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An Integrated Decision Making Model for Dynamic Pricing and Inventory Control of Substitutable Products Based on Demand Learning

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Abstract:

Purpose: This paper focuses on the PC industry, analyzing a PC supply chain system composed of one large retailer and two manufacturers. The retailer informs the suppliers of the total order quantity, namely Q, based on demand forecast ahead of the selling season. The suppliers manufacture products according to the predicted quantity. When the actual demand has been observed, the retailer conducts demand learning and determines the actual order quantity. Under the assumption that the products of the two suppliers are one-way substitutable, an integrated decision-making model for dynamic pricing and inventory control is established.

Design/methodology/approach: This paper proposes a mathematical model where a large domestic household appliance retailer decides the optimal original ordering quantity before the selling season and the optimal actual ordering quantity, and two manufacturers decide the optimal wholesale price.

Findings: By applying this model to a large domestic household appliance retail terminal, the authors can conclude that the model is quite feasible and effective. Meanwhile, the results of simulation analysis show that when the product prices of two manufacturers both reduce gradually, one manufacturer will often wait till the other manufacturer reduces their price to a crucial inflection point, then their profit will show a qualitative change instead of a real-time profit-price change.

Practical implications: This model can be adopted to a supply chain system composed of one large retailer and two manufacturers, helping manufacturers better make a pricing and inventory control decision.

Originality/value: Previous research focuses on the ordering quantity directly be decided. Limited work has considered the actual ordering quantity based on demand learning. However, this paper considers both the optimal original ordering quantity before the selling season and the optimal actual ordering quantity from the perspective of the retailer.

Keywords: promotion planning, substitutable products, variety choice, shelf-space allocation, inventory control

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1. Introduction

Currently, the computer industry in China is in a changing process and such characteristics of PC as a short lifecycle, frequent upgrades, multiple customer demands and demanding inventory management, entail the difficulty to achieve a perfect match between supply and demand and restrain the supply chain from responding quickly to new product needs, thus leading to a little or even minus profit of the PC industry supply chain. As a result, many firms have been transferring their focus from supply chain management to demand chain management, which is to implement effective inventory cost control and dynamic pricing for the purpose of coordinated operation of the supply chain by concentrating on factors that guide and affect demand.

PC is a type of short-lifecycle products. Studies of short-lifecycle products originated in the 1950s. Scarf (1959) used the dynamic pricing method to model a simple multi-period inventory pattern. This method requires the order quantity at the beginning of every period, and assumes that the stockholding cost, stockout cost and ordering cost are all linear and the demand is exponentially distributed. In addition, in the dynamic pricing method, the parameters are unknown and the distribution of parameters is updated by applying Bayesian Rules for every period. The proportion of short-lifecycle products in every period is increasing among the new products, accompanied by more diversified customer demands and fiercer competition. Accordingly, research relevant to short-lifecycle products is also on the rise and can be divided into three main areas: demand management, supply chain management (especially inventory management) and revenue management. In the area of demand management, studies focus on such topics as selection of demand prediction models, CPFR (cooperation plan, prediction and replenishment), quantification of Bullwhip Effect and value of information sharing; In the area of supply chain, related studies are about the multi-period dynamic inventory model, the inventory model of short-lifecycle products and the supply-chain contract of short-lifecycle products. In terms of the multi-period dynamic inventory model, related research includes analyzing historical data to update the Bayesian Model of demand distribution, the multi-period inventory model that inputs time series model (e.g. autoregression model ARMA), the dynamic inventory model under demand described by Markov Random process and dynamic inventory model based on demand prediction or prediction update. In terms of the inventory model of short-lifecycle products, studies are mainly composed of the inventory model of perishable products and that with two ordering chances. Studies on yield management mainly focus on dynamic pricing of demand prediction and dynamic pricing inventory decisions based on demand prediction study. Currently, models of short-lifecycle products tend to assume that there are updates for demand prediction and demand study, and there is a limit to the sales period. To be more specific, demand prediction describes how experiences, prediction models and appropriate methodologies can be used to forecast changes in market demand indicators and future trends, based on analysis of the relationship between past and present market demand conditions and factors that influence market demand.

It can be found that current models of short-lifecycle inventory management are of limited practicability because decision models fail to integrate with multiple decisions including demand prediction, pricing, inventory replenishment, multi-periods, nor do they find any effective technology to estimate demand distribution (parameters). This limitation entails that many models in this filed can only provide a theoretic perspective and there is no practical guide model yet.

This paper studies the PC industry and focuses on the supply chain comprised of two manufacturers, Lenovo and HP, and a large retailer. The retailer reserves total order quantity Q to Lenovo and HP before the selling season and the manufacturers of Lenovo and HP base their production on the reservation. In addition, after observing the actual demand quantity, the retailer decides the ordering quantity in line with research of demand and constructs a dynamic-pricing and inventory-control decision model on the assumption that two products provided by the two manufacturers are unilaterally substitutable. Finally, when this model is applied to a large domestic household appliance retailer, the result suggests that this model is feasible and effective. Furthermore, the sensitivity analysis also indicates that the decision-making model proposed in this paper can induce favorable a dynamic inventory and pricing strategy.

