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Demand Forecasting Using a Hybrid Model Based on Artificial Neural Networks: A Study Case on Electrical Products

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Abstract:

Purpose: This work aims to evaluate demand forecasting models to determine if using exogenous factors and machine learning techniques helps improve performance compared to univariate statistical models, allowing manufacturing companies to manage demand better.

Design/methodology/approach: We implemented a multivariate Auto-Regressive Moving Average with eXogenous input (ARMAX) statistical model and a Neural Network-ARMAX (NN-ARMAX) hybrid model for forecasting. Later, we compared both to a standard univariate statistical model to forecast the demand for electrical products in a Colombian manufacturing company.

Findings: The outcomes demonstrated that the NN-ARMAX model outperformed the other two. Indeed, demand management improved with the reduction of overstock and out-of-stock products.

Research limitations/implications: The findings and conclusions in this work are limited to Colombian manufacturing companies that sell electrical products to the construction industry. Moreover, the experts from the company that provided us with the data also selected the external factors based on their own experiences, i.e., we might have disregarded potential factors.

Practical implications: This work suggests that a model using neural networks and including exogenous variables can improve demand forecasting accuracy, promoting this approach in manufacturing companies dealing with demand planning issues.

Originality/value: The findings in this work demonstrate the convenience of using the proposed hybrid model to improve demand forecasting accuracy and thus provide a reliable basis for its implementation in supply chain planning for the electrical/construction sector in Colombian manufacturing companies.

Keywords: ARMAX, demand forecasting, neural network, NN-ARMAX

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1. Introduction

Manufacturing companies maintain their goal of managing demand uncertainty through forecasting methods tailored to the specific demand of the markets in which they operate (Gutierrez, Solis & Mukhopadhyay, 2008). This purpose is strategic because feeding their production planning systems with less uncertain demand forecast information brings:

1. More accurate investments,
2. A higher probability of achieving a high level of service to customers, and
3. Information that impacts profitability and business continuity objectives, at least indirectly.

Ultimately, companies seek to have the availability of products and the quantity required by the customer based on an adequate replenishment policy while avoiding negative impacts on planning, production capacity, and inventory management. (Kerkkänen, Korpela & Huiskonen, 2009; Ma, Wang, Che, Huang & Xu, 2013; Moroff, Kurt & Kamphues, 2021).

An accurate forecast and meeting demand requirements have an intuitive relationship. When demand exceeds forecasted levels, sales are lost, whereas when this falls below expectations, an overstock occurs, with associated maintenance costs (Kourentzes, Trapero & Barrow, 2020). It is challenging for companies to conduct efficient planning since the demand volatility that results in forecasting errors is often linked to the demand's dependence on exogenous and non-linear factors (Gonçalves, Cortez, Carvalho & Frazao,, 2021).

Colombian manufacturers may underestimate the importance of accurate demand forecasting in ensuring the success of their supply chains. As a result, advanced forecasting methods are commonly unknown (Silva & Rupasinghe, 2017). Demand management needs are more pertinent in developing countries due to global competition. Companies regularly apply univariate statistical methods to forecast demand because they are simple to understand and use (Fattah, Ezzine, Aman, El Moussami & Lachhab, 2018). These methods have some limitations when estimating demand (Fildes, Ma & Kolassa, 2019).

On the one hand, relying solely on historical demand data disregards the effects of exogenous variables that can explain demand behavior (Abolghasemi, Hurley, Eshragh & Fahimnia, 2020). On the other hand, they assume that time series have only linear components, although most real-time series have non-linear behavior. As a result, more sophisticated multivariate and machine-learning methods have been developed (Abbasimehr, Shabani & Yousefi, 2020).

Due to the inclusion of variables that have a specific association with demand behavior, multivariate statistical models allow the generation of far more accurate demand forecasts than their univariate counterparts. The multivariate statistical models require the correct variable selection to achieve a significant level of precision. However, one of these models' drawbacks is that they cannot detect non-linear patterns in time series, making them less accurate. Utilizing machine-learning techniques is one way to overcome these drawbacks, considering their usefulness and ubiquity in daily activities assisted by computers. With enough data and training, they potentially make more accurate predictions since they can learn and improve over time (Makridakis, Spiliotis & Assimakopoulos, 2018). Due to their capacity to learn from data, recognize linear and non-linear patterns, and improve models by including new attributes, neural networks have become significant in predicting applications (Benidis, Rangapuram, Flunkert, Wang, Maddix, Turkmen et al., 2020; Gutierrez et al., 2008). The purpose of this work is to conduct a comparative study of three demand forecasting approaches for electrical products in the construction sector in a Colombian manufacturing company, which are:

- Traditional univariate statistical model,
- Auto-Regressive Moving Average with eXogenous inputs (ARMAX) multivariate statistical model using relevant lags of external factors selected as exogenous variables, and
- Hybrid statistical multivariate/neural network NN-ARMAX model.

Finally, based on their performances, the best model should consider in the company to explain the demand planning process.

2. Related Research

2.1. Literature Review

Multiple researchers have used multivariate statistical and neural network models to improve the precision of demand forecasting for their products. The approaches used for demand forecasting are briefly summarized in this section, starting with traditional statistical models and ending with hybrid methods that combine statistical methods and neural network architectures. A multivariate adaptive regression spline statistical model (MARS) was proposed by Lu, Lee and Lian (2012) for the demand for computers in a Taiwanese wholesale company. They employed financial sector indicators and expert criteria to select the predictor variables. Initially, they compared the MARS model against six forecasting models using: support vector regression (SVR), multilayer perceptron neural network (MLP), cerebellar model articulation controller neural network (CMACNN), and their combinations with MARS. The results showed that the MARS had the best performance with a MAPE of 11%, and even the models combined with MARS had better results than the individual ones. Subsequently, they compared with other methods such as extreme learning machine (ELM), ELM-MARS, the autoregressive integrated moving average model (ARIMA), and multivariate linear regression (MLR). Once more, the MARS model outperformed these methods, demonstrating its suitability for predicting the demand for computers in wholesale businesses and its value in identifying the relevant forecasting variables.

Anggraeni, Vinarti and Kurniawati (2015) compared the level of precision of an ARIMA model and an ARIMA model with exogenous variables (ARIMAX) for demand forecasting for children's clothing in Indonesia. This study demonstrated that demand fluctuates during religious holidays, but the pattern of change was irregular, making it challenging to predict demand precisely. It also aimed to show how independent variables affected the demand. According to the autocorrelation function (ACF) and the partial autocorrelation function (PACF), the estimate for the ARIMA model was (2, 1, 0). The ARIMAX model used months preceding, during, and following the holidays as independent variables, yielding a model (3, 1, 0). The ARIMAX model was more assertive according to the performance metrics used: Akaike's information criterion (AIC), root mean square error (RMSE), and mean absolute percentage error (MAPE).

