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# Fill Rate Estimation in Periodic Review Policies with Lost Sales Using Simple Methods

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***Abstract:***

***Purpose:*** The exact estimation of the fill rate in the lost sales case is complex and time consuming. However, simple and suitable methods are needed for its estimation so that inventory managers could use them.

***Design/methodology/approach:*** Instead of trying to compute the fill rate in one step, this paper focuses first on estimating the probabilities of different on-hand stock levels so that the fill rate is computed later.

***Findings:*** As a result, the performance of a novel proposed method overcomes the other methods and is relatively simple to compute.

***Originality/value:*** Existing methods for estimating stock levels are examined, new procedures are proposed and their performance is assessed.

***Keywords:*** fill rate, lost sales, periodic review policy

## 1. Introduction and Literature Review

Inventory models in the lost sales context are harder to formulate than backordering models because the loss of unfulfilled demand is more difficult to represent and manage as pointed out by Bijvank, Huh, Janakiraman and Kang (2014) and Zipkin, 2008a, 2008b. However, the lost sales case is quite common in sectors and industries where customers are not prepared for waiting when a stockout occurs as for example in the retail industry or in ecommerce.

Three papers can be found in the lost sales context that provide a way to estimate the probabilities of the on-hand stock levels although their aim is quite different. For example, Cardós, Miralles and Ros (2005) propose the only known method that provides an exact method to compute the Cycle Service Level in a discrete lost sales demand context and also provides the exact on-hand stock levels. This method, named *Exact Method* further on, is based on the probability transition matrixes of the on-hand stock levels from the beginning of each cycle to its end; the convergence of the resulting Markov chain provides the on-hand probability vector at the beginning of the cycle. Cardós and Babiloni (2011) derive an approximation of the Cycle Service Level assuming that there are no stockouts during the lead time so that the probability vector of the on-hand stock levels at the beginning of the cycle is computed as in the backordering case. This method, named *M1 method* henceforth, can be used for any discrete demand distribution. Bijvank and Johansen (2012) propose an approximation procedure for computing the average on-hand stock when demand is compound Poisson. This method, named *B&J method* below, basically starts from the on-hand stock probabilities as in the backlog case but multiplied by a correction factor in order to provide the average stock obtained applying the Little's Law.

The aim of this paper is to derive and evaluate procedures to compute the fill rate based on: (a) an estimation of the on-hand stock levels at the beginning of the cycle; (b) in the lost sales context and with any discrete demand distribution; and (c) easy to implement in practical environments.

The rest of the paper is organized as follows. Section 2 presents the basic notation and assumptions, Section 3 proposes four new methods to estimate the on-hand stock level probabilities at the beginning of the cycle and Section 4 evaluates the performance of the existing and new methods when used to estimate the fill rate. Finally, Section 5 highlights the conclusions of this work.

## 2. Notation and Assumptions

Periodic review policies place replenishment orders every R time periods such that the on-hand stock plus outstanding orders reach the order-up-to level S. The order is received L time periods later. Figure 1 shows an example (a) where no stockout occurs and another example (b) showing a stockout. The notation in Figure 1 and in the rest of the paper is as follows:

S = order-up-to level,

R = review period and replenishment cycle,

L = lead time for the replenishment order,

$OH_t$ = on-hand stock at time t,

$D_t$ = accumulated demand during t consecutive periods,

$X^+$ = maximum {X, 0} for any expression X,

$f_t(\cdot)$ = probability mass function of demand at t,

$F_t(\cdot)$ = cumulative distribution function of demand during t periods.