The remainder of this paper is structured as follows. Section 2 provides an overview of a literature review on demand forecasting and inventory control. Section 3 presents the model assumption and symbol description. Section 4 describes our integrated decision model of substitutable-product demand learning, dynamic pricing and stock controlling. Detailed results of the empirical analysis are provided in Section 5. Finally, in Section 6, we conclude the discussions in the paper and propose an agenda for future research in the area.

2. Literature Review

In the current literature context, most papers on demand prediction have been on information updates and profit maximizations. For instance, Zhang, Zhu and Gou (2017) put most emphasis on the effects of information sharing on price strategy. Besides, there are several models and methods developed but not suitable for PC industry. Chen, Chao and Ahn (2019) combine demand learning into the nonparametric algorithm. Just In Time (JIT) is more suitable for those long-life commodity or those one that worth waiting for (White & Prybutok, 2001). Katehakis, Yang and Zhou (2020) studies the pricing and inventory problem for nonperishable products considering the random demand. However, very few of them has studied specific demand distributions. Bradford and Sugrue (1990) were the first to apply the Bayesian Method to inventory theories for demand prediction. They focus on products with one or two periods and use the negative binomial distribution to solve optimal inventory strategies. Jorgensen, Kort and Zaccour (1999) introduces prediction factors into demand and production. They describe "demand prediction" as the current demand affected by previous demands, which means the current demand ratio is on the basis of a combination of both prices and accumulative sales. Almost at the same time, Federgruen and Heching (1999) point out that the periodic price and its distribution can be specified and quantified as a function and establish a model to compute the deduced optimal strategies. Gao and Yu (2009) put forward the inventory model of short-lifecycle products. It involves an inventory model with two ordering chances that appeals to many scholars. Gurnani and Tang (1999), under the assumption that demand is known, discuss distributors' optimal ordering strategy when they have two ordering chances; Chen and Xu (2001) come up with an improved rapid reaction strategy that has two ordering and two production chances, and they compare it with traditional production and operation models. Petruzzi and Dada (2002) analyze whether the parameters affecting the demand could be endogenous. Meanwhile, they bring about the probability of eliminating the negative influence from those uncertain parameters through learning. A variant of newsvendor model with price effects puts insight into the interrelationship between dynamic stocking and pricing, which is a cyclic check model, where the retailers allow reordering with changeable prices in every period. Chen and David (2004) introduce a new concept of the symmetric k-concave functions and apply it to characterize the optimal strategy with the assumption of random demand. Rad, Khoshalhan and Setak (2014) assume that the demand rate is sensitive to the price and propose an integrated production-inventory-marketing model.

Afterwards, a demand-based fluid model is proposed by Adida and Perakis (2006) to deal with the uncertainty with no backorders. They assume that the demand is a linear of the price, the inventory cost is also linear, the production cost is an increasing convex function of production rate and all the coefficients are time-related. According to Li, Xiong, Zhou and Xiong (2013), the fuzziness and randomness can be quantified to mixed variables and there are three forms of programming solver. The numerical result of their method suggests that this method can well solve the dynamic pricing problem. Moreover, Rana and Oliveira (2014) points out that utilizing the enhancing learning algorithm to learn pricing for related demands in order to maximize the inventory profit. Adida and Perakis (2010) combine game theory and Nash equilibrium with pricing problem under competition and uncertainty, which is popular in duopoly. It is the first paper that brings the competition and demand uncertainty together with the dynamic nature of the pricing problem. Nevertheless, Kuyumcu and Popescu (2006) consider the pricing and inventory control problem in case of substitutions. They come up with a concave quadratic objective function with linear constraints. According to Duong, Wood and Wang (2015), the inventory control problem among perishable and substitutable products can be solved by a centralized two-echelon model, which is extended from application for blood center. Kaya and Ghahroodi (2018) review the inventory system periodically with an age and price dependent random demand for perishable products to establish inventory and pricing strategies.

With the acceleration of product lifecycle, many scholars have directed their attention to research on short-lifecycle products and most of them concentrate on perishable products which belong to short-lifecycle ones (Fisher, Rajaram & Raman, 2001). The conclusion that Gompertz model is proved effective for demand forecasting rather than classical models has been drawn, which sets the foundation in this field. Mishra (2013) considers the fact that the deterioration rate can be managed by the technology and develops a model to decide an optimal replenishment period, shortage period and this technology cost. These studies are of some significance for the subject of this paper – dynamic inventory and pricing management of PC, but there are still problems to solve due to the differences between these two kinds of products, especially differences about demand characteristics.

3. Model Description

This paper studies the PC industry and focuses on the supply chain comprised of two manufacturers, Lenovo and HP, and a large retailer, which is a well-known domestic household appliances retailer. The retailer orders the PC products from the two manufacturers and then sells them to consumers. Demand is considered as one of the most important element in order quantity. In real life, the retailer must predict the demand to prepare the stock for the selling season. Based on historical information, the retailer firstly predicts the demand and considers the wholesale prices to decide the original order quantity Q to Lenovo and HP before the selling season. However, demand is uncertain. Therefore, after observing the actual demand in selling seasons, the retailer carries out demand learning and decides the actual order quantity q, which is also relevant to the original order quantity Q. If the original order quantity Q is overestimated by the retailer, the manufactures are likely to suffer from a loss because of excessive production. In order to maintain cost-effective, the manufacturer imposes the punishment cost on the retailer. Thus, the punishment cost of h will be incurred to the retailer for every unit of overproduction. On the other hand, when the actual order quantity q cannot meet the actual demand, an out-of-stock arises, leading the retailer to substitute the product. While a variety of substitution structures occur in actual application, in this study we will focus on one-way substitution, which is common in electronic product manufacturing industry (Bassok, Anupindi & Akella, 1999). For example, high-end Product 1 from the Manufacturer 1 (i.e., the HP manufacturer), can substitute low-end Product 2 from the Manufacturer 2 (i.e. the Lenovo manufacturer), but not vice versa. We assume that both Product 1 and Product 2 comply with the normal distribution. The symbols used in the model are shown in Table 1.