To forecast demand for twelve SKUs from a U.K. cider company, Kourentzes and Petropoulos (2016) used a model known as the Multivariate Multiple Aggregation Prediction Algorithm with Exogenous Variables (MAPAx). Kourentzes, Petropoulos and Trapero (2014) previously developed a univariate MAPA model that combined several temporal aggregations and forecasted each one using the ETS (error, trend, and seasonality) statistical method. The predicted coefficients are then combined to form the final model. They compared the proposed model with other univariate and multivariate models: MAPA, ETS, Linear Regression, and ETSx (ETS with exogenous variables). The obtained results demonstrated the superiority of the MAPAx model over the other methods. Furthermore, using promotion data improved the forecasting accuracy compared to its univariate counterpart.

Gonçalves et al. (2021) applied an ARIMAX multivariate model, as well as machine-learning models such as MLP neural network, support vector regression (SVR), and random forests (RF), to weekly demand forecasting in a Portuguese production plant dedicated to electronic devices for automobiles at various stages of the product life cycle. The authors also referenced univariate models: Naïve, ARIMA, and Theta. That work considered up to 15 lags weeks for the ARIMAX model. The number of layers selected was seven, according to the secondary information obtained for the MLP. Based on proposed metrics such as inventory cost, overstock, and fill rate, their findings indicated that multivariate models outperform univariate models regardless of the cycle stage in which the product was. Furthermore, the ARIMAX model has higher precision in the early stages of a product's life cycle, while machine-learning methods performed better in the subsequent; they concluded that this is due to a lack of training data early in a product's life cycle. Finally, the results showed that the multivariate models reduced inventory costs, with the MLP model outperforming the others regarding overstock and average fill rate.

Kotsialos, Papageorgiou and Poulimenos (2005) compared a simple exponential smoothing (ES) with an MLP neural network for demand forecasting at two German retail companies. They used neural networks to address classical forecasting problems such as seasonality, long forecasting horizons, and new products. The authors concluded that neural network methods produced better results, though these were not statistically significant enough to justify their use over traditional statistical techniques like ES models. Carbonneau, Laframboise and

Vahidov (2008) used MLP, Recurrent Neural Networks (RNN), and Support Vector Machines (SVM) to forecast demand for foundry products in Canada and counteracted the "bullwhip" effect. They concluded that RNN and SVM outperformed traditional statistical techniques.

Abbasimehr et al. (2020) used a Long Short-Term Memory Neural Network (LSTM) for demand forecasting in a furniture company, arguing that a simple neural network architecture presents generalization problems when the data is highly variable. The authors proposed a grid search method for optimizing network hyperparameters to improve the model's precision. They compared the LSTM model to statistical models such as ARIMA, ES, and machine-learning models like MLP, SVM, and monolayer LSTM. Their model outperformed the SVM and MLP models in terms of symmetric mean absolute percentage error (SMAPE) and RMSE, demonstrating the superiority of machine learning over statistical models.

Kim, Jeong and Bae (2019) compared LSTM and ARIMA in order to determine which forecasting technique would work best for use in smart manufacturing. They discussed how LSTM eventually outperformed ARIMA.Despite the evolution of NN methods such as LSTM, hybrid approaches to forecasting accuracy are still used because they are more accurate and sufficient than single methods (Rosienkiewicz, 2021).

Yucesan, Gul and Celik (2018) compared ARIMAX, MLP, and an ARIMAX-MLP hybrid method for demand forecasting in a company's furniture in Turkey. The first step in the hybrid model is to calculate the model's residuals, which contain the non-linearity of the data; the second step is to add the residuals as separate inputs to an MLP network. The authors achieved the best results for five product families, with MAPE values less than 10%, and were superior to statistical and neural network methods (Fu, Chien & Lin, 2018). Aburto and Weber (2007) also proposed a hybrid model for forecasting demand for supermarket products in Chile using the residuals of an ARIMAX model and seasonality as the input to an MLP network (SARIMAX-MLP). Their MAPE and normalized mean square error (NMSE) values showed the hybrid model performed better than statistical models.

However, studies on demand forecasting for manufactured products in the electricity and construction sectors are scarce; the best that we know, Escavy, Herrero, Trigos and Sanz-Perez (2020) attempted to estimate the annual demand for aggregates in Spain, which are part of the dynamics of the construction sector. They used simple linear regressions to determine the relationship between aggregate production and each independent variable. The authors performed a multiple linear regression with all the independent variables to find a better fit. They discovered that the variables with the highest correlation were a) Demographic: the population reaching the age of emancipation, net immigration, and population growth; b) Economics: unemployment and GDP; and c) Construction (higher correlation): residential construction licenses, the added value of construction in the economy, weight of construction in the added value of the economy, and public works tendering.

2.2. Demand Forecasting

Forecasting is predicting one or more future events. They are required in various fields, such as science, engineering, and business, because they can provide relevant information for decision-making and developing plans to face future uncertainties (Montgomery, Jennings & Kulahci, 2008). Demand forecasting uses historical data and other variables to estimate customers' future demand for products or services at a specific time. Businesses use demand forecasting to schedule production, control stocks, for determining staffing needs, product pricing, and market potential (Aksoy & Guner, 2015).

As a result, various studies are focusing on improving demand forecasting efficiency by increasing predictions' accuracy, particularly in the long term. Demand forecasting can use qualitative and quantitative methods. Qualitative methods are subjective methods that rely on the judgment of experts. They are frequently used when there is little or no historical data, as with new products. Typically, marketing research strategies use surveys of potential customers or experiences with similar products; the Delphi method, developed by the RAND corporation in 1967, is one of the most formal and well-known qualitative methods (Montgomery et al., 2008). Although it is possible to perform some information analysis, these methods rely on expert judgment and lack data-driven decision models (Wang & Chen, 2019). Quantitative methods are based on statistics and use time series from historical demand data. According to Dellino, Laudadio, Mari, Mastronardi and Meloni (2018), one approach consists of identifying a mathematical model that describes the stochastic processes of data to predict future values through statistical analysis. Some quantitative methods are causal or multivariate because they include exogenous variables that affect demand, such as economic and demographic factors, demand for other products, price changes, and promotions, among other factors (Vidal, 2010).

