i.e. small changes in one individual's consumption may lead to big changes in another individual's allocated cost).

- An individual's allocated cost is not a continuous function of the total energy generated by the facility $E_A^{produced}$ (i.e. small changes in the energy generated by the whole community may lead to big changes in some individuals' allocated cost).

4.6. Systems with a Battery

In this section we consider systems with a battery (see Figure 6).

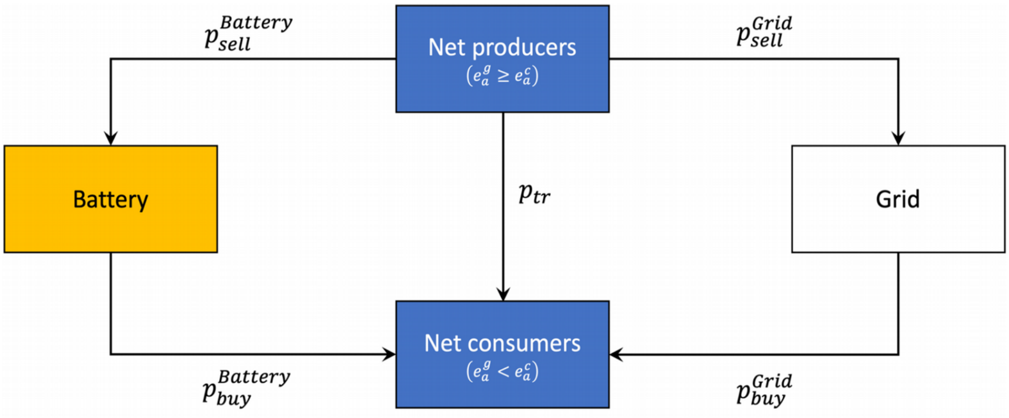

Figure 6. Illustration of a price-based allocation system assuming there is a battery

The first thing that must be decided at the time of managing the battery is the magnitude to optimize. A natural magnitude to minimize is the cost of energy for the whole community. This problem is not trivial, as it generally depends on the amount of energy that will be generated by the facility ($E^g_{A,t}$), the individuals' future consumption ($E^c_{A,t}$), and the future prices at which energy can be bought and sold to the grid ($p_{buy,t}^{Grid}$ and $p_{sell,t}^{Grid}$), see e.g. Xydas, Qadrdan, Marmaras, Cipcigan, Jenkins and Ameli (2017). A possible approach to optimizing this cost consists in forecasting the value of these uncertain magnitudes for future time slots and, based on these estimations, dynamically compute the values of the prices at which individuals should trade energy with the battery at every future time slot t ($p_{sell,t}^{Battery}$, $p_{buy,t}^{Battery}$) and the energy flows that minimize the cost.

As for the distribution of savings (or profits) generated by the battery, we believe that the most natural approach is to see the battery as an independent firm, even if it is financed by the members of the community. In this case, the distribution of profits should be made proportional to ownership shares, just like in any other firm. Naturally, the ownership of the battery does not have to match the ownership of the facility.

5. Other Allocation Systems

5.1. Introduction

In this section, we present some alternative allocation systems, and compare them with the family of price-based allocation systems presented in the previous section. To better illustrate the properties of each allocation system, we assume there is no battery in the community.

5.2. Bill Sharing

5.2.1. Introduction

Under the *bill-sharing* allocation system (Grzanic et al., 2021; Long et al., 2017; Zhou, Wu & Long, 2018), in every time slot, if the total energy produced by the facility exceeds the total consumption of the community ($E^g_A > E^c_A$), then the excess is sold to the grid, and the income obtained is shared among the members proportional to their overproduction $[e^g_a - e^c_a]^+$ (so only net producers obtain some income). On the other hand, if the community needs to buy energy from the grid ($E^c_A > E^g_A$), then the cost of this bill is distributed among the members proportional to their overconsumption $[e^c_a - e^g_a]^+$ (so only net consumers pay the bill). This is the so-called *bill sharing method net*, put forward by Grzanic et al. (2021) as an upgrade to the *bill sharing method* proposed by Long et al. (2017).

5.2.2. Definition

We assume that $p_{buy,t}^{Facility} < p_{buy,t}^{Grid}$ (otherwise, the situation is substantially simpler), so the community uses all the energy produced by the facility first and then, if needed, buys any shortage from the grid. There are no cash flows between the members of the community, and:

- If $E^g_{A,t} > E^c_{A,t}$, the income obtained from the grid is allocated proportional to $[e^g_a - e^c_a]^+$. Thus, the total cost for member *a* would be

$$cost_{a,t}^{BS} = e_{a,t}^{g} \cdot p_{buy,t}^{Facility} - \frac{[e_a^g - e_a^c]^+}{\sum_{i \in A}[e_i^g - e_i^c]^+} \cdot p_{sell,t}^{Grid} \cdot (E_{A,t}^g - E_{A,t}^c).$$

- If $E^c_{A,t} > E^g_{A,t}$, the cost of the bill with the grid is allocated proportional to $[e^c_a - e^g_a]^+$. Thus, the total cost for member *a* would be

$$cost_{a,t}^{BS} = e_{a,t}^{g} \cdot p_{buy,t}^{Facility} + \frac{[e_a^c - e_a^g]^+}{\sum_{i \in A}[e_i^c - e_i^g]^+} \cdot p_{buy,t}^{Grid} \cdot (E_{A,t}^c - E_{A,t}^g).$$

5.2.3. Properties

It is not difficult to see that, under this allocation system, net producers are generally better off outside the community (Grzanic et al., 2021). As a matter of fact, this system could be interpreted as a *price-based* allocation system where energy traded inside the community is given at no cost (p_{tr}=0), which implies that net producers are worse off inside the community if any amount of energy is traded internally. In our definition of *price-based* allocation systems, we assumed $0 \leq p_{sell}^{Grid} < p_{tr} < p_{buy}^{Grid}$, so the *bill-sharing* allocation system (p_{tr}=0) does not fit into our *price-based* framework.

The *bill-sharing* allocation system satisfies:

- Property 2: "No Pareto improvement is possible".
- Property 4: "Individuals' allocated cost is a continuous function".

The *bill-sharing* allocation system does **not** satisfy:

- Property 1: "Participation is beneficial". (Net producers are worse off if there is at least one net consumer)
- Property 3: "There is a perfect rank correlation between individuals' allocated cost and their consumption above their allocated production ($e^c_{a,t} - e^g_{a,t}$) in every compensation time slot".
- Property 5: "Every individual's allocated cost is a strictly increasing function of that individual's consumption". Under the *bill-sharing* allocation system, individuals' allocated cost is increasing, but not strictly increasing. As an example, note that if $E^c_{A,t} > E^g_{A,t}$, net producers with $e^g_a > e^c_a$ may marginally increase their consumption (so they are still net producers), without increasing their allocated cost.