Subscript 1 and 2 stands for Manufacturers 1 and 2 respectively.

Decision variables	Definition
Q_1, Q_2	The original ordering quantity before the selling season
q_1, q_2	The actual ordering quantity ($q \leq Q$)
w_1, w_2	The wholesale price per unit
Parameters	**Definition**
x_1, x_2	Observed demand
h_1, h_2	Punishment cost per unit
p_1, p_2	The retail price per unit
c_1, c_2	The manufacturing cost per unit
π_r	The retailer's profit
π_{s_1} π_{s_2}	The manufacturer's profit

Table 1. Notation

4. The Integrated Decision Model of Substitutable Product Demand Learning, Dynamic Pricing and Stock Controlling

4.1. Basic model - The integrated decision-making model of one-product demand learning, dynamic pricing and inventory control

Before building the integrated decision-making model of substitutable products, we consider the situation with only one manufacturer. For convenience, the construction of the following model begins with the revenue function at the end retailer of the supply chain and is passed to the revenue function of the manufacturer. It should be noted that there is no inventory cost because our model is based on the newsboy model for perishable products, in which the inventory cost is not considered.

As shown in Equation (1), the retailer's profit equals their revenue minus the purchasing cost and the punishment cost due to a lower ordering quantity compared to expectations, in which the sales revenue is multiplying the selling price by the lower of actual ordering quantity q and observed demand x, therefore

$$\max_{q\le Q} \pi_r(q,Q,w) = p\cdot\min(q,x) - qw - h(Q-q) \tag{1}$$

As the original order quantity before the selling season Q and the actual order quantity q are both random variables, the retailer's profit is also a random variable. Additionally, for the reason that retailer's actual ordering quantity q is directly relevant to their observed demand and the original ordering quantity Q before seasons, to maximize Equation (1), we conclude that

$$q(Q,w) = \begin{cases} \min(x,Q) & w-h>0 \\ Q & w-h<0 \end{cases} \tag{2}$$

Using Equation (2) into Equation (1), then

$$\max_{Q} \pi_r(Q,w) = E\{p\cdot\min(q(Q,w),x) - q(Q,w)w - h(Q-q(Q,w))\} \tag{3}$$

In Equation (3), E stands for expected value.

Equation (4) presents the manufacturer's profit function, which is also a random variable due to the random distribution of Q and q. The function equals the manufacturers' sales revenue plus the punishment benefit minus the producing cost, in which the sales revenue is the selling price multiplied by the retailer's actual ordering quantity q; the manufacturer's profit is the function of Q and w. However, the retailer's original ordering quantity Q before seasons is determined by the wholesale price w, so

$$\max_{w} \pi_s(w) = E\{w\cdot q(Q(w),w) + h(Q(w) - q(Q(w),w))\} - c\cdot Q(w) \tag{4}$$

I. When $w - h < 0$, because $\frac{\partial^2 \pi_r}{\partial^2 Q} = -p\cdot f(Q) < 0$, we calculate the first derivative of Q in Equation (2) and assume it zero to find the optimal Q. In the following equation, the expectation E in Equation (3) is replaced with the probability density functions $f(x)$ and $f(Q)$. $F(Q)$ is the probability distribution function derived from $f(Q)$.

$$\begin{aligned} \frac{\partial \pi_r}{\partial Q} &= p\cdot(Q\cdot f(Q) + \int_Q^{+\infty} f(x)dx - Q\cdot f(Q)) - w \\ &= p\cdot(1-F(Q)) - w = 0 \end{aligned}$$

then

$$Q^* = F^{-1}(1-\frac{w}{p}) \tag{5}$$

Using Equation (5) into Equation (4), we can learn

$$\begin{aligned} \max_{w} \pi_s(Q,w) &= E(w\cdot Q) - c\cdot Q \\ &= w\cdot Q - c\cdot Q \\ &= (w-c)\cdot F^{-1}(1-\frac{w}{p}) \end{aligned} \tag{6}$$

II. When $w - h > 0$, because $\frac{\partial^2 \pi_r}{\partial^2 Q} = -(p-w+h)f(Q) < 0$, so we can calculate the first derivative of Q in Equation (2) and assume it zero to find the optimal Q, that is

$$\frac{\partial \pi_r}{\partial Q} = (p-w+h)(1-F(Q)) - h = 0$$

therefore

$$Q^* = F^{-1}(\frac{p-w}{p-w+h}) \tag{7}$$

Using Equation (7) in Equation (4), we can get

$$\begin{aligned} \max_{w} \pi_s(Q,w) &= E\{w \cdot \min(Q,x) + h(Q - \min(Q,x))\} - c \cdot Q \\ &= hQ + (w-h)E\{\min(Q,x)\} - cQ \\ &= hQ + (w-h)\cdot(\int_0^Q x \cdot f(x)dx + Q \cdot \int_Q^{+\infty} f(x)dx) - cQ \\ &= (h-c)Q + (w-h)\cdot(\int_0^Q x \cdot f(x)dx + Q \cdot \int_Q^{+\infty} f(x)dx) \end{aligned} \tag{8}$$