2.3. ARMAX/ARIMAX Models

In the 1970s, Box and Jenkins investigated techniques based on linear mathematical models representing autoregressive and moving-average processes. Several experiments demonstrated that these techniques accurately capture the dynamics of linear time series (Tealab, 2018). According to Anggraeni et al. (2015), there are four types of univariate models: autoregressive (AR), moving average (MA), autoregressive moving average (ARMA), and autoregressive integrated moving average (ARIMA). Montgomery et al. (2008) stated that in stationary time series, the values exhibit stable and invariant statistical behavior (Pantoja, 2004). Thus, a time series is the average of the previous values plus a linear combination of each value's deviations, also known as the moving average (MA). Finite MA models assume that the oldest deviations have no relevance to the time series after a lag q, so the effect of the deviations may decay exponentially with time, with older deviations having lower weights than the most recent ones. As a result, this effect implies that previous series values contributed to the current value, acting as an autoregressive model (AR) (Pantoja, 2004). When an AR model is insufficient to explain the exponential decay of the effect of the deviations, the model can be adjusted by including MA terms, yielding an autoregressive moving average model ARMA. An appropriate way to build a model is by analyzing the behavior of the autocorrelation function (ACF) and the partial autocorrelation function (PACF). Although Montgomery et al. (2008) recommend using statistical software to test various models and select the best one that fits the data. Generally, non-stationary time series exhibit homogeneous behavior over time. A non-stationary time series y_t is homogeneous if its first difference $y_t - y_{t-1} = (1 - B)y_t$ and higher order differences $(1 - B)^d y_t$ can produce a stationary time series. If its d-th difference produces a stationary ARMA (p, q) process, it is referred to as an ARIMA (p, d, q) model, with the term "integrated" d. The ARIMAX model has the representation (p, d, q, r), which combines the autoregressive model (AR) of order p, the integration (I) with a degree of differentiation d (which is 0 in the ARMAX model), the moving average (MA) of degree q, and the exogenous variables $X(r)$, where r denotes the maximum number of variables included in the model (Gonçalves et al., 2021). The expression for an ARMAX model (p, q, r) is described in (1).

$$y_t = \delta + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{r} \omega_j x_j + \sum_{k=1}^{q} \theta_k e_{t-k} \quad (1)$$

Where:

y_t: Dependent variable
δ: Constant
ϕ_i: Autorregresive coefficient
y_{t-i}: Autorregresive variable
ω_j: Exogenous variable coefficient
x_j: Exogenous variable
θ_k: Moving average coefficient
e_{t-k}: Moving average variable

2.4. Artificial Neural Networks in Statistical Regression Models

Artificial neural networks (ANNs) are widely used to implement time series forecasting models due to their versatility and ability to identify non-linear patterns in time series (Abbasimehr et al., 2020; Syam & Sharma, 2018). According to Nørgaard, Ravn, Poulsen and Hansen (2000), in a hybrid statistical/neural network approach, the input structure of linear models can be used to select the proper input variables for the model, while the internal architecture of the model can be a feedforward ANN. If an ARMAX model is used to select network inputs, the model is known as NN-ARMAX, which stands for neural network ARMAX. Although the neural network performs a non-linear function, the predictor will have feedback when the regressors are selected, as in an ARMAX model. Previous prediction errors are affected by the output and fed back into the network as model inputs, as illustrated in Figure 1.

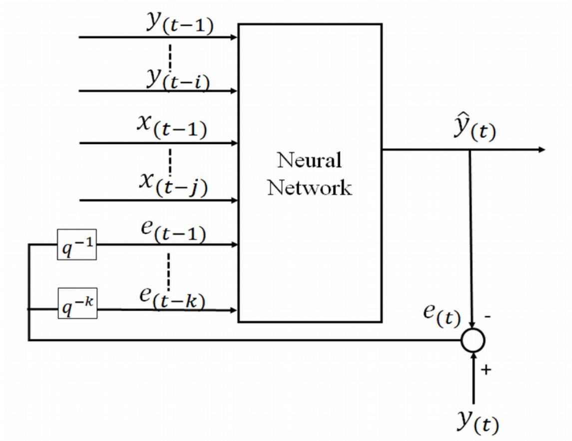

Figure 1. NN-ARMAX model (Nørgaard et al., 2000)

3. Methodology

Figure 2 illustrates the proposed methodology. Data gathering and treatment were done by analyzing sales data and defining the external factors affecting demand for selecting the SKUs and the exogenous variables for developing the models. There was also a descriptive statistical analysis of demand for the selected products. The current forecasting model of the company study case was the univariate statistical base model. We used a weighted moving average method for SKUs with a coefficient of variation less than 30%, and for other SKUs, a linear regression model. We identified the independent variables of the ARMAX model by observing the AR and MA processes involved in the demand data and including the relevant lags of the external factors as exogenous variables. Then, we implemented an NN-ARMAX model using the same independent variables of the ARMAX model. Their parameters were optimized during training by adjusting the hyperparameters and generating new moving average values. The results obtained in the dependent variable (demand forecasts) are compared for evaluation of the models. The one with the most significant positive economic impact, i.e., the one that reduces overstock and out-of-stock products, was selected.