5.3. Surplus-Based Allocation Systems

5.3.1. Introduction

As in *price-based* allocation systems, in *surplus-based* allocation systems, net producers and net consumers trade energy in the internal market (see Figure 7). This internal market generates a certain surplus $Surplus_A^{Tr}$ that was computed in Section 4.2.2 and equals $(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}) \cdot E_{A,t}^{tr}$. Different instances of allocation systems within the family of *surplus-based* allocation systems follow different rules to distribute this surplus among the members of the community; some of these instances may not use the concept of the transfer price p_{tr}.

In the following sections, we discuss two specific instances of the *family of surplus-based* allocation systems: distribution of surplus proportional to facility ownership shares α_a (which does not use any transfer price) and distribution of surplus proportional to transferred energy (which is a particular case of a price-based allocation system where $p_{tr,t} = (p^{Grid}_{buy,t} + p^{Grid}_{sell,t})/2$).

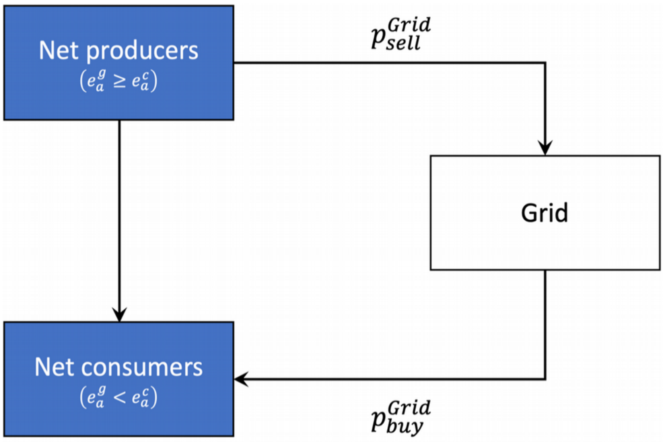

Figure 7. Illustration of the *family of surplus-based* allocation systems

5.3.2. Distribution of Surplus Proportionally to Facility Ownership Share

This allocation system distributes the surplus generated in the internal market proportionally to facility ownership shares α_a. This implies that the individual allocated cost for member a would be:

$$cost^{SP\alpha}_{a,t} = cost^{NoTr}_{a,t} - \alpha_a \cdot Surplus^{Tr}_{A,t} = cost^{NoTr}_{a,t} - \alpha_a \cdot \left(p^{Grid}_{buy,t} - p^{Grid}_{sell,t}\right) \cdot E^{tr}_{A,t}$$

This allocation system does not fit into the price-based framework described in Section 4. To see this, consider a member a such that $e^c{}_a = e^g{}_a$. Under a price-based allocation system, the costs for such a member a would be $cost^{Tr}_a = cost^{NoTr}_a = p^{Facility}_{buy} \cdot e^g_a$. However, under the allocation system that distributes surplus according to α_a, this member would generally enjoy a lower allocated cost, since it would get a fraction α_a of the surplus generated in the internal market. This member would benefit from trading even though she does not trade.

The allocation system that distributes surplus proportionally to facility ownership share satisfies:

- Property 1: "Participation is beneficial"
- Property 2: "No Pareto improvement is possible".
- Property 4: "Individuals' allocated cost is a continuous function".
- Property 5: "Every individual's allocated cost is a strictly increasing function of that individual's consumption".

The allocation system that distributes surplus proportionally to facility ownership share does **not** satisfy:

- Property 3: "There is a perfect rank correlation between individuals' allocated cost and their consumption above their allocated production ($e^c{}_{a,t} - e^g{}_{a,t}$) in every compensation time slot".

Interestingly, the fulfilment of properties 1, 2, 4 and 5 does not depend on the specific choice of α_a as the criterion to distribute the surplus. Thus, any arbitrary way of distributing the surplus would also fulfil those properties. Some ways of distributing the surplus may even add Property 3 to the repertoire (see next section).

5.3.3. Distribution of Surplus Proportional to Transferred Energy

This allocation system distributes the surplus generated in the internal market proportional to the amount of energy traded in the internal market $e_{a,t}^{transferred}$. This implies that the individual allocated cost for member a would be:

$$cost_{a,t}^{SPetr} = cost_{a,t}^{NoTr} - \frac{e_{a,t}^{tr}}{2 \cdot E_{A,t}^{tr}} \cdot Surplus_{A,t}^{Tr}$$

(Note that $\sum_{i \in A} e_{i,t}^{tr} = 2 \cdot E_{A,t}^{tr}$) In the appendix we prove that this allocation system is equivalent to the price-based allocation system where $p_{tr} = (p_{buy,t}^{Grid} + p_{sell,t}^{Grid})/2$, which is a particular case of the price setting mechanism analyzed in Section 4.4, with λ=1/2.

6. Case Study

In this section we present a fictional case study to illustrate our main theoretical results, and to gain some further insight on the different factors that affect the savings achieved by P2P-ET under different allocation systems. The information included in this section –plus some extra information provided in Appendix D– is sufficient to replicate this case study.

6.1. The Setting

We consider a community of 100 prosumers with two possible daily energy consumption profiles: Gauss (G) and anti-Gauss (aG) (Figure 8). Our case study covers a whole year (8760 hours), throughout which these daily consumption patterns repeat. To explore the impact of **heterogeneity in energy consumption profiles**, we study three different scenarios:

- 10G-90aG: 10% of prosumers have a Gauss energy consumption profile, and 90% of prosumers have an anti-Gauss profile.
- 30G-70aG: 30% of prosumers have a Gauss energy consumption profile, and 70% of prosumers have an anti-Gauss profile.
- 50G-50aG: 50% of prosumers have a Gauss energy consumption profile, and 50% of prosumers have an anti-Gauss profile.

As for the energy generation, we consider a PV facility that is shared equally by the prosumers (α_a=1/100 for all members a). The annual profile of PV energy generation has been obtained from the European Commission's "PVGIS" tool (Huld, Müller & Gambardella, 2012). This annual generation profile has been scaled to consider **different percentage coverages of the annual energy consumption of the whole community**. As an example, Figure 8 shows two profiles of the energy generated by the PV facility on one specific day, after scaling, so the energy generated over the year by the PV covers the 50% or the 100% of the annual consumption of the community.