4.2. The Integrated Decision Model of Substitutable-Product Demand Learning, Dynamic Pricing and Stock Controlling

Based on the basic model, we consider that there are two manufacturers simultaneously providing substitutable but not identical PC products to the retailer, then Equation (1) can be transformed into the following

$$\begin{aligned} &\max_{\substack{q_1 \le Q_1 \\ q_2 \le Q_2}} \pi_r(q_1, Q_1, w_1, q_2, Q_2, w_2) \\ &= p_1 \cdot \min(q_1, x_1) - q_1 w_1 - h_1(Q_1 - q_1) \\ &+ p_2 \cdot \min(q_2 + (q_1 - x_1)^+, x_2) - q_2 w_2 - h_2(Q_2 - q_2) \end{aligned} \tag{9}$$

where $(q_1 - x_1)^+ = \max\{ q_1 - x_1, 0\}$, it represents the number of Product 1 can be used to replace Product 2. The similar expressions in following equations have the same mathematical implication.

According to Equation (2), Equation (9) can be transformed to Equation (10). We can learn that the retailer's profit is the function of the original ordering quantity Q_1, Q_2 they promise the two manufacturers before seasons and their wholesale prices w_1, w_2, therefore

$$\begin{aligned} \max_{Q_1,Q_2} \pi_r(Q_1, w_1, Q_2, w_2) = E\{&p_1 \cdot \min(q_1(Q_1,w_1), x_1) - q_1(Q_1,w_1)w_1 - h_1(Q_1 - q_1(Q_1,w_1))\} \\ &+ E\{p_2 \cdot \min(q_2(Q_2,w_2) + (q_1(Q_1,w_1) - x_1)^+, x_2) \\ &- q_2(Q_2,w_2)w_2 - h_2(Q_2 - q_2(Q_2,w_2))\} \end{aligned} \tag{10}$$

Equation (4) can be transformed to two profit functions of the two manufacturers. It should be pointed out that, since this paper and model is from the retailer's perspective, due to the one-way substitutable relationship between two products, there is no direct game between two manufacturers. Their profits are functions relevant to Q and w. However, the retailer's original ordering quantity Q before seasons is determined by the wholesale price w, so

$$\begin{aligned} \max_{w_1} \pi_{s_1}(w_1, w_2) = E\{&w_1 \cdot q_1(Q_1(w_1,w_2), w_1) + h_1(Q_1(w_1,w_2) \\ &- q_1(Q_1(w_1,w_2), w_1))\} - c_1 \cdot Q_1(w_1,w_2) \end{aligned} \tag{11a}$$

$$\begin{aligned} \max_{w_2} \pi_{s_2}(w_1, w_2) = E\{&w_2 \cdot q_2(Q_2(w_1,w_2), w_2) + h_2(Q_2(w_1,w_2) \\ &- q_2(Q_2(w_1,w_2), w_2))\} - c_2 \cdot Q_2(w_1,w_2) \end{aligned} \tag{11b}$$

Considering $w_i > h_i$ $(i = 1,2)$, where punishment is below the wholesale price, it is also reasonable that

I. when $Q_1 \le x_1$, $Q_2 \le x_2$, $\pi_r = p_1 \cdot q_1 - q_1 \cdot w_1 - h_1(Q_1 - q_1) + p_2 \cdot q_2 - q_2 \cdot w_2 - h_2(Q_2 - q_2)$. Then, $q_1 = Q_1$, $q_2 = Q_2$.

II. when $Q_1 \le x_1$, $Q_2 > x_2$, $\pi_r = p_1 \cdot q_1 - q_1 \cdot w_1 - h_1(Q_1 - q_1) + p_2 \cdot x_2 - q_2 \cdot w_2 - h_2(Q_2 - q_2)$. Now, $q_1 = Q_1$, $q_2 = x_2$.

III. When $Q_1 \le x_1$, $q_1 < x_1$, obviously π_r strictly increases with q_1, so we only need to discuss the optimum solution in the $x_1 \le q_1 \le Q_1$ range, currently

$$\begin{aligned}\pi_r &= p_1x_1 - q_1w_1 - h_1(Q_1 - q_1)\\ &+p_2\cdot\min(q_2+q_1-x_1,x_2)-q_2w_2-h_2(Q_2-q_2)\end{aligned}$$

The Feasible region can be further divided into two parts: $0 \le q_2 \le x_1 + x_2 - q_1$, $Q_2 \ge q_2 > x_1 + x_2 - q_1$.

a. When $Q_2 \ge q_2 > x_1 + x_2 - q_1$,

$$\begin{aligned}\pi_r &= p_1x_1 - q_1w_1 - h_1(Q_1 - q_1)\\ &+p_2\cdot\min(q_2+q_1-x_1,x_2)-q_2w_2-h_2(Q_2-q_2)\end{aligned}$$

Since π_r apparently strictly decreases with q_2, it is impossible to calculate the optimum solution within the range of $Q_2 \ge q_2 > x_1 + x_2 - q_1$, so we can only discuss the $q_2 \le x_1 + x_2 - q_1$ range.