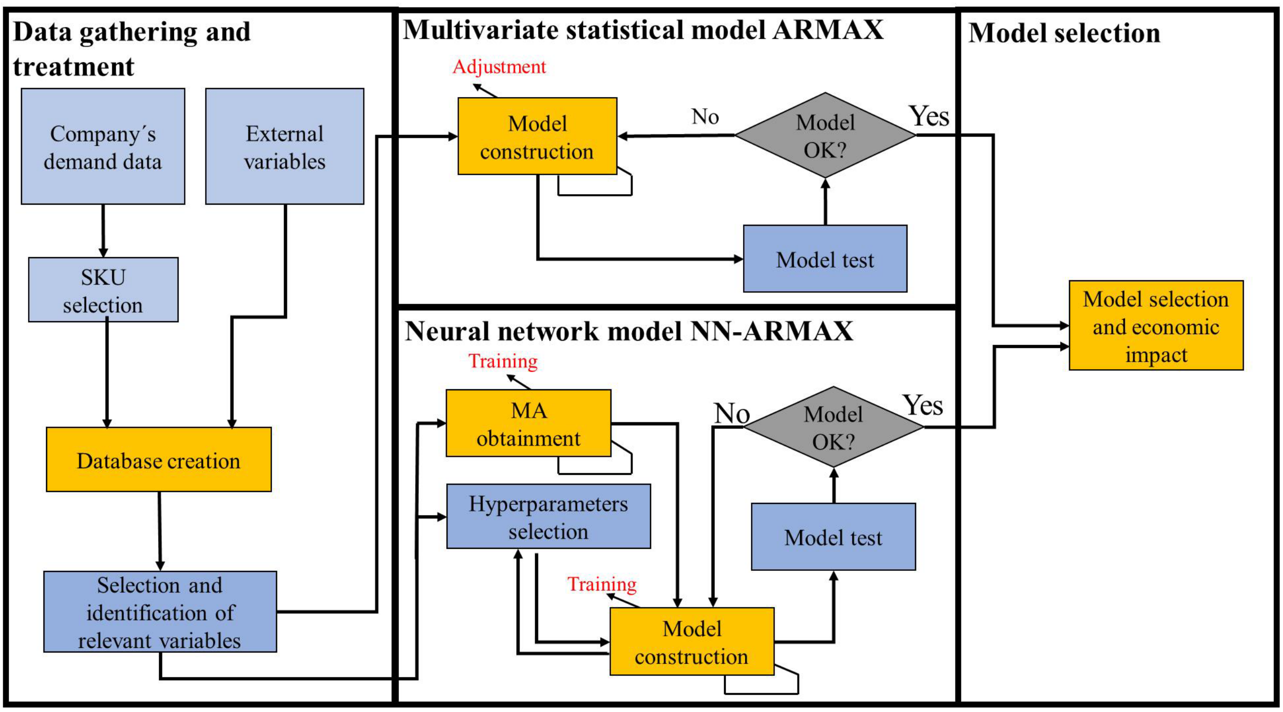

Figure 2. Methodology

3.1. Demand and Sales Datasets

The company study case provided monthly demand and sales data for make-to-stock SKUs for three products named A, B, and C from 2013 to 2020. These electrical goods for the construction industry were selected using Pareto analysis based on sales volume and marginal revenue. Table 1 lists the external factors considered in this study. They were determined using expert criteria from the company's Sales and Demand Planning areas. The factors were selected based on those considered when making forecasting adjustments using the enterprise resource planning system (ERP). The data for the selected external factors were provided from historical company information and public access datasets from the Internet.

The exogenous variables used as independent variables for the models were selected using a partial cross-correlation analysis (CCF) to identify those lags in external factors that showed a relevant correlation with the demand data for products A, B, and C over 95% of the confidence interval. In addition, we discarded external factors correlated with others to avoid collinearity.

Notation	External factors	Units	Source
x_0	Price rise announcement		Company
x_1	Metal price cash settlement (LME_MP_CS)	USD/t	London Metal Exchange (LME)
x_2	Metal price 3 months (LME_MP_3M)	USD/t	London Metal Exchange (LME)
x_3	Metal Stock (LME_M_Stock)	t(thousands)	London Metal Exchange (LME)
x_4	Total approved construction licenses (Total_Lic)	m^2	Colombian Chamber of construction (CAMACOL)
x_5	Mid-market rate (MMR) USD/COP	%	Banco de la República

Table 1. External factors considered in this work

3.2. Construction and Evaluation of the ARMAX Model

To create an ARMAX model, we first used the Dickey-Fuller test to see if the time series of selected products were stationary. Using the autocorrelation function (ACF) and the partial autocorrelation function (PACF), we determined whether autoregressive and moving average processes exist to identify the independent variables AR (autoregressive) and MA (moving average) of the model. We also used product demand data and exogenous variables selected through partial cross-correlation analysis as training data from January 2013 to September 2019. The best-fitting ARMAX model is selected based on the *p*-value and R^2 obtained from test data. Finally, the assumptions of normality and independence of residuals were validated using the Anderson-Darling and Ljung-Box tests, respectively. Figure 3 depicts the model construction sequence.

3.3. Construction and Evaluation of the Hybrid Model

We built the NN-ARMAX model with the previously developed ARMAX model, so we used the same autoregressive lags and previously identified relevant exogenous variables as independent variables of the model. The information was divided into two categories: training data (from January 2013 to September 2019) and test data (from October 2019 to December 2020). We developed a method for calculating the moving average; during training, we run the pre-trained model to obtain predictions and errors, which are then fed back into the input data as MA to make a new prediction and obtain new errors. It offers two advantages: first, it reduces errors to zero; second, it increases input data by joining various input data versions with various moving average values, a process known as data augmentation. Figure 4 illustrates this procedure.

This process was repeated iteratively by including an early stopping method to avoid overfitting until the training data achieved high R^2 and low MSE values. We selected the model with the best performance using the test set; however, a poor performance required repeating the training process. We also included a hyperparameters grid search to find the best-performing model. Figure 5 shows these steps.

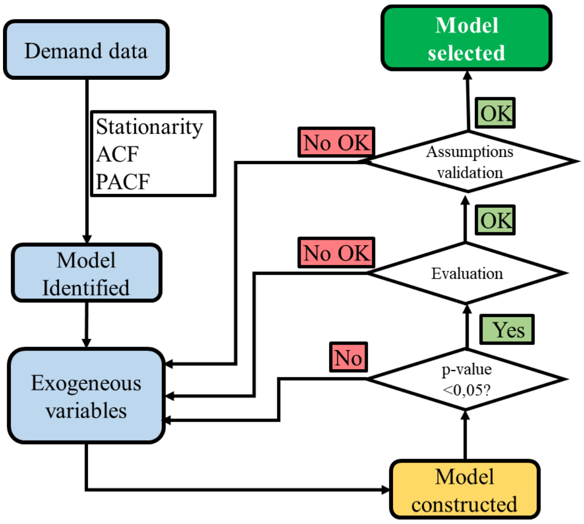

Figure 3. Construction sequence for ARMAX model

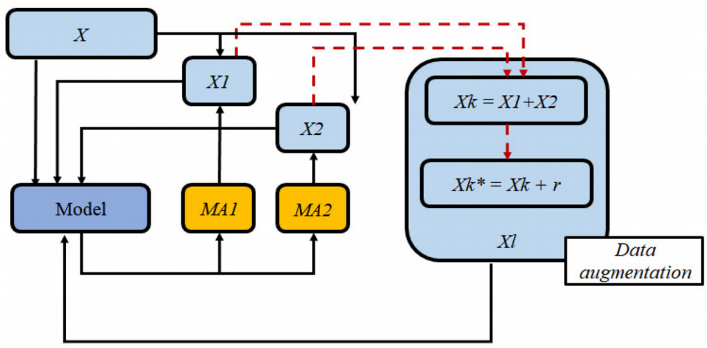

Figure 4. Moving average obtainment and data augmentation procedure

3.4. Model Comparison and Selection

With all models for each product, we compared the performance using the metrics presented in Table 2, where y_i is the real value and $\hat{y}_i$ is the obtained value in the i-th sample (dependent variable).

The selected model had the lowest average overstock and out-of-stock between October 2019 and December 2020, hoping to impact the replenishment positively.