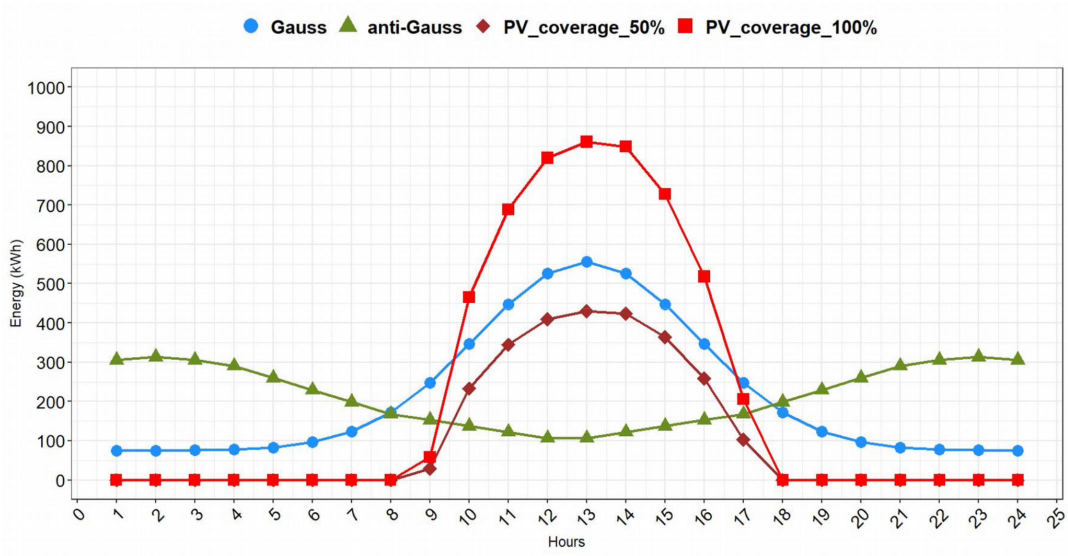

Figure 8. Graphical representation of the two daily energy consumption profiles considered (Gauss and anti-Gauss) and of two PV energy daily generation profiles (covering 50% and 100% of total consumption)

We also consider **two scenarios for the buy-sell spread** $\left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right)$:

- Low_Spread: $p_{buy,t}^{Grid} = 0.3€/\text{kWh}$ and $p_{sell,t}^{Grid} = 0.1€/\text{kWh}$.
- High_Spread: $p_{buy,t}^{Grid} = 1€/\text{kWh}$ and $p_{sell,t}^{Grid} = 0.1€/\text{kWh}$.

Finally, we analyze the following **allocation systems**:

- No_Trade: There is no internal energy trade within the community
- Mid-price: Price-based allocation system with $p_{tr} = \left(p_{buy,t}^{Grid} + p_{sell,t}^{Grid}\right)/2$.
- Bill_Sharing: bill sharing allocation system, as defined in Section 5.2.
- Surplus-Based: Surplus-based allocation system where the share of the surplus given to prosumer with ID i is proportional to i. E.g., prosumer with ID 10 gets ten times more share of the surplus than prosumer with ID 1.

6.2. Results

6.2.1. Heterogeneity in Energy Consumption Profiles

Internal trading can only take place if there are both net producers and net consumers in a time slot. In other words, there must be both a) prosumers with consumption above their allocated PV generation and b) prosumers with consumption below their allocated PV generation. Thus, generally, internal trading –and the savings it provides– are greater the greater the heterogeneity of consumption profiles, and when PV energy coverages are intermediate (they cannot be too low, since everyone would be a net consumer, or too high, since everyone would be a net producer).

Figure 9 illustrates these insights for our community of 100 prosumers in a High_Spread scenario, comparing the situations with and without internal trading. Specifically, Figure 9 shows the savings provided by the PV facility under two scenarios: one where internal trading is not allowed (No_Trade) and another one where the Mid-price allocation system is in place (Mid-price). We can see that savings provided by internal trading (i.e. the difference between the two lines in each graph) are greatest in the scenario with the greatest heterogeneity of consumption profiles (50G-50aG) and for intermediate PV converge rates (between 20% and 60%).

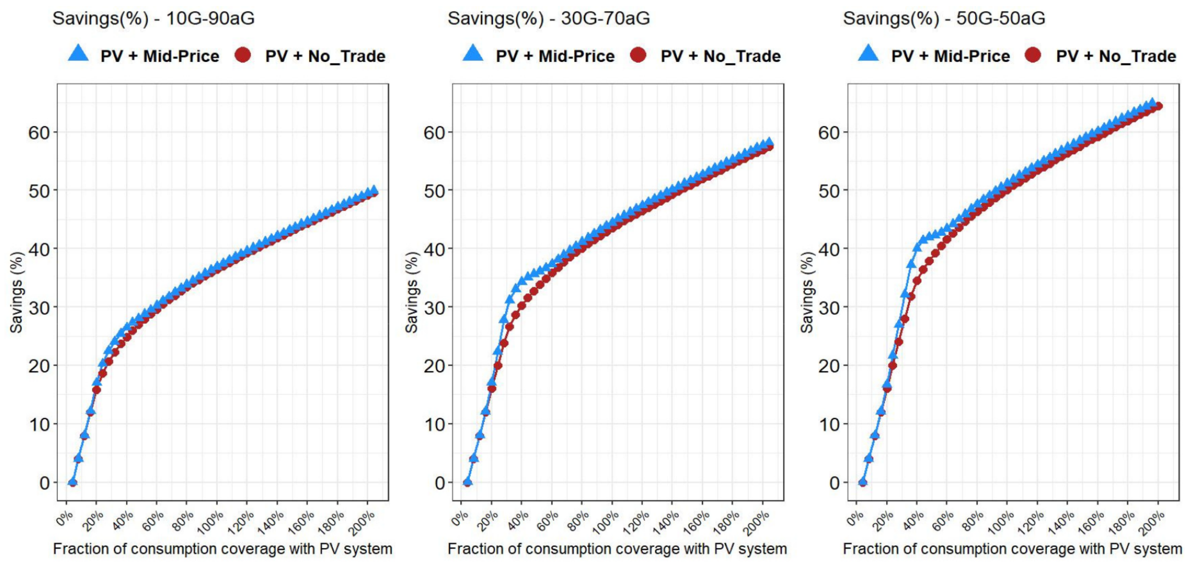

Figure 9. Savings provided by the PV facility in a community of 100 prosumers in a High_Spread setting under three different scenarios: 10G-90aG (left), 30G-70aG (center), and 50G-50aG (right). The two lines in each plot correspond to a) a situation where internal trading is not allowed (No_Trade) and b) a situation where the Mid-price allocation system is in place (Mid-price)

6.2.2. Buy-Sell Spread

As we have seen in Section 4.2, the buy-sell spread $\left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right)$ plays a major role in the magnitude of the surplus generated by internal trading. In fact, this surplus is proportional to the buy-sell spread. This is illustrated in Figure 10 for the same community of 100 prosumers considered in Figure 9 under different consumption scenarios. Also, like Figure 9, Figure 10 shows that the surplus generated in the internal market is greatest in the scenario with the greatest heterogeneity of consumption profiles (50G-50aG) and for intermediate PV coverage rates.