b. When $0 \le q_2 \le x_1 + x_2 - q_1$ (now it is required that $q_1 \le x_1 + x_2$, otherwise $x_1 + x_2 - q_1 \le 0$, the solution to q_2 is an empty set),

$$\begin{aligned}\pi_r &= p_1x_1 - q_1w_1 - h_1(Q_1 - q_1) + p_2(q_2+q_1-x_1) - q_2w_2 - h_2(Q_2-q_2)\\ &= (p_1-p_2)x_1 - h_1Q_1 - h_2Q_2 + (p_2-(w_1-h_1))q_1 + (p_2-(w_2-h_2))q_2\end{aligned}$$

Apparently, π_r increases with q_2, $q_2 = \min\{x_1 + x_2 - q_1, Q_2\}$, therefore

$$\begin{aligned}\pi_r &= (p_1-p_2)x_1 - h_1Q_1 - h_2Q_2\\ &\quad+(p_2-(w_1-h_1))q_1 + (p_2-(w_2-h_2))\min\{x_1+x_2-q_1,Q_2\}\end{aligned}$$

There, $x_1 \le q_1 \le \min\{x_1 + x_2, Q_1\}$.

Discussion will be based on three situations:

I. When $w_1 - h_1 \ge p_2$, π_r decreases with q_1, hence, $q_1 = x_1$, $q_2 = \min\{x_1 + x_2 - q_1, Q_2\} = \min\{x_2, Q_2\}$;

II. When $w_2 - h_2 \le w_1 - h_1 \le p_2$, if $x_1 \le q_1 \le x_1 + x_2 - Q_2$, π_r increases with q_1, otherwise π_r decreases with q_1, therefore $q_1 = \min\{x_1 + (x_2 - Q_2)^+, Q_1\}$, $q_2 = \min\{x_1 + x_2 - q_1, Q_2\} = \min\{x_2, Q_2\}$, where $(x_2 - Q_2)^+ = \max\{x_2 - Q_2, 0\}$, it represents the number of Product 2 that needs to be replaced with Product 1.

III. When $w_1 - h_1 \le w_2 - h_2$, $q_2 = \min\{x_1 + x_2 - q_1, Q_2\} = \min\{(x_1 + x_2 - Q_2)^+, Q_2\}$.

To conclude based on the three above mentioned situations I、II、III, the optimum strategy is shown below.

Three situations	The optimum strategy
$w_1 - h_1 \ge p_2$	$q_1 = \min\{x_1, Q_1\}$, $q_2 = \min\{x_2, Q_2\}$
$w_2 - h_2 \le w_1 - h_1 \le p_2$	$q_1 = \min\{x_1 + (x_2 - Q_2)^+, Q_1\}$, $q_2 = \min\{x_2, Q_2\}$
$w_1 - h_1 \le w_2 - h_2$	$q_1 = \min\{x_1 + x_2, Q_1)\}$, $q_2 = \min\{(x_2 - (Q_2 - x_1)^+)^+, Q_2\}$

Table 2. Conclusion for optimum strategy of three mentioned situations

I. When $w_1 - h_1 \ge p_2$,

$$\begin{aligned}&\max_{Q_1,Q_2} \pi_r(Q_1,w_1,Q_2,w_2)\\ &= E\{p_1\cdot\min(q_1(Q_1,w_1),x_1) - q_1(Q_1,w_1)w_1 - h_1(Q_1 - q_1(Q_1,w_1))\}\\ &\quad+E\{p_2\cdot\min(q_2(Q_2,w_2)+(q_1(Q_1,w_1)-x_1)^+,x_2)\\ &\quad-q_2(Q_2,w_2)w_2 - h_2(Q_2-q_2(Q_2,w_2))\}\\ &= E\{p_1\cdot\min(Q_1,x_1) - w_1\min(Q_1,x_1) - h_1(Q_1-\min(Q_1,x_1))\}\\ &\quad+E\{p_2\cdot\min(Q_2,x_2) - w_2\min(Q_2,x_2) - h_2(Q_2-\min(Q_2,x_2))\}\\ &= (p_1+h_1-w_1)E\min(Q_1,x_1) + (p_2+h_2-w_2)E\min(Q_2,x_2)\\ &-h_1Q_1 - h_2Q_2\end{aligned}$$

II. When $w_2 - h_2 \le w_1 - h_1 \le p_2$,

$$\begin{aligned}
&\max_{Q_1,Q_2} \pi_r(Q_1,w_1,Q_2,w_2) \\
&= E\{p_1 \cdot \min(q_1(Q_1,w_1),x_1) - q_1(Q_1,w_1)w_1 - h_1(Q_1 - q_1(Q_1,w_1))\} \\
&\quad + E\{p_2 \cdot \min(q_2(Q_2,w_2) + (q_1(Q_1,w_1)-x_1)^+, x_2) \\
&\quad - q_2(Q_2,w_2)w_2 - h_2(Q_2 - q_2(Q_2,w_2))\} \\
&= E\{p_1 \cdot \min(Q_1,x_1) - (w_1 - h_1)\min\{x_1 + (x_2 - Q_2)^+, Q_1\} - h_1 Q_1\} \\
&\quad + E\{p_2 \cdot \min(Q_2 + (Q_1 - x_1)^+, x_2) - (w_2 - h_2)\min\{x_2, Q_2\} - h_2 Q_2\}
\end{aligned}$$