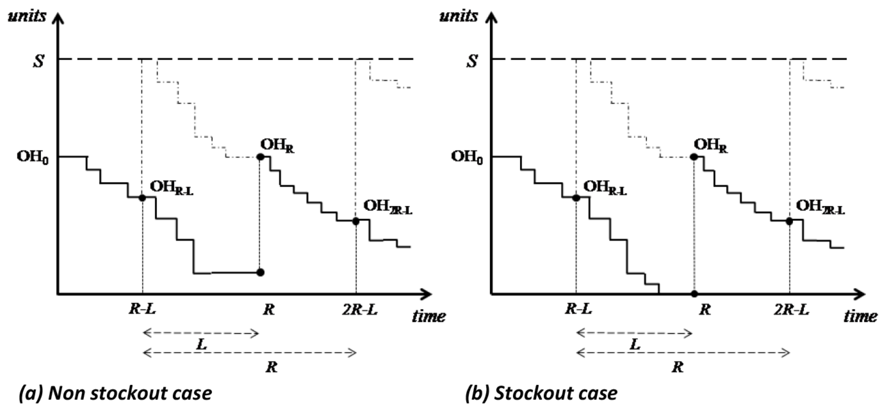


Figure 1. Periodic review system with lost sales with and without stockout

General assumptions of this paper are: (i) time is discrete and is organized in a numerable and infinite succession of equally spaced instants; (ii) the lead time and the review period are constant and known; (iii) there is never more than one outstanding order, leading to L<R; (iv) the replenishment orders are received at the end of the period; (v) demand is fulfilled with the on-hand stock at the beginning of the period; (vi) demand is stationary and i.i.d. and follows any discrete distribution function; and (vii) unfulfilled demand is lost.

## 3. Proposed Methods

### 3.1. Fill Rate Estimation

Guijarro, Cardós and Babiloni (2012) compute the fill rate using a classical approach such as

$$\beta = 1 - \frac{\sum_{i=0}^{S} P\left(OH_R = i\right) \cdot \sum_{j=i+1}^{\infty} \left(j-i\right) \cdot f_R\left(j\right)}{\sum_{j=1}^{\infty} j \cdot f_R\left(j\right)} \tag{1}$$

However, we use the expression below because of its superior numerical stability and adherence to the fill rate definition itself

$$\beta = \frac{\sum_{i=0}^{S} P\left(OH_R = i\right) \cdot \sum_{j=1}^{i} \left\{ j \cdot f_R\left(j\right) + i\left[1 - F_R\left(i\right)\right] \right\}}{\sum_{j=1}^{\infty} j \cdot f_R\left(j\right)} \tag{2}$$

### 3.2. On-hand Estimation Methods

*M1 method* assumes no stockout during the cycle, so the on-hand stock probabilities at the beginning of the cycle are

$$\begin{aligned} &OH_{R-L} = \left[OH_0 - D_{R-L}\right]^+ \\ &OH_R = \left[OH_0 - D_R\right]^+ + S - OH_{R-L} = S - D_L \\ &P(OH_R = i) = \begin{cases} f_L(S-i) & i = 1,\ldots,S \\ 1 - F_L(S-1) & i = 0 \end{cases} \end{aligned} \tag{3}$$

These probabilities can be expressed as a vector

$$\overline{P(OH_1)} = \begin{pmatrix} 1 - F_L(S-1) & f_L(S-1) & \ldots & f_L(0) \end{pmatrix} \tag{4}$$

*M2 method*, the first proposed method, is the opposite of *M1 method* because now we assume that there is a stockout as soon as possible, before the stock is reviewed

$$OH_{R-L} = \left[OH_0 - D_{R-L}\right]^+ = 0 \Rightarrow OH_R = S \tag{5}$$

$$\overline{P(OH_2)} = \begin{pmatrix} 0 & \ldots & 0 & 1 \end{pmatrix} \tag{6}$$

Based on *M1* and *M2 methods*, *M3 method* proposes another estimate of the probability vector based on the probability of occurring the assumptions of *M2*

$$P(OH_0 - D_{R-L} \leq 0) = P(D_{R-L} \geq S) = 1 - F_{R-L}(S-1) = \beta \tag{7}$$

$$\overline{P(OH_3)} = \beta \cdot \overline{P(OH_2)} + (1-\beta) \cdot \overline{P(OH_1)} \tag{8}$$

*M4 method* is based on improving this idea using the probability of *M1* assumptions

$$\begin{aligned} P(OH_0 \geq D_R) &= \sum_{i=0}^{S} P(OH_0 \geq D_R) P(OH_0 = i) = \\ &= \sum_{i=1}^{S} F_R(i) \cdot f_L(S-i) + F_R(0) \cdot [1 - F_L(S-1)] = \alpha \end{aligned} \tag{9}$$

$$\overline{P(OH_4)} = \frac{\beta}{\alpha+\beta} \cdot \overline{P(OH_2)} + \frac{\alpha}{\alpha+\beta} \cdot \overline{P(OH_1)} \tag{10}$$