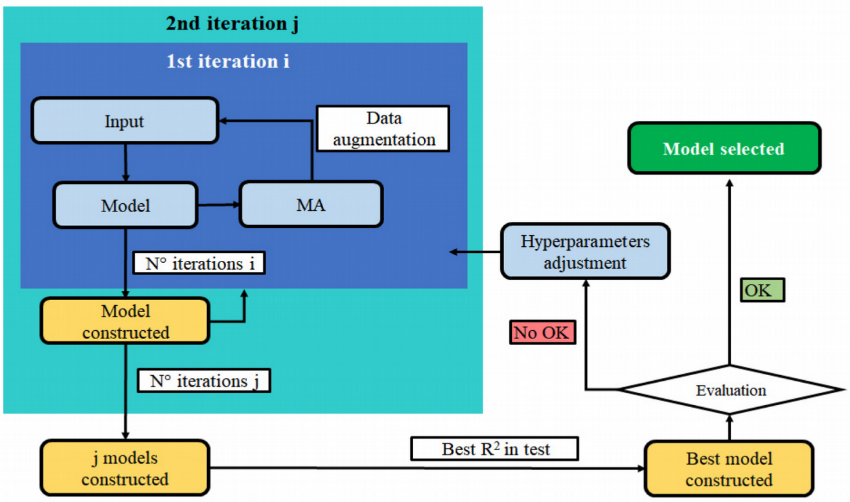

Figure 5. Construction sequence for NN-ARMAX model

Performance metrics	
Mean absolute percentage error (MAPE)	$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{\lvert y_i - \hat{y}_i\rvert}{y_i} * 100\%$
Root mean square error (RMSE)	$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$
Coefficient of determination (R^2)	$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2}$

Table 2. Performance metrics to compare models

4. Results and Discussion

Figure 6 shows the time series plots of the three products' demands. In general, the time series exhibit high variability, as evidenced by the pronounced peaks and valleys depicted in the plots; additionally, the peaks and valleys for the three products coincide in the same periods, implying that the factors influencing demand behavior must be the same. Table 3 displays the descriptive statistics of demand for the three products. Product A had the highest average demand and dispersion, while product B had the lowest. Product C had the lowest average demand and the highest dispersion. The three products had a high coefficient of variation (CV), indicating that they correspond to high variability or volatility demands (Abolghasemi, Beh, Tarr & Gerlach, 2020; Abolghasemi, M., Hurley et al. 2020). Product B showed the slightest variation with a CV of 0.298. Product C had the highest CV of 0.379 despite having the lowest dispersion. This analysis indicates that the demand for the three products is highly variable, making it difficult to forecast accurately.

Tables 4, 5, and 6 show the selected exogenous variables with their respective CCF coefficients for products A, B, and C. The relevant exogenous variables for products A and B are metal stock, total approved construction licenses, and mid-market rate (MMR) COP/USD variation. The relevant exogenous variables for product C are the metal price at three months (we discarded the price of the metal cash settlement because they are highly correlated), the metal stock, total approved construction licenses, and MMR COP/USD variation. Based on the correlation, the price rise announcement and metal prices variables do not have enough weight in the demand forecasting models. Although products A and B have the same external factors, they do not have the same lags. Product C, unlike products A and B, has a relevant correlation with metal prices at three months; this behavior is explained by the fact that product C is more complex than the other two products, as evidenced by the descriptive analysis. That would indicate that more variables may be required to explain such behavior.

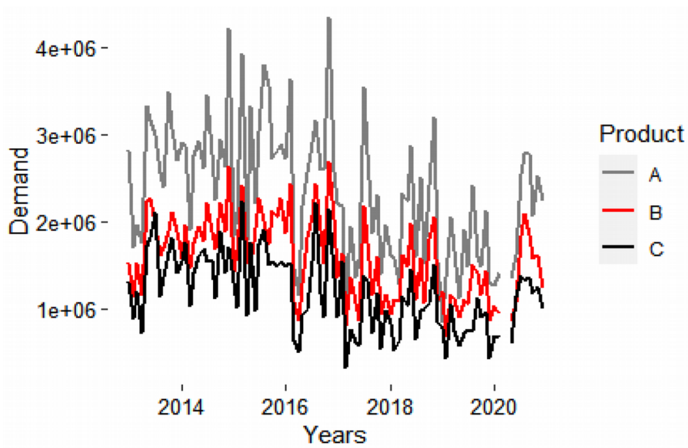

Figure 6. Time series plots of demands

	Product A	Product B	Product C
Mean	2,348,626	1,588,955	1,179,849
Standard deviation	764,078	473,627	446,701
Minimum	850,100	690,800	322,800
Q1	1,663,800	1,151,600	831,100
Median	2,354,050	1,586,000	1,145,100
Q3	2,864,100	1,946,800	1,520,300
Maximum	4,346,900	2,684,400	2,235,800
Mean absolute Deviation	820,916	624,916	544,633
Interquartile range	1,181,425	790,100	680,925
Coefficient of variation	0.325	0.298	0.379

Table 3. Descriptive statistics of demands

Variable	Lag	Denomination	Coefficient	CI (95%)
x_3	t-1	LME_M_Stock(t-1)	0.259	0.218
	t-2	LME_M_Stock(t-2)	0.285	
	t-3	LME_M_Stock(t-3)	0.243	
	t-4	LME_M_Stock(t-4)	0.225	
x_4	t-1	Total_Lic(t-1)	0.313	
	t-2	Total_Lic(t-2)	0.394	
	t-5	Total_Lic(t-5)	0.313	
	t-7	Total_Lic(t-7)	0.293	
	t-8	Total_Lic(t-8)	0.395	
	t-10	Total_Lic(t-10)	0.300	
	t-11	Total_Lic(t-11)	0.398	
x_5	t-11	Var_MMR(t-11)	0.328	

Table 4. Exogenous variables identified for product A

Variable	Lag	Denomination	Coefficient	CI (95%)
x_3	t-1	LME_M_Stock(t-1)	0.251	0.218
x_4	t-1	Total_Lic(t-1)	0.284	
	t-2	Total_Lic(t-2)	0.416	
	t-5	Total_Lic(t-5)	0.244	
	t-6	Total_Lic(t-6)	0.241	
	t-7	Total_Lic(t-7)	0.298	
	t-8	Total_Lic(t-8)	0.415	
	t-9	Total_Lic(t-9)	0.271	
	t-10	Total_Lic(t-10)	0.265	
	t-11	Total_Lic(t-11)	0.407	
	t-12	Total_Lic(t-12)	0.277	
x_5	t-11	Var_MMR(t-11)	0.346	
	t-12	Var_MMR(t-12)	0.304	