III. When $w_1 - h_1 \le w_2 - h_2$,

$$\begin{aligned}
&\max_{Q_1,Q_2} \pi_r(Q_1,w_1,Q_2,w_2) \\
&= E\{p_1 \cdot \min(q_1(Q_1,w_1),x_1) - q_1(Q_1,w_1)w_1 - h_1(Q_1 - q_1(Q_1,w_1))\} \\
&\quad + E\{p_2 \cdot \min(q_2(Q_2,w_2) + (q_1(Q_1,w_1)-x_1)^+, x_2) \\
&\quad - q_2(Q_2,w_2)w_2 - h_2(Q_2 - q_2(Q_2,w_2))\} \\
&\max_{Q_1,Q_2} \pi_r(Q_1,w_1,Q_2,w_2) \\
&= E\{p_1 \cdot \min(q_1(Q_1,w_1),x_1) - q_1(Q_1,w_1)w_1 - h_1(Q_1 - q_1(Q_1,w_1))\} \\
&\quad + E\{p_2 \cdot \min(q_2(Q_2,w_2) + (q_1(Q_1,w_1)-x_1)^+, x_2) \\
&\quad - q_2(Q_2,w_2)w_2 - h_2(Q_2 - q_2(Q_2,w_2))\}
\end{aligned}$$

It is hard to find the analytical solution of Q1 and Q2, because their formulas are very complicated. Therefore, the optimized approximation solution is obtained by using some optimization algorithms.

5. Empirical Analysis and Sensitivity Analysis

This model uses actual sales data of PC products over the past 15 month (68 weeks in total) from a well-known household appliance retailer in Shanghai. The specific property information of two products is shown in Table 3.

We select two similar products from HP and Lenovo respectively as our research subjects, among which Product 1 represents HP's product and Product 2 represents Lenovo's product. When Product 2 is out of stock, the retailer would like to sell high-quality Product 1 at the price of Product 2. The specific property information of these two products is presented in Table 3. And the initial prices of Product 1 and Product 2 are ¥5030 and ¥4600. The basic parameters of two products are shown in Table 4.

Brand	HP	Lenovo
CPU	AMD Sempron 3200+ 1.8 GHz	AMD Athlon64 3000+ 1.8Ghz
RAM	512M	256M
Graphic Card	Integrated graphics card	NVIDIA GeForce 6200TC 128M
Hard Disk	80g/7200	80g/7200
Initial Price	¥5,030	¥4,600

Table 3. Property information of Product 1 and Product 2

	Parameter	Product 1	Product 2
Demand	k_i	1939.14	365.04
	α_i	0.54	0.45
Cost	C_i	$0.80 * w_1$	$0.85 * w_1$
	h_i	$0.20 * \overline{P}_1$	$0.15 * \overline{P}_2$
	B_i	10000	10000

Table 4. Information about

The main method used for model parameter estimation is demand-fitting method which is based on attribute analysis (Gao, Shi & Liu, 2012). The parameters above are based on the sales data of products from the first period to the seventeenth period. In terms of the decision-making power, the manufacturer decides the wholesale price w_i; the retailer decides the original ordering quantity before the selling season Q, and the actual ordering quantity q. Besides, the h_i and $\overline{P}_i$ are decided by the negotiation between manufacturers and retailers.

When Product 2 of Manufacturer 2, namely Lenovo manufacturer is out of stock, it can be substituted by Product 1 from Manufacturer 1, namely HP manufacturer. Using the two parameters above into the model, we present the calculation results as the results of sensitivity analysis after dynamic regulation of different products' prices. It is worth noting that the retailer and the two manufacturers are at a loss, that is, the profit is negative due to the fact that the appliance store is at the period of fighting for PC's market share.

5.1. The Manufacturer's Profit Change Based on Different Price Combinations of Two Products

We can see that as HP and Lenovo gradually reduce the price of PC, the computer makers of these two brands are at a loss in Figure 1 and Figure 2.

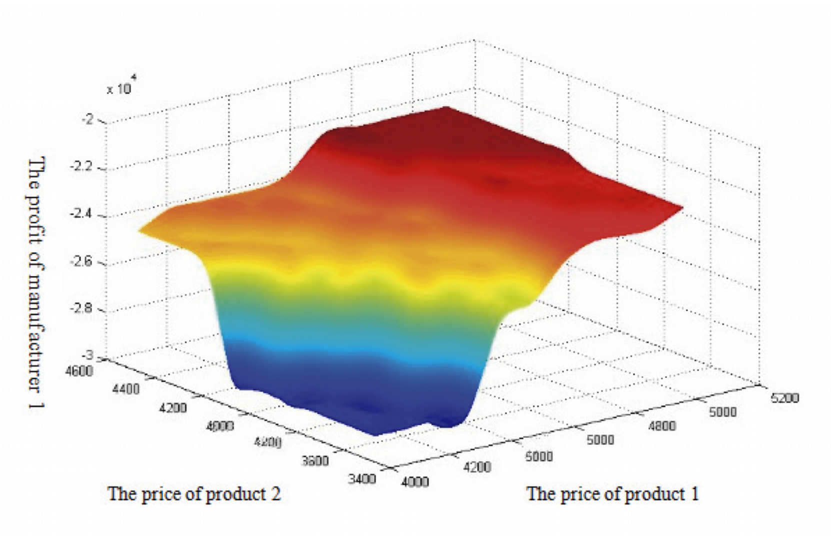

Figure 1. The curve of Manufacturer 1's profit change with Lenovo computer and HP computer prices