Finally, *M5 method* simplifies the calculation of *a* and *β* assuming that *a=β* and *a+β=1* so that

$$\overline{P(OH_5)} = \frac{1}{2} \cdot \overline{P(OH_2)} + \frac{1}{2} \cdot \overline{P(OH_1)} \tag{11}$$

## 4. Experimental Evaluation

The rationale for every method is quite different, so it is necessary to assess their accuracy in terms of their deviations from the exact fill rate. We perform an extensive experiment including smooth, intermittent, lumpy and erratic demand but also a wide combination of stock policy parameters as seen in Table 1. This dataset provides 12,348 cases whose fill rate estimates are represented in Figure 2.

| **Demand distribution** | | |
|---|---|---|
| Poisson | $\lambda$ | 0.01, 0.1, 0.5, 1, 2, 5, 10 |
| Negative Binomial | $r$ | 0.1, 0.25, 0.5, 0.75, 1, 1.25, 1.5, 3 |
| | $\theta$ | 0.1, 0.25, 0.3, 0.4, 0.5, 0.75, 0.9 |
| **Inventory system** | | |
| R | | 2, 3, 5, 10, 15, 20, 30 |
| L | | 1, 3, 5, 10, 15, 20 |
| S | | 1, 3, 5, 10, 15, 20, 30 |