Table 5. Exogenous variables identified for product B

Variable	Lag	Denomination	Coefficient	CI (95%)
x_2	t-5	LME_MP_3M(t-5)	0.228	0.218
	t-8	LME_MP_3M(t-8)	0.238	
	t-9	LME_MP_3M(t-9)	0.244	
	t-10	LME_MP_3M(t-10)	0.228	
x_3	t-1	LME_M_Stock(t-1)	0.290	
	t-2	LME_MP_Stock(t-2)	0.333	
	t-3	LME_MP_Stock(t-3)	0.286	
	t-4	LME_MP_Stock(t-4)	0.266	
	t-5	LME_MP_Stock(t-5)	0.236	
x_4	t-1	Total_Lic(t-1)	0.374	
	t-2	Total_Lic(t-2)	0.377	
	t-5	Total_Lic(t-5)	0.254	
	t-7	Total_Lic(t-7)	0.290	
	t-8	Total_Lic(t-8)	0.268	
	t-9	Total_Lic(t-9)	0.424	
	t-10	Total_Lic(t-10)	0.223	
	t-11	Total_Lic(t-11)	0.263	
x_5	t-1	Var_MMR(t-1)	0.355	
	t-11	Var_MMR(t-11)	0.224	
	t-12	Var_MMR(t-12)	0.268	

Table 6. Exogenous variables identified for product C

Table 7 shows that the time series demand for products A, B, and C is stationary. Figure 7 also illustrates that the behavior in the ACF and PACF correlograms combines exponential decay and damped sinusoidal patterns, allowing us to conclude that moving average and autoregressive terms exist. To construct the forecasting model for the three products, we used an ARMA (1,1) model. The best model considering the *p*-value of the coefficients of the exogenous variables was the ARMAX model, with the exogenous variables *Total_Lic(t*−8*)* and *Total_Lic(t*−11*)* for product A; *Total_Lic(t*−7*)*, *Total_Lic(t*−11*)*, and *Var_MMR(t*−12*)* for products B and C. Table 8 shows the values of the coefficients and their corresponding *p*-value for each term of the fitted ARMAX model.

Product	p-value
A	0.021
B	0.020
C	0.010

Note: null hypothesis (non-stationary) is rejected if *p*-value <0.05

Table 7. Dickey-Fuller test results

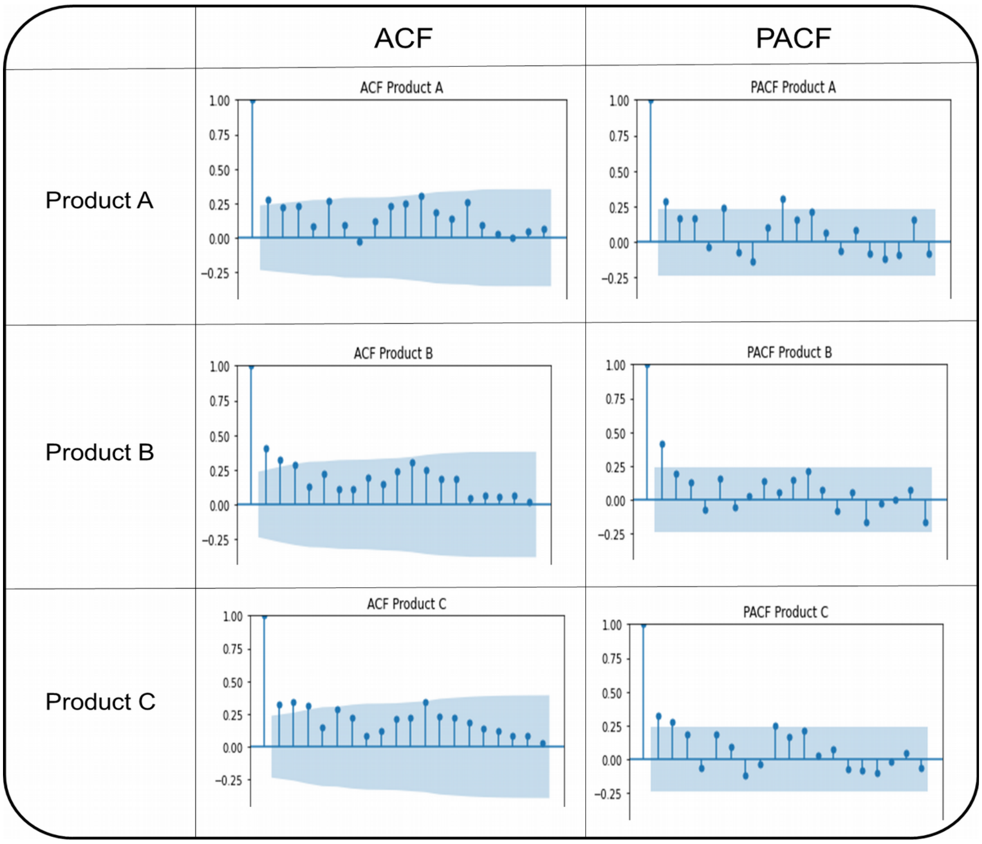

Figure 7. ACF and PACF correlograms

Prod.	Term	Description	Result	*p*-value
A	ϕ_1	Autoregressive coefficient in lag (t-1)	-0.8150	0.006
	θ_1	Coefficient of moving average in lag (t-1)	0.8729	0.001
	ω_1	Coefficient of Total_Lic in lag (t-8)	0.6319	0.000
	ω_2	Coefficient of Total Lic in lag (t-11)	0.6813	0.000
B	ϕ_1	Autoregressive coefficient in lag (t-1)	-0.9897	0.000
	θ_1	Coefficient of moving average in lag (t-1)	1.0873	0.000
	ω_1	Coefficient of Total_Lic in lag (t-7)	0.3460	0.000
	ω_2	Coefficient of Total_Lic in lag (t-11)	0.5443	0.000
	ω_3	Coefficient of Total_Lic in lag (t-12)	3.191×10^{-4}	0.000
C	ϕ_1	Autoregressive coefficient in lag (t-1)	0.9498	0.000
	θ_1	Coefficient of moving average in lag (t-1)	-1.0790	0.000
	ω_1	Coefficient of Total_Lic in lag (t-7)	0.2359	0.021
	ω_2	Coefficient of Total_Lic in lag (t-11)	0.3461	0.001
	ω_3	Coefficient of Total_Lic in lag (t-12)	3.016×10^{-4}	0.006

Table 8. Coefficients of ARMAX model for products A, B, and C

Equation of ARMAX model for product A:

$$y_t = \phi_1 y_{t-1} + \theta_1 e_{t-1} + \omega_1 x_4(t-8) + \omega_2 x_4(t-11) \tag{2}$$

Equation of ARMAX model for products B and C:

$$y_t = \phi_1 y_{t-1} + \theta_1 e_{t-1} + \omega_1 x_4(t-7) + \omega_2 x_4(t-11) + \omega_3 x_5(t-12) \quad (3)$$

Where:

y_t: Output (dependent variable)
y_{t-1}: Autorregressive variable
e_{t-1}: Moving average variable
$x_j(t-i)$: Exogenous variable j in lag i

The ARMAX model was more accurate than the base model for the three products, as shown by the performance metrics in Table 11. In summary, we demonstrated that using multivariate statistical models such as ARMAX improves demand forecasting performance over univariate models. Furthermore, when working with linear forecasting models, we found that data from construction licenses and MMR USD/COP variation are relevant to the family of products used in this study. In contrast, the influence of variables related to raw material price and stock is negligible in the proposed linear model. It demonstrates that the economic dynamics of the construction sector, such as the number of approved construction licenses and the MMR USD/COP, are the primary external factors driving demand for the selected electrical products. These variables significantly impact decision-making for the start of construction projects, their progress, and their completion, influencing the decision to purchase the selected electrical products.