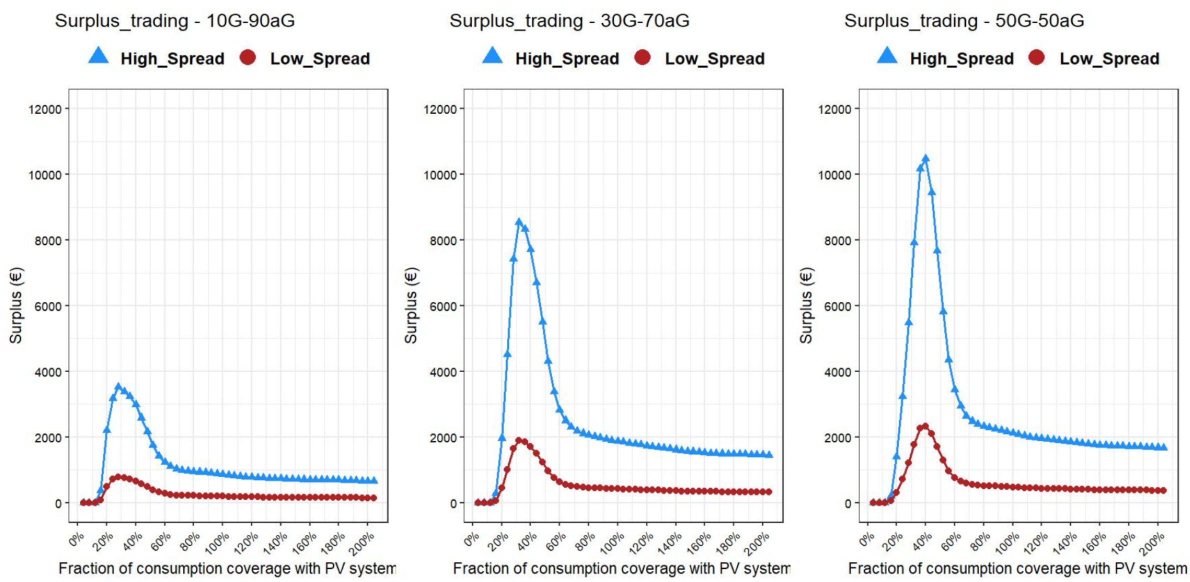

Figure 10. Surplus generated by internal trading in a community of 100 prosumers under three different scenarios: 10G-90aG (left), 30G-70aG (center), and 50G-50aG (right). The two lines in each plot correspond to a) a situation where the buy-sell spread equals 0.2 (Low_Spread) and b) a situation where the buy-sell spread equals 0.9 (High_Spread)

6.2.3. Allocation Systems

Our final experiment compares the different allocation systems from the perspective of each individual prosumer. Each prosumer is identified with a different ID number. Prosumers with ID number in the range [1,50] have a Gauss consumption profile, while prosumers with ID number in the range [51,100] have an anti-Gauss consumption profile. Figure 11 shows the cost allocated to each individual under different allocation systems: No_Trade, Mid-price, Bill_Sharing, and Surplus-Based.

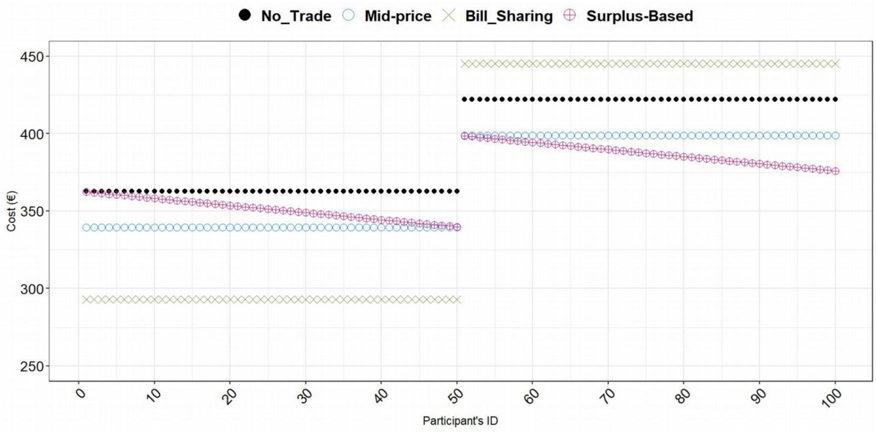

Figure 11. Individual cost allocated to each prosumer under different allocation systems: No_Trade, Mid-price, Bill_Sharing, and Surplus-Based. The consumption profile scenario is 50G-50aG, the PV coverage is 40%, and the spread scenario is Low_Spread

As expected, allocation systems satisfying Property 1 ("Participation is beneficial") assign a cost lower than the cost when trading is not allowed (No_Trade) to every prosumer. The only allocation system that does not satisfy this property is the Bill_Sharing, which penalizes net producers (in this case, the greatest net producers are the prosumers with an anti-Gauss consumption profile, ID in [51,100]). Note also that the absolute reduction in cost with respect to the No_Trade situation (i.e. the allocated surplus) is the same for every prosumer under the Mid-price allocation system, as proved in Section 4.4. In contrast, under the Surplus-Based allocation system, the surplus is distributed proportional to ID number, so the reduction in cost is greater the greater the prosumer's ID number.

7. Conclusions

In this paper, we have analyzed different allocation systems for energy communities. We have initially focused on a family of allocation systems that determines energy flows and distributes costs and benefits by setting different prices for the energy traded between the elements of the community in each time slot. Different ways of setting the transfer price at which members of the community trade energy among them give rise to different instances within this family of *price-based allocation systems*.

We have proved that every instance of this family of *price-based allocation systems* satisfies two desirable properties (i.e. "Participation is beneficial" and "No Pareto improvement is possible"), and we have established some conditions that guarantee the fulfilment of three additional desirable properties (e.g. "Every individual's allocated cost is a strictly increasing function of that individual's consumption").

We have also compared the family of *price-based* allocation systems with two alternatives: the *bill-sharing* allocation system, and the family of *surplus-based* allocation systems. Interestingly, there is one particular allocation system that belongs both to the family of *price-based* allocation systems and to the family of *surplus-based* allocation systems (Figure 12). This allocation system distributes the surplus generated in the internal market proportional to the amount of energy traded in the internal market by setting the transfer price to the average between the selling and the purchasing price of energy imposed by the external grid. This allocation system satisfies five properties that are particularly interesting for the management of energy communities.