Table 1. Dataset of the experiment

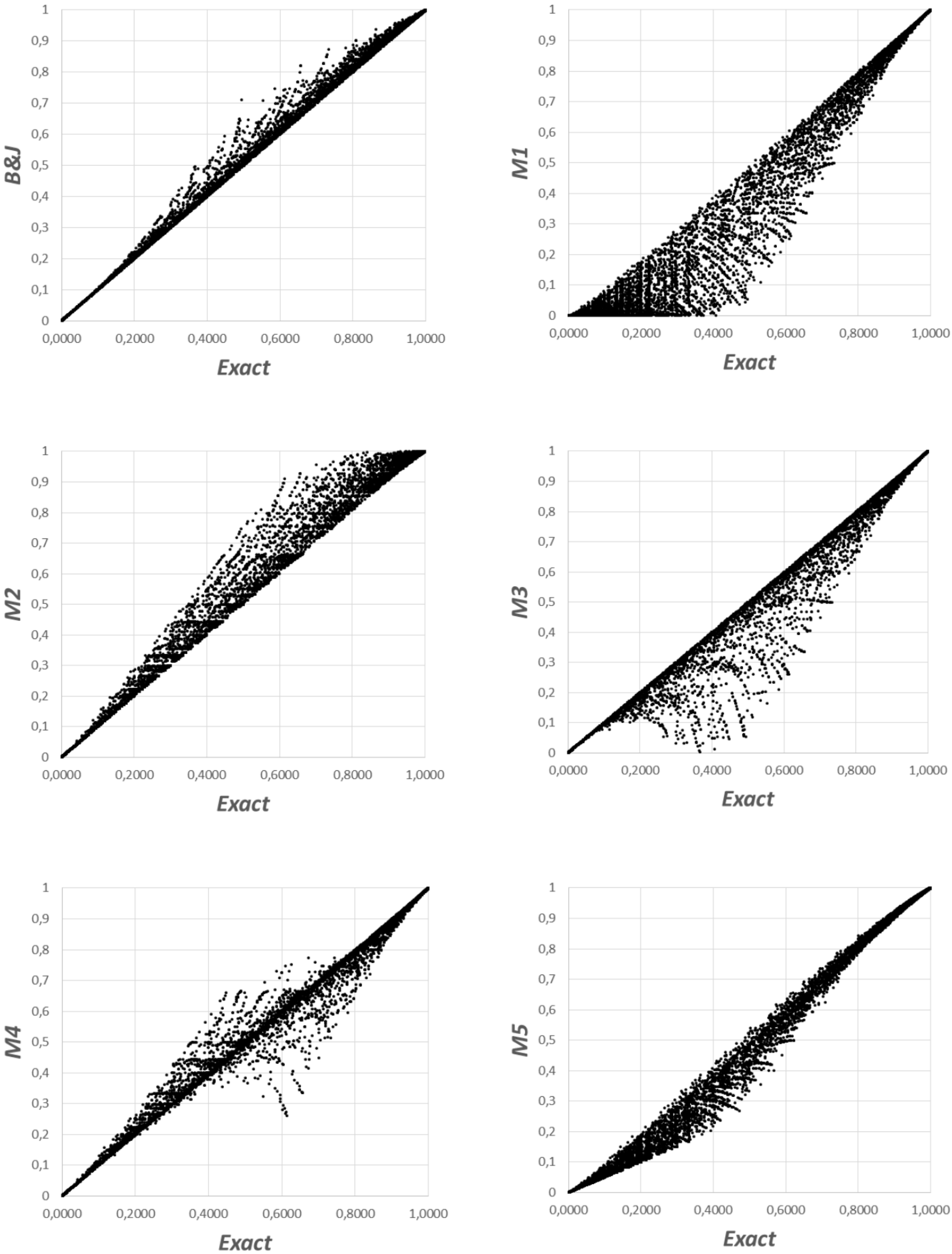


Figure 2. Fill rate estimates against the exact value for the considered methods

Additionally, Table 2 analyze the average, standard, maximum and minimum deviations for the estimation methods. This analysis just includes cases with a fill rate between 0.50 and 0.99 because cases out of this range have no practical interest.

| FR | B&J | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|---|
| 0.50-0.55 | 0.0238 | -0.1689 | 0.0707 | -0.0813 | 0.0003 | -0.0491 |
| 0.55-0.60 | 0.0271 | -0.1557 | 0.0803 | -0.0867 | -0.0185 | -0.0377 |
| 0.60-0.65 | 0.0250 | -0.1303 | 0.0721 | -0.0696 | -0.0288 | -0.0291 |
| 0.65-0.70 | 0.0271 | -0.1124 | 0.0859 | -0.0722 | -0.0422 | -0.0132 |
| 0.70-0.75 | 0.0278 | -0.0936 | 0.0824 | -0.0632 | -0.0426 | -0.0056 |
| 0.75-0.80 | 0.0232 | -0.0647 | 0.0717 | -0.0450 | -0.0363 | 0.0035 |
| 0.80-0.85 | 0.0208 | -0.0474 | 0.0684 | -0.0368 | -0.0316 | 0.0105 |
| 0.85-0.90 | 0.0153 | -0.0267 | 0.0505 | -0.0208 | -0.0193 | 0.0119 |
| 0.90-0.95 | 0.0100 | -0.0122 | 0.0349 | -0.0105 | -0.0103 | 0.0113 |
| 0.95-0.99 | 0.0016 | -0.0011 | 0.0063 | -0.0010 | -0.0010 | 0.0026 |

(a) Average Deviation

| FR | B&J | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|---|
| 0.50-0.55 | 0.0279 | 0.1042 | 0.0639 | 0.0969 | 0.0501 | 0.0417 |
| 0.55-0.60 | 0.0316 | 0.1004 | 0.0708 | 0.1022 | 0.0644 | 0.0379 |
| 0.60-0.65 | 0.0289 | 0.0883 | 0.0709 | 0.0890 | 0.0771 | 0.0328 |
| 0.65-0.70 | 0.0309 | 0.0753 | 0.0658 | 0.0783 | 0.0669 | 0.0258 |
| 0.70-0.75 | 0.0280 | 0.0621 | 0.0592 | 0.0651 | 0.0581 | 0.0211 |
| 0.75-0.80 | 0.0200 | 0.0437 | 0.0523 | 0.0463 | 0.0446 | 0.0141 |
| 0.80-0.85 | 0.0192 | 0.0333 | 0.0423 | 0.0347 | 0.0319 | 0.0120 |
| 0.85-0.90 | 0.0127 | 0.0181 | 0.0309 | 0.0189 | 0.0187 | 0.0090 |
| 0.90-0.95 | 0.0086 | 0.0089 | 0.0196 | 0.0093 | 0.0092 | 0.0060 |
| 0.95-0.99 | 0.0028 | 0.0021 | 0.0085 | 0.0020 | 0.0020 | 0.0032 |