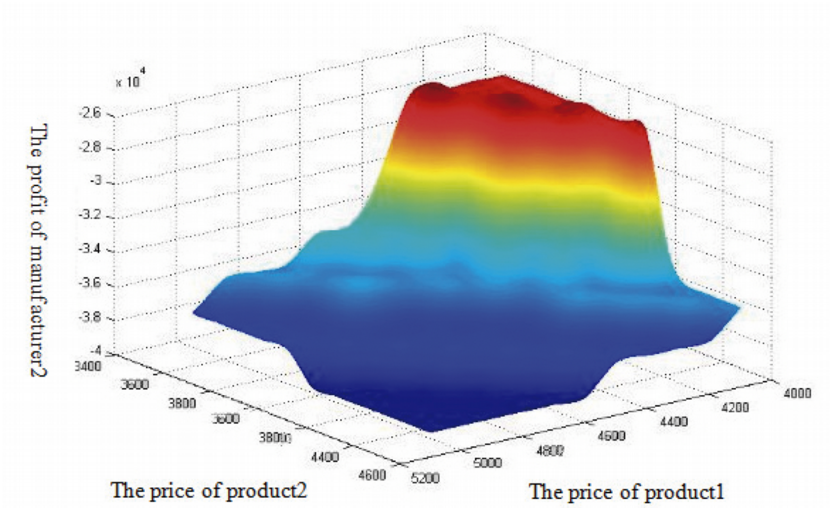

Figure 2. The curve of Manufacturer 2's profit change with Lenovo computer and HP computer prices

The difference is, the profit of HP's computer maker experiences a downward trend while that of Lenovo's computer maker undergoes a gradual rise. The detailed information about key nodes is presented in Table 5 and Table 6.

The price of Product1	5030	4730	4430
The first turning point of profit reduction of Manufacturer1	4000	4300	4100
The percentage of profit reduction	2.05%	2.05%	2.60%
The second turning point of price reduction of Manufacturer1	-	3900	3800
The percentage of profit reduction	-	2.60%	3.16%
The third turning point of profit reduction of Manufacturer1	-	-	3600
The percentage of profit reduction	-	-	2.36%

Table 5. The situation of Manufacturer1's profit change with the Lenovo and HP computer prices

The price of Product1	5030	4730	4430
The first turning point of profit increase of Manufacturer 2	4000	4300	4100
The percentage of the increase of profit	4.213%	4.213%	4.069%
The second turning point of profit increase of Manufacturer 2	-	3900	3800
The percentage of profit increase	-	4.069%	3.896%
The third turning point of profit increase of Manufacturer2	-	-	3600
The percentage of profit increase	-	-	7.145%

Table 6. The situation of Manufacturer2's profit change with the Lenovo and HP computer prices

5.2. The Total Amount of Retailers' Order and its Effect on Profit Under Different Price Combinations of Two Products

We can see from Figure 3 and Figure 4 that as HP and Lenovo gradually reduce the price of PC, Q2 falls accordingly, while Q1 does not change obviously. The detailed information about key nodes is presented in Table 7 and Table 8.

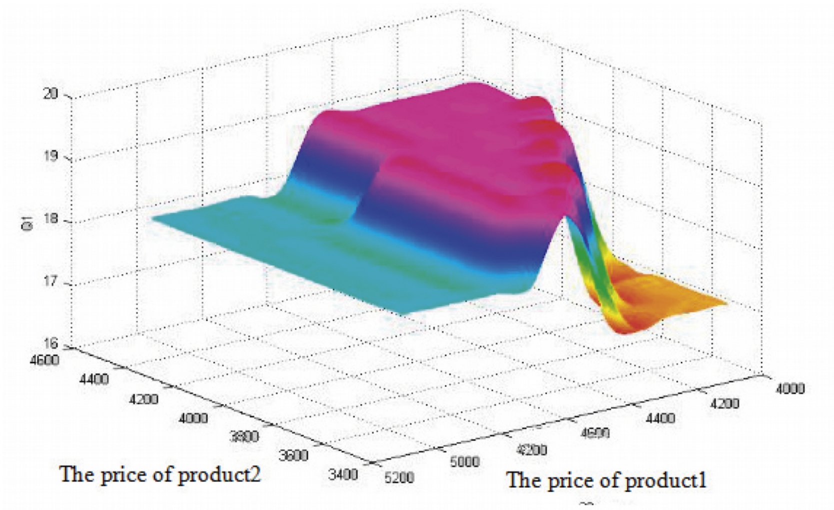

Figure 3. The curve of the total order quantity Q1 that the retailer reserves to Manufacturer 1 before the selling season as Lenovo and Hewlett Packard computer prices change