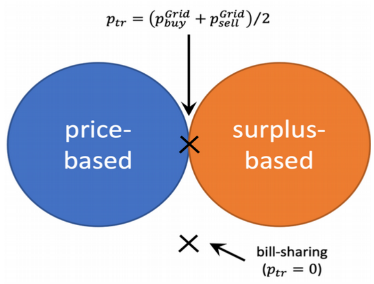

Figure 12. Schematic representation of the different allocation systems analyzed in this paper and the relationship between them

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References

Ceglia, F., Marrasso, E., Pallotta, G., Roselli, C., & Sasso, M. (2022). The State of the Art of Smart Energy Communities: A Systematic Review of Strengths and Limits. *Energies*, 15(9). https://doi.org/10.3390/en15093462

De Almansa, M., Campos, F.A., Doménech, S., & Villar, J. (2019). Residential DER cooperative investments. *International Conference on the European Energy Market, EEM*. https://doi.org/10.1109/EEM.2019.8916424

Domènech-Monfort, M., De Jesús, C., Wanapinit, N., & Hartmann, N. (2022). A Review of Peer-to-Peer Energy Trading with Standard Terminology Proposal and a Techno-Economic Characterisation Matrix. *Energies*, 15(23), 9070. https://doi.org/10.3390/en15239070

European Commission (2019). The European Green Deal. *European Commission*, 53(9), 24. https://doi.org/10.1017/CBO9781107415324.004

European Federation of Citizen Energy Cooperatives (2021). *Annual report 2021*. Available at: https://www.rescoop.eu/uploads/REScoop-Annual-Report-2021.pdf

European Parliament. (2018). *Directive (EU) 2018/2001 of the European Parliament and of the Council of 11 December 2018 on the promotion of the use of energy from renewable sources*. Available at: https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=uriserv:OJ.L_.2018.328.01.0082.01.ENG

European Parliament (2019). *Directive (EU) 2019/944 of the European Parliament and of the Council of 5 June 2019 on common rules for the internal market for electricity and amending Directive 2012/27/EU*. Available at: https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:02019L0944-20220623

Eurostat (2021). *House or flat: where do you live? - Products Eurostat News – Eurostat*. Available at: https://ec.europa.eu/eurostat/web/products-eurostat-news/-/ddn-20210521-1

Eurostat (2022). *Electricity production capacities for renewables and wastes (EU27, Solar, 2012-2021)*. Available at: https://ec.europa.eu/eurostat/databrowser/view/nrg_inf_epcrw/default/line?lang=en

Faria, P., Barreto, R., & Vale, Z. (2019). Demand Response in Energy Communities Considering the Share of Photovoltaic Generation from Public Buildings. *SEST 2019 - 2nd International Conference on Smart Energy Systems and Technologies*. https://doi.org/10.1109/SEST.2019.8849082

Goia, B., Cioara, T., & Anghel, I. (2022). Virtual Power Plant Optimization in Smart Grids: A Narrative Review. *Future Internet*, 14(5). https://doi.org/10.3390/fi14050128

Grzanic, M., Morales, J.M., Pineda, S., & Capuder, T. (2021). Electricity Cost-Sharing in Energy Communities under Dynamic Pricing and Uncertainty. *IEEE Access*, 9, 30225-30241. https://doi.org/10.1109/ACCESS.2021.3059476

Heinisch, V., Odenberger, M., Göransson, L., & Johnsson, F. (2019). Prosumers in the electricity system-household vs. system optimization of the operation of residential photovoltaic battery systems. *Frontiers in Energy Research*, 6(January). https://doi.org/10.3389/fenrg.2018.00145

Henni, S., Staudt, P., & Weinhardt, C. (2021). A sharing economy for residential communities with PV-coupled battery storage: Benefits, pricing and participant matching. *Applied Energy*, 301(March), 117351. https://doi.org/10.1016/j.apenergy.2021.117351

Herenčić, L., Kirac, M., Keko, H., Kuzle, I., & Rajšl, I. (2022). Automated energy sharing in MV and LV distribution grids within an energy community: A case for Croatian city of Križevci with a hybrid renewable system. *Renewable Energy*, 191, 176-194. https://doi.org/10.1016/J.RENENE.2022.04.044

Hernandez-Matheus, A., Löschenbrand, M., Berg, K., Fuchs, I., Aragüés-Peñalba, M., Bullich-Massagué, E. et al. (2022). *A systematic review of machine learning techniques related to local energy communities,* 170, 112651. https://doi.org/10.1016/j.rser.2022.112651

Honarmand, M.E., Hosseinnezhad, V., Hayes, B., Shafie-Khah, M., & Siano, P. (2021). An Overview of Demand Response: From its Origins to the Smart Energy Community. *IEEE Access*, 9, 96851-96876. https://doi.org/10.1109/ACCESS.2021.3094090

Huld, T., Müller, R., & Gambardella, A. (2012). A new solar radiation database for estimating PV performance in Europe and Africa. *Solar Energy*, 86(6), 1803-1815. https://doi.org/10.1016/J.SOLENER.2012.03.006

International Energy Agency (2022). *World Energy Outlook 2022*. Available at: https://www.iea.org/reports/world-energy-outlook-2022

Jasiński, J., Kozakiewicz, M., & Sołtysik, M. (2021). Determinants of energy cooperatives' development in rural areas–evidence from Poland. *Energies*, 14(2). https://doi.org/10.3390/en14020319

Javadi, M.S., Gough, M., Nezhad, A.E., Santos, S.F., Shafie-khah, M., & Catalão, J.P.S. (2022). Pool trading model within a local energy community considering flexible loads, photovoltaic generation and energy storage systems. *Sustainable Cities and Society*, 79. https://doi.org/10.1016/J.SCS.2022.103747

Long, C., Wu, J., Zhang, C., Thomas, L., Cheng, M., & Jenkins, N. (2017). Peer-to-peer energy trading in a community microgrid. *2017 IEEE Power & Energy Society General Meeting* (1-5). https://doi.org/10.1109/PESGM.2017.8274546

Maldet, M., Revheim, F. H., Schwabeneder, D., Lettner, G., del Granado, P.C., Saif, A. et al. (2022). Trends in local electricity market design: Regulatory barriers and the role of grid tariffs. *Journal of Cleaner Production*, 358, 131805. https://doi.org/10.1016/J.JCLEPRO.2022.131805

Moret, F., & Pinson, P. (2019). Energy Collectives: A Community and Fairness Based Approach to Future Electricity Markets. *IEEE Transactions on Power Systems*, 34(5), 3994-4004. https://doi.org/10.1109/TPWRS.2018.2808961

Muhsen, H., Allahham, A., Al-Halhouli, A., Al-Mahmodi, M., Alkhraibat, A., & Hamdan, M. (2022). Business Model of Peer-to-Peer Energy Trading: A Review of Literature. *Sustainability*, 14(3), 1616. https://doi.org/10.3390/su14031616

Mulder, G., Six, D., Claessens, B., Broes, T., Omar, N., & Mierlo, J. Van. (2013). The dimensioning of PV-battery systems depending on the incentive and selling price conditions. *Applied Energy*, 111, 1126-1135. https://doi.org/10.1016/j.apenergy.2013.03.059

Norbu, S., Couraud, B., Robu, V., Andoni, M., & Flynn, D. (2021). Modelling the redistribution of benefits from joint investments in community energy projects. *Applied Energy*, 287(January), 116575. https://doi.org/10.1016/j.apenergy.2021.116575

Paudel, A., Chaudhari, K., Long, C., & Gooi, H.B. (2019). Peer-to-peer energy trading in a prosumer-based community microgrid: A game-theoretic model. *IEEE Transactions on Industrial Electronics*, 66(8), 6087-6097. https://doi.org/10.1109/TIE.2018.2874578

Pindyck, R.S., & Rubinfeld, D.L. (2017). *Microeconomics*. Pearson Series in Economics.