We use the demand in period $t-1$ (autoregressive variable) and the exogenous variables chosen through CCF in the NN-ARMAX hybrid model. We used a grid-searching approach to determine the hyperparameters, similar to Sharma and Singhal (2019), to maximize R^2 during the training phase. Table 9 shows the values of the selected hyperparameters. A single hidden layer was insufficient during the model's development because it achieved a low R^2 value (less than 0.8), so we added a second layer. The number of neurons in the hidden layers increases the network's precision, with 20 neurons per layer providing satisfactory performance. We also included an adaptive learning rate with an initial value of 0.9, and the results improved. We used a validation fraction of 30% due to the significant variation in training and testing data, which allowed for better results when we checked the test data. L2 regularization, ReLU activation function, and the quasi-newton solver were among the neural network hyperparameters.

Hyperparameters	Value
Number of hidden layers	2
Number of neurons in each hidden layer	20
Initial learning rate	0.9
Batch size (include all data)	1000
Maximum number of iterations	10
Tolerance	0.001
Validation fraction	0.3
Alpha (L2 regularization parameter)	0.0001
Activation function	ReLU
Solver	lbfgs

Table 9. Hyperparameters selected for the NN-ARMAX model

Table 10 shows the equations for the NN-ARMAX models. For the three performance metrics in the three products, the NN-ARMAX model performed noticeably better than the base forecasting model. Despite the variability of product A demand in the test data, the model's effectiveness identifies demand patterns and achieves higher forecasting accuracy than the base model. Product B, like product A, has a high variability; however, the model performed well in identifying the increase in demand in the second half of the year; despite showing specific difficulties with demand at the start of 2020, the forecast performance was comparable to that obtained with product A. Product C had the most challenging demand because its MAPE and R^2 values were lower than those of products A and B. Other unidentified external factors might be to blame.

Product	NN-ARMAX Equations	
A	$y_t = f(y_{t-1}, x_3(t-1), x_3(t-2), x_3(t-3), x_3(t-4), x_4(t-1), x_4(t-2), x_4(t-5), x_4(t-7), x_4(t-8), x_4(t-10), x_4(t-11), x_5(t-11), e_{t-1})$	(4)
B	$y_t = f(y_{t-1}, x_3(t-2), x_4(t-1), x_4(t-2), x_4(t-5), x_4(t-6), x_4(t-7), x_4(t-8), x_4(t-9), x_4(t-10), x_4(t-11), x_4(t-12), x_5(t-11), x_5(t-12), e_{t-1})$	(5)
C	$y_t = f(y_{t-1}, x_3(t-5), x_3(t-8), x_3(t-9), x_3(t-10), x_4(t-1), x_4(t-2), x_4(t-3), x_4(t-4), x_4(t-5), x_5(t-1), x_5(t-2), x_5(t-3), x_5(t-5), x_5(t-7), x_5(t-8), x_5(t-9), x_5(t-10), x_5(t-11), x_5(t-1), x_5(t-11), x_5(t-12), e_{t-1})$	(6)

Table 10. Equations for NN-ARMAX model

Where:

y_t: Output (dependent variable)
y_{t-1}: Autorregresive variable
e_{t-1}: Moving average variable
$x_j(t-i)$: Exogenous variable j in lag i

The outstanding performance of the NN-ARMAX model is due to the inclusion of autoregressive and moving average terms identified in the ARMAX model as independent variables or inputs. It shows the great advantage of combining statistical and machine-learning methods to develop forecasting models, even for demands with high variability. Furthermore, the addition of exogenous variables improved forecasting precision. Unlike the ARMAX model, which only allowed the use of a few variables due to the limitations of linear models, the NN-ARMAX model allowed the use of all variables without restriction. It allows for extracting more information from available data than statistical models.

We used the performance metrics for products A, B, and C to compare the results of the ARMAX and NN-ARMAX models, as shown in Table 11. Figure 8 illustrates the demand trend graph in a solid black line, the ARMAX model in a dashed red line, and the NN-ARMAX model in a dashed blue line. The NN-ARMAX model is more closely related to the demand line than the ARMAX model. We demonstrate that the NN-ARMAX model can better represent the demand behavior for the MTS products commercialized by the company study case through a simple visualization exercise. Between October 2019 and June 2020, Product A demonstrates that the NN-ARMAX model fits the demand line better than the ARMAX model. As a result, demand rose significantly beginning in July 2020; this forecasting stage was difficult for all three models because the abrupt change was caused by external factors not considered in this study, possibly a bullwhip effect caused by the COVID-19 pandemic. Despite this shift in behavior, the ARMAX and NN-ARMAX models performed better in terms of adaptation due to the inclusion of autoregressive and moving average terms. The same analysis applies to product B; we observed the same increase in demand beginning in July 2020, and the NN-ARMAX model is the one that best adapts to the behavior of demand throughout the evaluation period. After June 2020, Product C was the most difficult to forecast. Because a significant deviation is visible in both predictions, neither the ARMAX nor the NN-ARMAX models can adapt to the demand appropriately. The NN-ARMAX model stands out for being able to better adjust to demand prior to June 2020. This difficulty in forecasting product C is partly explained by the effect of the COVID pandemic and by the fact that it has the highest coefficient of variability of the three products.

According to the results, the NN-ARMAX hybrid model performed the best in the three metrics for the three products, demonstrating a clear superiority of this approach. Table 12 shows the three forecasting models' overstock and out-of-stock simulation results. Compared to the base model, the NN-ARMAX model had a moderately better performance in out-of-stock products with an 11% reduction for product C; however, the NN-ARMAX model for products A and B had a significant reduction in out-of-stock with values of 100% and 85%, respectively. The NN-ARMAX model outperformed the base model in overstock in the three products.