(b) Standard Deviation

| FR | B&J | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|---|
| 0.50-0.55 | 0.1334 | -0.0076 | 0.2618 | 0.0000 | 0.1437 | 0.0378 |
| 0.55-0.60 | 0.1607 | -0.0074 | 0.2605 | 0.0000 | 0.1434 | 0.0437 |
| 0.60-0.65 | 0.1311 | -0.0053 | 0.2966 | 0.0000 | 0.1361 | 0.0396 |
| 0.65-0.70 | 0.1641 | -0.0066 | 0.2700 | 0.0000 | 0.0996 | 0.0415 |
| 0.70-0.75 | 0.1387 | -0.0049 | 0.2555 | 0.0000 | 0.0379 | 0.0434 |
| 0.75-0.80 | 0.0982 | -0.0025 | 0.2019 | 0.0000 | 0.0050 | 0.0416 |
| 0.80-0.85 | 0.0933 | -0.0007 | 0.1756 | 0.0000 | 0.0000 | 0.0429 |
| 0.85-0.90 | 0.0727 | -0.0006 | 0.1256 | 0.0000 | 0.0000 | 0.0329 |
| 0.90-0.95 | 0.0411 | -0.0001 | 0.0883 | 0.0000 | 0.0000 | 0.0252 |
| 0.95-0.99 | 0.0220 | 0.0000 | 0.0434 | 0.0000 | 0.0000 | 0.0157 |

(c) Maximum Deviation

| FR | B&J | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|---|
| 0.50-0.55 | 0.0000 | -0.4221 | 0.0000 | -0.3946 | -0.2257 | -0.1766 |
| 0.55-0.60 | 0.0000 | -0.3903 | 0.0003 | -0.3885 | -0.2885 | -0.1664 |
| 0.60-0.65 | 0.0000 | -0.3598 | 0.0031 | -0.3542 | -0.3541 | -0.1223 |
| 0.65-0.70 | 0.0000 | -0.3231 | 0.0009 | -0.3231 | -0.3230 | -0.0851 |
| 0.70-0.75 | 0.0000 | -0.2524 | 0.0028 | -0.2524 | -0.2523 | -0.0848 |
| 0.75-0.80 | 0.0000 | -0.1849 | 0.0050 | -0.1849 | -0.1849 | -0.0266 |
| 0.80-0.85 | 0.0000 | -0.1453 | 0.0063 | -0.1395 | -0.1299 | -0.0131 |
| 0.85-0.90 | 0.0000 | -0.0884 | 0.0064 | -0.0883 | -0.0880 | -0.0061 |
| 0.90-0.95 | 0.0000 | -0.0417 | 0.0036 | -0.0417 | -0.0417 | -0.0012 |
| 0.95-0.99 | 0.0000 | -0.0144 | 0.0000 | -0.0144 | -0.0144 | 0.0000 |

(d) Minimum Deviation

Table 2. Deviations of the estimation methods

## 5. Conclusions

The analysis of the experimental data, assuming that a deviation of about 0.01 is acceptable, shows the following results:

1. *B&J method* present a very good overall performance and can be used when *FR>0.90* but probably this method requires the highest computational effort.
2. *M1 method* always underestimates the fill rate and it can be used when *FR>0.95*.
3. *M2 method* always overestimates the fill rate and it can also be used when *FR>0.95*.
4. *M3 method* behaves like *M1 method* but improves its performance so that it can be used when *FR>0.90*.
5. *M4 method* underestimates but also overestimates the fill rate, but it can also be used when *FR>0.90*.
6. *M5 method* underestimates and overestimates the fill rate, but its accuracy is the best so that it can be used even when *FR>0.65*.

Therefore, *M5 method* outperforms the other alternative methods because of its low average deviation, low standard deviation and ease of calculation. In fact this research shows that fill rate can be estimated with high accuracy using a simple method instead of applying the complex calculations needed for the exact method.

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