Pörtner, H.O., Roberts, D.C., Tignor, M., Poloczanska, E.S., Mintenbeck, K., Alegría, A. et al. (2022). Summary for Policymakers. In *Climate Change 2022: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change* (3-33). Cambridge University Press, Cambridge, UK and New York, NY, USA.

Reis, I.F.G., Gonçalves, I., Lopes, M.A.R., & Henggeler-Antunes, C. (2021). Business models for energy communities: A review of key issues and trends. *Renewable and Sustainable Energy Reviews*, 144, 111013. https://doi.org/10.1016/j.rser.2021.111013

US Congress (2022). *Public Law 117-169 (Aug. 16, 2022)*. Available at: https://www.congress.gov/bill/117th-congress/house-bill/5376/text

Xydas, E., Qadrdan, M., Marmaras, C., Cipcigan, L., Jenkins, N., & Ameli, H. (2017). Probabilistic wind power forecasting and its application in the scheduling of gas-fired generators. *Applied Energy*, 192, 382-394. https://doi.org/10.1016/J.APENERGY.2016.10.019

Zhou, Y., Wu, J., & Long, C. (2018). Evaluation of peer-to-peer energy sharing mechanisms based on a multiagent simulation framework. *Applied Energy*, 222, 993-1022. https://doi.org/10.1016/J.APENERGY.2018.02.089

Zhou, Y., Wu, J., Long, C., & Ming, W. (2020). State-of-the-Art Analysis and Perspectives for Peer-to-Peer Energy Trading. *Engineering*, 6(7), 739-753. https://doi.org/10.1016/J.ENG.2020.06.002

Appendices

A. Propositions About Price-Based Allocation Systems

Proposition 1 (Property 1: "Participation is beneficial"): Every member is better off under a price-based allocation system than in a situation where every member a gets $e^{g}_{a,t}$, and no energy is traded among the members.

Proof: The proposition states that $cost_a^{NoTr} \geq cost_a^{Tr}$ and there is the possibility that $cost_a^{NoTr} > cost_a^{Tr}$. In Section 4.2.4, we have seen that, for net producers: $cost_{a\in NP,t}^{NoTr} - cost_{a\in NP,t}^{Tr} = e_{a,t}^{tr} \cdot (p_{tr,t} - p_{sell,t}^{Grid}) \geq 0$. This inequality is strict whenever member a trades in the internal market, i.e., in every time slot t such that $e^{tr}_{a,t} > 0$. Analogously, for net consumers: $cost_{a\in NC,t}^{NoTr} - cost_{a\in NC,t}^{Tr} = e_{a,t}^{tr} \cdot (p_{buy,t}^{Grid} - p_{tr,t}) \geq 0$. And the inequality is strict whenever the member trades in the internal market.

Proposition 2 (Property 2: "No Pareto improvement is possible"): Any price-based allocation system guarantees that the resulting allocation of costs and benefits is Pareto optimal.

Proof: Given that $p_{buy}^{Facility} < p_{buy}^{Grid}$, the most efficient outcome for the community as a whole is to use up as much of the energy produced internally as possible. This is guaranteed under any price-based allocation system since $p_{buy}^{Facility} < p_{sell}^{Grid} < p_{tr} < p_{buy}^{Grid}$, i.e. all net producers will try to sell their overproduction $[e^{g}_{a} - e^{c}_{a}]^{+}$ in the internal market (since $p_{buy}^{Facility} < p_{sell}^{Grid} < p_{tr}$) and all net consumers will try to buy their overconsumption $[e^{c}_{a} - e^{g}_{a}]^{+}$ in the internal market (since $p_{tr} < p_{buy}^{Grid}$). Achieving this guarantees that the final outcome will be Pareto optimal regardless of how the surplus is distributed, since any distribution of a fixed quantity is Pareto optimal (i.e. it is impossible to make an agent better off without making another agent worse off; it is a zero-sum game).

Proposition 3 (Property 3: "Fairness"): Under any price-based allocation system, if $p_{buy}^{Facility}$ is sufficiently low, there is a perfect rank correlation between individuals' allocated cost and their overconsumption defined as $(e^c{}_{a,t} - e^g{}_{a,t})$ in every compensation time slot.

Proof: We prove the statement only for net consumers (for whom $(e^c{}_{a,t} - e^g{}_{a,t})$ is positive). The proof for net producers (for whom $(e^c{}_{a,t} - e^g{}_{a,t})$ is negative) is analogous. First, note that, for net consumers:

$$e_{a\in NC,t}^{tr} = \frac{E_{A,t}^{tr}}{\sum_{i\in A}\left[e_{i,t}^{c} - e_{i,t}^{g}\right]^{+}}\left(e_{a,t}^{c} - e_{a,t}^{g}\right) \quad \text{and} \quad 0 \leq \frac{E_{A,t}^{tr}}{\sum_{i\in A}\left[e_{i,t}^{c} - e_{i,t}^{g}\right]^{+}} \leq 1$$

The allocated cost for net consumers is (see Section 4.2.3):

$$\begin{aligned} cost_{a\in NC,t}^{Tr} &= e_{a,t}^{g} \cdot p_{buy,t}^{Facility} + e_{a,t}^{tr} \cdot p_{tr,t} + \left(e_{a,t}^{c} - e_{a,t}^{g} - e_{a,t}^{tr}\right) \cdot p_{buy,t}^{Grid} \\ &= e_{a,t}^{g} \cdot p_{buy,t}^{Facility} + e_{a,t}^{tr} \cdot \left(p_{tr,t} - p_{buy,t}^{Grid}\right) + \left(e_{a,t}^{c} - e_{a,t}^{g}\right) \cdot p_{buy,t}^{Grid} = \\ &= e_{a,t}^{g} \cdot p_{buy,t}^{Facility} + \left(e_{a,t}^{c} - e_{a,t}^{g}\right)\left(\frac{E_{A,t}^{tr}}{\sum_{i\in A}\left[e_{i,t}^{c} - e_{i,t}^{g}\right]^{+}} \cdot \left(p_{tr,t} - p_{buy,t}^{Grid}\right) + p_{buy,t}^{Grid}\right) \\ &= e_{a,t}^{g} \cdot p_{buy,t}^{Facility} + \left(e_{a,t}^{c} - e_{a,t}^{g}\right)\left(p_{buy,t}^{Grid} \cdot \left(1 - \frac{E_{A,t}^{tr}}{\sum_{i\in A}\left[e_{i,t}^{c} - e_{i,t}^{g}\right]^{+}}\right) + p_{tr,t}\frac{E_{A,t}^{tr}}{\sum_{i\in A}\left[e_{i,t}^{c} - e_{i,t}^{g}\right]^{+}}\right) \end{aligned}$$

Note that, at any given time slot t, the term $\left(p_{buy,t}^{Grid} \cdot \left(1 - \frac{E_{A,t}^{tr}}{\sum_{i\in A}\left[e_{i,t}^{c} - e_{i,t}^{g}\right]^{+}}\right) + p_{tr,t}\frac{E_{A,t}^{tr}}{\sum_{i\in A}\left[e_{i,t}^{c} - e_{i,t}^{g}\right]^{+}}\right)$ is the same for every member of the community, and it is non-negative so, for sufficiently low $p_{buy,t}^{Facility}$, there is a perfect rank correlation between $cost_{a\in NC,t}^{Tr}$ and $(e^c{}_{a,t} - e^g{}_{a,t})$.