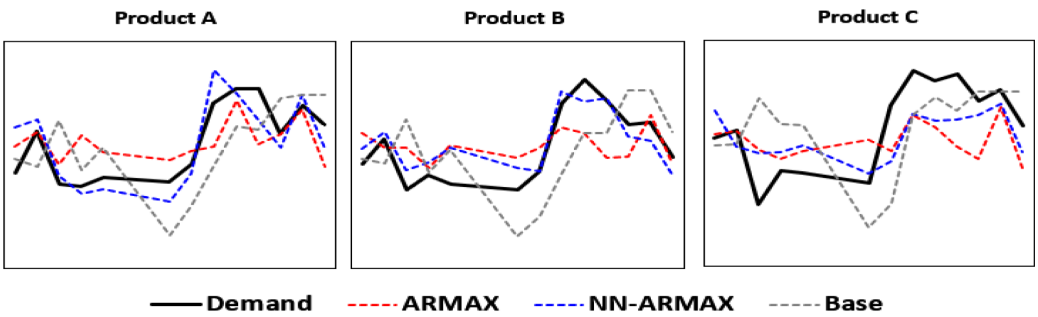

Figure 8. Forecast comparison plots of the models

		Base	ARMAX	NN-ARMAX
R^2	Product A	-0.19	0.30	0.65
	Product B	-0.37	0.17	0.75
	Product C	-0.34	0.06	0.39
RMSE	Product A	610,982	469,035	328,985
	Product B	461,991	359,861	195,232
	Product C	353,123	295,601	237,467
MAPE	Product A	30.99%	21.27%	14.73%
	Product B	32.40%	24.53%	14.39%
	Product C	37.60%	28.57%	24.28%

Table 11. Comparison of demand forecasting performance

Prod.	Indicator	Base	ARMAX	NN-ARMAX	NN-ARMAX vs. Base	NN-ARMAX vs. ARMAX
A	Overstock	463,592	392,900	402,700	13%	**-2%**
	Out-of-stock	173,408	98,500	0	100%	100%
B	Overstock	307,908	380,738	293,562	5%	23%
	Out-of-stock	136,831	53,754	20,000	85%	63%
C	Overstock	238,946	203,008	209,215	12%	**-3%**
	Out-of-stock	47,254	52,315	52,315	**-11%**	0%
Total	Overstock	1,010,446	976,646	905,477	10%	7%
	Out-of-stock	357,492	204,569	72,315	80%	65%

Table 12. Overstock and out-of-stock results and reduction percentage of the NN-ARMAX model compared to the other models.

When we compare the NN-ARMAX model to the ARMAX model, we found that the ARMAX model achieves a higher, although slight, reduction of overstock of 2% and 3%, respectively, in products A and C. In contrast, the NN-ARMAX model achieves a reduction of 23% in product B. Regarding out-of-stock, it is worth noting that there is no out-of-stock product for product A during the simulation period, demonstrating the enormous potential of implementing multivariate and machine learning forecasting models to improve the company's level of service. We noticed a 63% reduction in out-of-stock for product B. In the case of product C, however, the level of out-of-stock obtained is the same for both. That explains the close relationship between the performance of both forecasts, particularly in MAPE.

When we consider the aggregate results for the three products, the NN-ARMAX model achieves a 10% reduction in overstock compared to the base model and a 7% reduction for the ARMAX model. That represents a reduction in inventory maintenance and non-turnover capital work costs. The NN-ARMAX had a 65% out-of-stock rate in the overall average of the three products. This improvement allowed us to conclude that implementing the NN-ARMAX model has a positive economic impact on the increase in monthly incomes because out-of-stock due to forecasting accounts for approximately 46.5% of non-fulfillment of orders, which can be associated with lost sales.

5. Conclusions and Future Works

We presented a model to forecast the demand for electrical products in Colombia's construction industry. The model performed better than the company's current base forecasting model and had a favorable economic impact on demand management. Initially, we considered the external factors determined by the company to develop a multivariate statistical forecasting model. Later, we implemented a hybrid multivariate statistical and neural network forecasting model, outperforming the multivariate statistical and base models.

We build forecasting models for three relevant products selected based on annual sales volume and marginal revenue. Demand data of the selected products A, B, and C revealed that they exhibit similar behavior due to the influence of the same factors and show high variability.

The external factors were selected using the cross-correlation function, which identified the exogenous variables as independent variables for the proposed models. This procedure determined that the price announcement factor has a low correlation with demand. In contrast, the factors of total construction licenses approved, TRM USD/COP variation, price, and raw material stock showed relevant lags correlated with product demand. All products presented stationarity in the time series. Because their ACF and PACF displayed an exponential decay and damped sinusoidal behavior, autoregressive and moving average terms were included in the model as independent variables.

Furthermore, the inclusion of exogenous variables improved forecasting accuracy. The relevant variables for the ARMAX model were construction licenses and the MMR USD/COP variation. Other variables related to raw material price and the stock had no significance despite having relevant lags identified with the CCF function.

These findings confirm that the construction sector's economic dynamics directly impact the demand for the electrical products considered for this study. That makes sense because more approved construction licenses imply a higher short-term demand for construction products. Indeed, the MMR USD/COP impacts the economy by determining when to buy goods and services; the MMR affects the starting of construction work, its progress, and completion, which impacts the electrical product purchases. This study determined that only lags greater than seven months are relevant to demand, which makes sense given that the works can take six months or longer to begin once the licenses are approved.

Subsequently, the developed ARMAX model was used to create a hybrid model known as NN-ARMAX. During the data augmentation, we included an autoregressive variable in the input data to obtain the moving average variable through error predictions and input feedback. The hyperparameters were adjusted until the performance metrics yielded the best results in the test data. In this case, we discovered that the NN-ARMAX model outperformed the base model for demand forecasting for all products. Thus, we showed the advantages of autoregressive and moving average variables for demand forecasting using hybrid statistical/machine learning models.

Another reason the NN-ARMAX model outperformed the ARMAX model was the limitations on the exogenous variables allowed based on the *p*-value of the coefficients and the fulfillment of assumptions in the ARMAX model. Unlike the ARMAX model, the NN-ARMAX model can include all variables identified with the CCF function, allowing the model to extract more information from the available data and identify patterns in demand that linear or statistical models cannot. As a result, NN-ARMAX models had a more significant positive economic impact on demand management.

With the NN-ARMAX model, we got an overstock reduction of 10% and 7% compared with the base and the ARMAX models, respectively; this represents a reduction in inventory maintenance and working capital costs. Furthermore, we obtained a reduction in out-of-stock products of 80% and 65%, respectively, compared to the base and ARMAX models; this increases the company's monthly incomes because out-of-stock products account for approximately 46,5% of non-fulfillment orders.

This work can serve as a reference point for all those interested in using neural networks to improve univariate statistical demand models for demand forecasting. The methodology allows for identifying the relevant exogenous, autoregressive, and moving average terms by ensuring the best model through performance metrics obtained in the training and testing procedures.

Declaration of Conflicting Interests

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