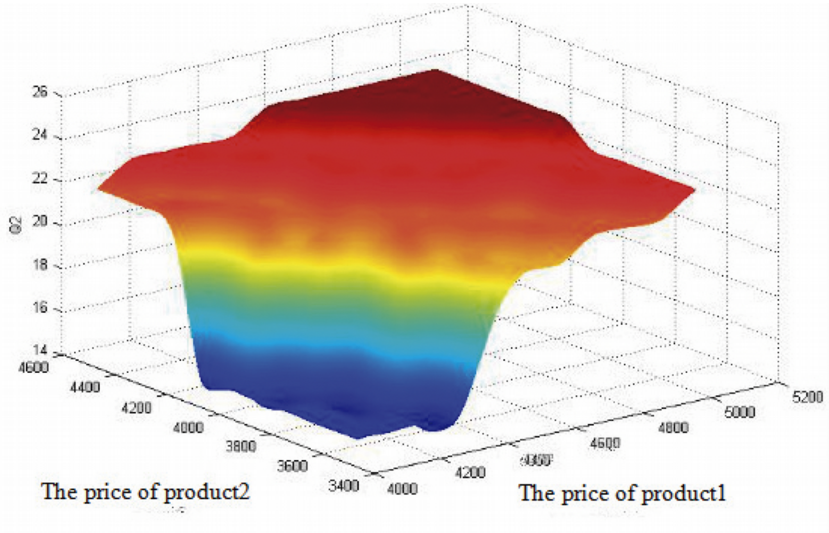

Figure 4. The curve of the total order quantity Q2 that the retailer reserves to Manufacturer 2 before the selling season as Lenovo and Hewlett Packard computer prices change

The price of Product1	5030	4730	4430
The first turning point of Q1 reduction	-	-	3600
The percentage of Q1 reduction	-	-	1
The second turning point of Q2 reduction of Manufacturer 1	-	-	-
The percentage of Q1 reduction	-	-	-
The third turning point of Q1 reduction of Manufacturer 1	-	-	-
The percentage of Q1 reduction	-	-	-

Table 7. The change of Q1 with the change of Lenovo and HP computer prices

The price of Product1	5030	4730	4430
The first turning point of Q2 reduction of Manufacturer 2	4000	4300	4100
The number of Q2 reduction	1	1	1
The second turning point of Q2 reduction of Manufacturer 2	-	3900	3800
The number of Q2 reduction	-	1	1
The third turning point of Q2 reduction of Manufacturer 2	-	-	3600
The number of Q2 reduction	-	-	2

Table 8. The change of Q2 with the change of Lenovo and HP computer prices

Figure 5 presents the information about profit based on different combinations of Q1 and Q2.

6. Conclusion and Management Implications

This paper studied the personal computer industry and focused on the supply chain composed of two manufacturers and one retailer, and finally put forward one kind of integrated decision-making model for dynamic inventory and dynamic pricing which put alternative PC products into consideration. By applying this model to a large domestic household appliance retail terminal, we can conclude based on the calculation results that the model is quite feasible and effective. The results of simulation analysis provide us with management implications in the following two aspects:

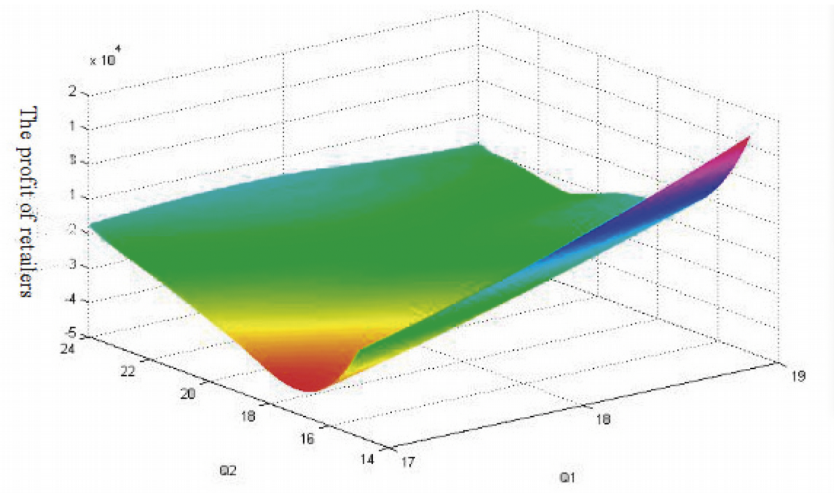

Figure 5. The curve of the retailer's profit change with the change of Q1 and Q2

1. From Simulation Analysis 1, when the computer prices of Lenovo and Hewlett Packard both reduce gradually, Hewlett Packard computer manufactures' profits decrease but at the same time Lenovo computer manufacturers' profits increase. In other words, one party will often wait till the other party reduces its price to a crucial inflection point, then its profit will show a qualitative change instead of a real-time profit-price change. Therefore, it is important for manufacturers to make dynamic pricing decisions in a fully competitive retail environment, and make pricing adjustment to meet the key node so that this adjust will have an influence on demand and profit.
2. From Simulation Analysis 2, when the computer prices of Lenovo and Hewlett Packard change, Q1 experiences a phasic change while Q2 is very sensitive to the price change, with a massive volume change of floating up and down. Therefore, when the retailers place orders to the manufacturers of Hewlett Packard before the sales season, they can sign more rigid contracts with a certain floating rate. However, when the retailers order Lenovo products, which are regarded as low-end products, to the manufacturers of Lenovo before the sales season, they have to sign more flexible contracts to ensure that the price fluctuations will not have an effect on profit, or sign dynamic contracts which allow dynamic updates and adjustments for total quantity as long as the total contract amount remains unchanged.

This research can be further extended by considering demand learning or by combining the lifecycle features and the integrated decision-making research of demand function's dynamic inventory and dynamic pricing.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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