Proposition 4 (Property 4: "Smoothness"): Under any price-based allocation system where p_{tr} is a continuous function, individuals' allocated cost is a continuous function.

Proof: Given that $e^{tr}{}_a$ is a continuous function, it is straightforward to see that, if p_{tr} is continuous, then both $cost_{a\in NP}^{Tr}$ and $cost_{a\in NC}^{Tr}$ are also continuous.

Proposition 5 (Property 5: "Environmental friendliness"): Under any price-based allocation system where p_{tr} does not depend on energy consumption, every individual's allocated cost is a strictly increasing function of that individual's consumption.

Proof: We prove the statement only for net consumers. The proof for net producers is analogous. The allocated cost for net consumers is (see Section 4.2.3):

$$cost_{a\in NC,t}^{Tr} = e_{a,t}^{g} \cdot p_{buy,t}^{Facility} + e_{a,t}^{tr} \cdot p_{tr,t} + \left(e_{a,t}^{c} - e_{a,t}^{g} - e_{a,t}^{tr}\right) \cdot p_{buy,t}^{Grid}$$

Assuming p_{tr} does not depend on energy consumption, we have that:

$$\frac{\partial cost_{a\in NC,t}^{Tr}}{\partial e_{a,t}^{c}} = \frac{\partial e_{a,t}^{tr}}{\partial e_{a,t}^{c}} \cdot p_{tr,t} + \frac{\partial\left(e_{a,t}^{c} - e_{a,t}^{tr}\right)}{\partial e_{a,t}^{c}} \cdot p_{buy,t}^{Grid} = p_{buy,t}^{Grid} - \frac{\partial e_{a,t}^{tr}}{\partial e_{a,t}^{c}} \cdot \left(p_{buy,t}^{Grid} - p_{tr,t}\right)$$

Since $0 \leq \frac{\partial e_{a,t}^{tr}}{\partial e_{a,t}^{c}} \leq 1$ and $\left(p_{buy,t}^{Grid} - p_{tr,t}\right) < p_{buy,t}^{Grid}$, we have that $\frac{\partial cost_{a\in NC,t}^{Tr}}{\partial e_{a,t}^{c}} > 0$.

B. Propositions about the *Bill-Sharing* Allocation System

Proposition 6 (Property 2: "No Pareto improvement is possible"): The *bill-sharing* allocation system guarantees that the resulting allocation of costs and benefits is Pareto optimal.

Proof: In the *bill-sharing* allocation system, the community will use the cheapest source of energy, i.e. the facility if $p_{buy,t}^{Facility} \leq p_{buy,t}^{Grid}$ or the grid if $p_{buy,t}^{Facility} > p_{buy,t}^{Grid}$. As explained in the proof of Proposition 2, this guarantees that the final outcome will be Pareto optimal.

Proposition 7 (Property 4: "Smoothness"): The *bill-sharing* allocation system guarantees that individuals' allocated cost is a continuous function.

Proof: The individual allocated cost for member a under the *bill-sharing* allocation system is:

$$cost_{a,t}^{BS} = \begin{cases} e_{a,t}^{g} \cdot p_{buy,t}^{Facility} - \dfrac{\left[e_a^g - e_a^c\right]^+}{\sum_{i\in A}\left[e_i^g - e_i^c\right]^+} \cdot p_{sell,t}^{Grid} \cdot \left(E_{A,t}^g - E_{A,t}^c\right) & \text{if } E_{A,t}^g > E_{A,t}^c \\ e_{a,t}^{g} \cdot p_{buy,t}^{Facility} & \text{if } E_{A,t}^g = E_{A,t}^c \\ e_{a,t}^{g} \cdot p_{buy,t}^{Facility} + \dfrac{\left[e_a^c - e_a^g\right]^+}{\sum_{i\in A}\left[e_i^c - e_i^g\right]^+} \cdot p_{buy,t}^{Grid} \cdot \left(E_{A,t}^c - E_{A,t}^g\right) & \text{if } E_{A,t}^g < E_{A,t}^c \end{cases}$$

This is a continuous function.

Proposition 8 (Weak version of Property 5: "Environmental friendliness"): The *bill-sharing* allocation system guarantees that every individual's allocated cost is a weakly increasing function of that individual's consumption, but it is not a strictly increasing function of that individual's consumption.

Proof: We prove the statement only for net producers for whom $[e^g_a - e^c_a]^+ = e^g_a - e^c_a > 0$. The proof for other members is analogous. The formula for member a's allocated cost under the *bill-sharing* allocation system $(cost_{a,t}^{BS})$ can be found in the proof of Proposition 7. The partial derivative of net producer a's allocated cost $cost_{a\in NP,t}^{BS}$ with respect to her consumption $e^c_{a,t}$ is:

$$\frac{\partial cost_{a\in NP,t}^{BS}}{\partial e_{a,t}^c} = \\ = \begin{cases} p_{sell,t}^{Grid} \cdot \left(\dfrac{\sum_{i\in A}\left[e_i^g - e_i^c\right]^+ - \left[e_a^g - e_a^c\right]^+}{\left(\sum_{i\in A}\left[e_i^g - e_i^c\right]^+\right)^2}\left(E_{A,t}^g - E_{A,t}^c\right) + \dfrac{\left[e_a^g - e_a^c\right]^+}{\sum_{i\in A}\left[e_i^g - e_i^c\right]^+} \right) & \text{if } E_{A,t}^g > E_{A,t}^c \\ 0 & \text{if } E_{A,t}^g \leq E_{A,t}^c \end{cases}$$

It is easy to check that this partial derivative is non-negative over its domain.

C. Propositions about Surplus-Based Allocation Systems

Let β_{at} [0,1] be the fraction of surplus allocated to member a in time slot t.

Proposition 9 (Property 1: "Participation is beneficial"): Every member is better off under a surplus-based allocation system than in a situation where every member a gets $e^g_{a,t}$, and no energy is traded among the members.

Proof: The individual allocated cost under any surplus-based allocation system is never greater than the individual cost incurred if no exchange of energy is allowed:

$$cost_{a,t}^{SP\alpha} = cost_{a,t}^{NoTr} - \beta_{a,t} \cdot Surplus_{A,t}^{Tr} = cost_{a,t}^{NoTr} - \beta_{a,t} \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right) \cdot E_{A,t}^{tr} \geq cost_{a,t}^{NoTr}$$

Besides, if $\beta_{at} > 0$ and there is any exchange of energy within the community (i.e. $E^{tr}_{A,t} > 0$), the inequality above is strict, i.e. participation is strictly beneficial.

Proposition 10 (Property 2: "No Pareto improvement is possible"): Any surplus-based allocation system guarantees that the resulting allocation of costs and benefits is Pareto optimal.

Proof: The proof is analogous to proof of Proposition 6.

Proposition 11 (Property 4: "Smoothness"): Any surplus-based allocation system guarantees that individuals' allocated cost is a continuous function.

Proof: The individual allocated cost for member a under a surplus-based allocation system is:

$$cost_{a,t}^{SP\alpha} = cost_{a,t}^{NoTr} - \beta_{a,t} \cdot Surplus_{A,t}^{Tr} = cost_{a,t}^{NoTr} - \beta_{a,t} \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right) \cdot E_{A,t}^{tr}$$

This is a continuous function.

Proposition 12 (Property 5: "Environmental friendliness"): Any surplus-based allocation system guarantees that every individual's allocated cost is a strictly increasing function of that individual's consumption.

Proof: The formula for member a's allocated cost under any surplus-based allocation system is:

$$cost_{a,t}^{SP\alpha} = cost_{a,t}^{NoTr} - \beta_a \cdot Surplus_{A,t}^{Tr} =$$

$$= p_{buy,t}^{Facility} \cdot e_{a,t}^{g} + p_{buy,t}^{Grid} \cdot \left[e_{a,t}^{c} - e_{a,t}^{g}\right]^{+} - p_{sell,t}^{Grid} \cdot \left[e_{a,t}^{g} - e_{a,t}^{c}\right]^{+} - \beta_a \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right) \cdot E_{A,t}^{tr}$$

Note that, for net producers $e^{g}{}_{a,t} - e^{c}{}_{a,t} \geq 0$, we have:

$$cost_{a\in NP,t}^{SP\alpha} = p_{buy,t}^{Facility} \cdot e_{a,t}^{g} - p_{sell,t}^{Grid} \cdot \left(e_{a,t}^{g} - e_{a,t}^{c}\right) - \beta_a \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right) \cdot E_{A,t}^{tr}$$

Note also that, for net producers, $\frac{\partial E_{A,t}^{tr}}{\partial e_{a,t}^{c}} \leq 0$. Thus,

$$\frac{\partial cost_{a\in NP,t}^{SP\alpha}}{\partial e_{a,t}^{c}} = p_{sell,t}^{Grid} - \beta_a \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right) \cdot \frac{\partial E_{A,t}^{tr}}{\partial e_{a,t}^{c}} \geq p_{sell,t}^{Grid} > 0$$

For net consumers ($e^{c}{}_{a,t} - e^{g}{}_{a,t} \geq 0$), we have:

$$cost_{a\in NC,t}^{SP\alpha} = p_{buy,t}^{Facility} \cdot e_{a,t}^{g} + p_{buy,t}^{Grid} \cdot \left(e_{a,t}^{c} - e_{a,t}^{g}\right) - \beta_a \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right) \cdot E_{A,t}^{tr}$$

Note that $\frac{\partial E_{A,t}^{tr}}{\partial e_{a,t}^{c}} \leq 1$. Thus,

$$\frac{\partial cost_{a\in NC,t}^{SP\alpha}}{\partial e_{a,t}^{c}} = p_{buy,t}^{Grid} - \beta_a \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right) \cdot \frac{\partial E_{A,t}^{tr}}{\partial e_{a,t}^{c}} \geq p_{sell,t}^{Grid} > 0$$

Therefore, the partial derivative is strictly positive both for net producers and for net consumers.

Proposition 13 (A price- and surplus-based allocation system): The allocation system where surplus is distributed proportional to transferred energy is equivalent to the price-based allocation system with $p_{tr,t} = \left(p_{buy,t}^{Grid} + p_{sell,t}^{Grid}\right)/2$.

Proof: The allocated individual cost under the allocation system where surplus is distributed proportional to transferred energy is, by definition:

$$cost_{a,t}^{SPetr} = cost_{a,t}^{NoTr} - \frac{e_{a,t}^{tr}}{2 \cdot E_{A,t}^{tr}} \cdot Surplus_{A,t}^{Tr} = cost_{a,t}^{NoTr} - e_{a,t}^{tr} \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right)/2$$

The allocated individual cost under the price-based allocation system with $p_{tr,t} = \left(p_{buy,t}^{Grid} + p_{sell,t}^{Grid}\right)/2$ is, for net producers:

$$cost_{a\in NP,t}^{Tr} = cost_{a,t}^{NoTr} - e_{a,t}^{tr} \cdot \left(p_{tr,t} - p_{sell,t}^{Grid}\right) = cost_{a,t}^{NoTr} - e_{a,t}^{tr} \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right)/2 = cost_{a,t}^{SPetr}$$

And, for net consumers:

$$cost_{a\in NC,t}^{Tr} = cost_{a,t}^{NoTr} - e_{a,t}^{tr} \cdot \left(p_{buy,t}^{Grid} - p_{tr,t}\right) = cost_{a,t}^{NoTr} - e_{a,t}^{tr} \cdot \left(p_{buy,t}^{Grid} - p_{sell,t}^{Grid}\right)/2 = cost_{a,t}^{SPetr}$$

D. Specific Details of the Case Study

D.1. Energy Consumption Profiles

Hour	Gauss profile (W)	anti-Gauss profile (W)
1	75.05	305.42
2	75.21	313.06
3	75.82	305.42
4	77.75	290.15
5	83.12	259.61
6	96.14	229.07
7	123.44	198.52
8	172.69	167.98
9	248.42	152.71
10	346.01	137.44
11	447.80	122.17
12	526.41	106.90

Hour	Gauss profile (W)	anti-Gauss profile (W)
13	556.14	106.90
14	526.41	122.17
15	447.80	137.44
16	346.01	152.71
17	248.42	167.98
18	172.69	198.52
19	123.44	229.07
20	96.14	259.61
21	83.12	290.15
22	77.75	305.42
23	75.82	313.06
24	75.21	305.42

D.2. Energy Generation Profile

The generation profile of 1 kWp photovoltaic facility was obtained using the European Commission's "PVGIS" tool (Huld et al., 2012; https://re.jrc.ec.europa.eu/pvg_tools/en/) with the following input data:

Latitude (decimal degrees):	42.339
Longitude (decimal degrees):	-3.701
Elevation (m):	863
Radiation database:	PVGIS-SARAH
Slope:	30 deg
Azimuth:	0 deg
Nominal power of the PV system (c-Si) (kWp):	1.0
PV technology*	Crystalline silicon
System losses (%):	14.0